Place-valued Logics

around

Cybernetic Ontology, the BCL and AFOSR

Rudolf Kaehr

ThinkArt Lab Glasgow 2006

Place-valued logics around Cybernetic Ontology, the BCL and AFOSR

DERRIDA'S MACHINES PART III

BYTES & PIECES

of

PolyLogics, m-Lambda Calculi, ConTeXtures

Place-valued logics around Cybernetic Ontology, the BCL and AFOSR

© by Rudolf Kaehr

ThinkArt Lab Glasgow Hallowe'en 2006

"Interactivity is all there is to write about:

it is the paradox and

the horizon of realization."

Place-valued logics around Cybernetic Ontology, the BCL and AFOSR

In Honour of Rowena Swanson

"Harold's [Wooster] assistant Rowena Swanson become a real staunch supporter of Doug's [Doug Engelbart] work and when Harold would put Doug's proposal into a pile of proposals he was not in favor of - Rowena would come into the office after he'd gone home and move Doug's proposal into the pile of projects Harold favored. This was how touch-&-go it was. Thanks to Harold & Rowena he spent the next two years formulating a conceptual framework for his pursuit which he published in his seminal work, "*Augmenting Human Intellect: A Conceptual Framework"*, which is still the bible for using computers to augment our capability to solve problems together." http://www.invisiblerevolution.net/episodes.html

"Na, jedenfalls hat sich eine Frau gekümmert, die im *Air Force Office of Scientific Research* gearbeitet hat, [...]. Rowena Swanson, die hat einen Sinn für interessante und etwas *oddy* Leute gehabt, und hat meine Arbeit unterstützt. - Und die hat sich in den Gotthard Günther verliebt, also ich meine nicht erotisch, sondern: *Gott sei Dank, da ist einer, der redet solche Sachen, die kein Mensch versteht!*, also so etwa." http://www.vordenker.de/hvf/kl_gg_hvf_interview.pdf

"Die mehrwertigen Kalküls sind also nichts anderes als eine sinngemässe Übertragung des uns aus der Arithmetik längst geläufigen Begriffs des Stellenwertes auf das Gebiet der reinen Logik." Gotthard Günther, Die aristotelische Logik des Seins, 1959

1 General remarks on Gunther's place-value logics

Gunther was dealing with many-valued logics in the 50s. He produced some new philosophical interpretations for many-valuedness (cf. critics by Turquette). As Gunther himself mentions in a footnote, Campell jr. has given him a hint to understand many-valued logical functions as place-valued systems. This idea of a distribution of two-valued logics, performing a many-valued logic is focussing more the local aspects of many-valuedness in contrast to the more global interpretations of the single logical values considered before. Gunther's place-value logics can be understood as a compromise between Lukasievicz' concept and Emil Post's "product-logical" conception and technique of many-valuedness.

Gunther's place-value logics are not product logics but logical systems of distributed and mediated two-valued logics. Place-value systems had been developed by Gunther and Gunther/von Foerster at the BCL as truth-functional systems. That is, their semantics is based on mathematical functions over the set of values. The problem to solve was to find a correspondence between the functional concepts, that is, the total functions over the value sets and the single, local, distributed logical truth-functions (decomposition problem). The second problem to solve was to find a minimal set of local functions as a base system to construct by mediation all the total function of the many-valued logics, that is the $m^{m/n}$ functions. This problem was solved by the idea of local negations and by the morphograms as value-independent patterns of possible valuations. The result is that a set of 15 morphograms is necessary and sufficient to construct all logical truth-value functions of the total function, therefore it was called a *quindecimal place-valued logical system.* In concreto, this reduction of the value-sequences to a combination of

15 morphograms had to be adjusted with a lot of interpretations, functionally based on negations.

This approach worked fine for general many-valuedness, but was reduced to 2 and only 2 variables, that is, the concept worked only for unary and binary many-valued logical functions. Combinatorial work to this topics had been done by Na (1964), reconstructed by Mahler / Kaehr (1992) and some special possible solutions had been proposed. But the problem, never mentioned by Gunther or von Foerster et al., remained unsolved. Surely, a logic with 2 and only 2 variables is hardly a full fledged logic. This amazing situation has never disturbed the reception / exploitation of so called “polycontexturality” by sociologists and their adepts.

The failure of solving the decomposition problem (Krantz products?) for more than 2 variables is not a surprising situation. I don’t know of any solution to this problem from the side of so called trichotomic-triadic conceptualizations, say semiotics a la Pierce / Bense or Category Theory. They simply don’t mention it.

Furthermore, the mechanism of global and local viewpoints has never been formalized properly. The logical formalism was global, based on total functions, the understanding and interpretations of the results of transformations was local but descriptive, reflecting the underlying morphogrammatic structure of the global functions. Proof-theoretical considerations and a clear concept of semantics, esp. of semantic contradictions / refutations had been lacking, too.

1.1 The Information Sciences Directorate of the AFOSR

Ernst von Glasersfeld: Why I Consider Myself a Cybernetician

"In those days, the Information Sciences Directorate, a division of the U.S.Air Force Office of Scientific Research, was sponsoring research in many different areas, some of which, like computational linguistics, had only the vaguest connection with military objectives. The Directorate was run by Harold Wooster and Rowena Swanson, two outstanding individuals who were in many ways the opposite of what you have come to expect of administrators, let alone military administrators. They were both highly imaginative, widely read and cultivated, and enthusiastically open to new and controversial ideas."

Cybernetics & Human Knowing

A Journal of Second Order Cybernetics & Cyber-Semiotics, Vol. 1 no. 1 1992

http: / / www.imprint.co.uk / C&HK / vol1 / v1-1evg.htm

"Dass mir die notwendige Zeit für die Erweiterung des ursprünglichen Textes zur Verfügung stand, habe ich im wentlischen Dr. Harold Wooster und Mrs. Rowena Swanson im USAF-Office of Scientific Research und der unübertroffenen Grosszügigkeit zu danken, mit der sie meine Verpflichtungen interpretierten." Gotthard Günther, Vorwort zur 2. Auflage von *"Das Bewusstsein der Maschinen"* 1963.

http: / / www.kybernetiknet.de / ausgabe4 / pdf / GG_Vorwort_Bewusstsein.pdf

Swanson, Rowena. (1967). Information System Networks – Let’s Profit from What we Know. In George Schecter (Ed.), Information Retrieval. Washington D.C.: Thompson Book Company.

http: / / faculty.ivytech.edu / ~wmitchel / NASARECON.htm

At the same time as Gunther wanted to go beyond computation, modern computing was f(o)unded.

"Thanks to Harold & Rowena he spent the next two years formulating a conceptual framework for his pursuit which he published in his seminal work, "*Augmenting Human Intellect: A Conceptual Framework"*, which is still the bible for using computers to augment our capability to solve problems together."

http: / / www.invisiblerevolution.net / episodes.html

1.2 A revolution in logic?

Forget about the tedious problems of combinatorial analysis of place-valued logics and all the ambitious philosophical interpretations. What is the real impact of Gunther’s approach? And why is it so difficult to accept? What is the craziness that Rowena Swanson was so much intrigued?

It is very difficult to understand Gunther’s work because of its endless amalgamations and fusions with other scientific trends, like many-valuedness, dialectics, second-order-cybernetics, metaphysics of information, theory of living systems, cognitive science, deviant logics, triadic semiotics, trans-humanism, SF, paradigm change, system theory.

Gunther’s conceptual approach of *place-valued logics* is easy to understand, but nearly impossible to be accepted by mathematicians, philosophers and logicians.

In a subversive step of arithmetizing logic and logifying arithmetic, Gunther revolutionized the old Chinese / Indian concept of Zero and positionality to a mechanism of distribution and mediation of logical systems and of formal systems as such.

As we know, without the positionality system and its cipher Zero the whole Western science, technology and business wouldn’t exist. On the other hand, without Western alphabetism (atomism, linearity, ideality) the modern positional numeration system couldn’t have such a historic impact on technology and society in general.

> "Therefore, albeit the Hindus perfected one of the greatest discoveries in human history -- the zero, they could not realize its cosmic function as a mathematical tool of science."

Gunther’s approach is unseen subversiveness! Never happened in the last 5000 years. A concept, valuable inside a theory, i.e., in arithmetic is used / abused to place full logical theories in a distribution instead of numerals in a positional arithmetic. The part is treated as a whole, reversed, and moved from the arithmetic to the logical sphere.

Linear positionality

gegenseitiger Abhängigkeit sich befinden. *Eine mehrwertige Logik beschreibt ein solches Abhängigkeitssystem der möglichen Stellenwerte, die die klassische Logik in dem Reflexionssystem unseres Bewußtseins einnehmen kann.*

Das soll an dem einfachen Beispiel des binarischen Zahlensystems erläutert werden. Die einzige dabei gebrauchte positive Ziffer „1" hat eine doppelte Bedeutung. Erstens als Einheit und zweitens als Quantität, je nach ihrem Stellenwert. Wenn wir also schreiben:

0	=	0	110	=	6
1	=	1	111	=	7
10	=	2	1 000	=	8
11	=	3	1 001	=	9
100	=	4	1 010	=	10
101	=	5	1 011	=	11

usw.

so ist es immer dieselbe identische „1", die sich in verschiedenen Stellen mit verschiedenen (quantitativen) Bedeutungen wiederholt. Eine „1" an der ersten Stelle bedeutet 1; an der vierten Stelle aber bedeutet dieselbe Ziffer 8.

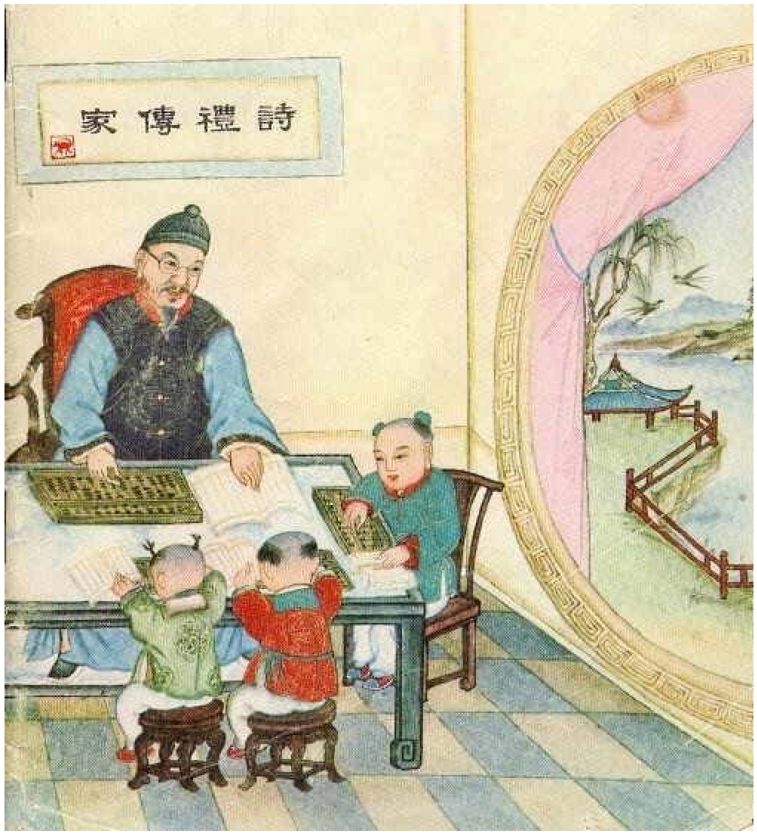
Eine mehrwertige Logik ist nun nichts anderes als ein System, das uns erlaubt, unserer einzigen, „wirklichen" Logik verschiedene Stellenwerte im System des Bewußtseins derart zu geben, daß jeder Stellenwert mit einer verschiedenen semantischen Bedeutung des sich so wiederholenden zweiwertigen Kalküls verbunden ist. Ein solches mehrwertiges System erlaubt dann den strukturellen *Zusammenhang* der verschiedenen zweiwertigen Erlebnisstufen des Bewußtseins abzulesen.

In his paper *"Die Aristotelische Logik des Seins und die nicht-Aritotelische Logik der Reflexion"1959*, Gunther has given an exposition of the results of his research about a logic of reflection in such a concise and clear way that it is nearly impossible to not to understand his approach.

But this exactly was the obstacle. How can we mix logic with the positional system of arithmetic?

And how can we succeed, later, from linearity to tabularity of a kenogrammatic positionality system?

The concept of zero was conceived by the Chinese then improved on by Hindus



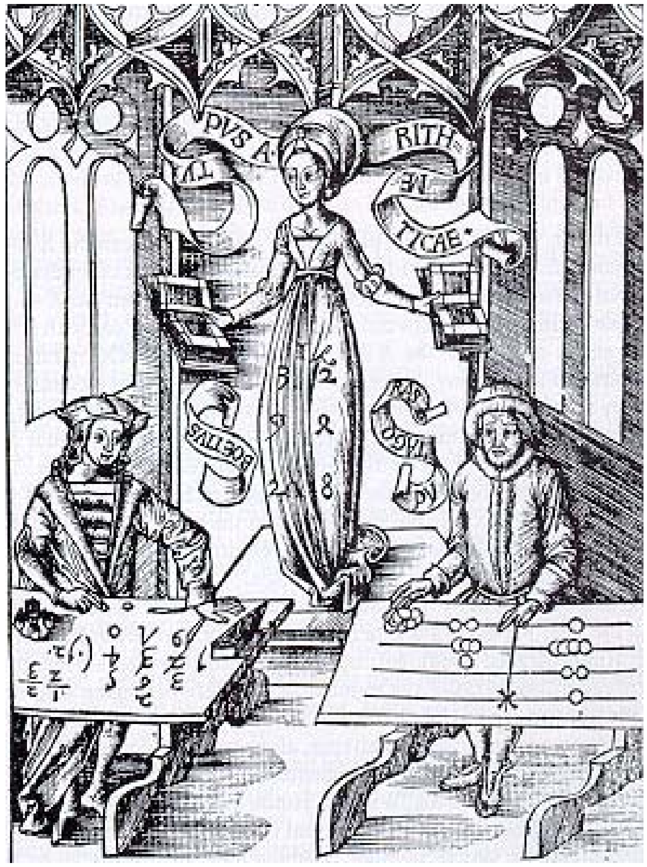
http://www.joernluetjens.de/sammlungen/abakus/abakus-en.htm

> "They [the Chinese] then invented symbols for the content of each column to replace drawing a picture of the number of beads. Having developed symbols to express the content of each column, they had to invent a symbol for the numberless content of the empty column -- that symbol came to be known to the Hindus as "sunya", and sunya later became "sifr" in Arabic; "cifra" in Roman; and finally "cipher" in English.
> Only an empty column of an abacus could possibly provide the human experience that called for the invention of the zero -- the symbol for "nothingness", and that discovery of the symbol for nothingness had an enormous significance upon subsequent humanity."
> http://www.gupshop.com

A better understanding of Gunther's approach to trans-classic logic can be found in the fact of Gunther's early studies of Sanskrit and Chinese language and philosophy. I propose that this had a much more profound influence on the "deep-structure" of his writing than anything consciously declared as Hegelian. Also an expert in German idealism, his interpretation of Hegel's Logik as a *positional* system of thought (Stellungen des Gedankens) in his dissertation 1933 was in fact a departure from traditional Western philosophy. To give time a place he needed to invent a *structural space* to place it. As a consequence, Gunther invented a series of radicalized negativity and zero. Kenograms as inscriptions of ultimate emptiness, surpassing nothingness, opened up the possibility to distribute formal systems as such, including their internal concepts of zero, nil, blank, zero-set and linearity, over a tabular kenogrammatic matrix. Positionality as introduced by the Abacus as a practical device (computer) had to be "tabularized" to enable reflectional and interactional mathematics, graphematics, of trans-computation as possible futures.

http://www.thinkartlab.com/CCR/rudys-chinese-challenge.html

Algorist versus Abacist



http://library.thinkquest.org/22584/emh1100.htm

Are we not in a similar situation today? After the decline of the paradigm of algorithmic programming a new round has to be opened with "*interactionists*".

Interactionality, reflectionality and complexity of computation managed by the impotent and chaotic methods based on linear arithmetic and bivalent logic? The un-denied success of this paradigm is based on a self-destroying exploitation of natural and human resources. Not long ago, Medieval European scientists and mathematicians had been victims of their dysfunctional methods based on a Christian refutation of the Arabic positionality system.

Keith J Devlin:

"In the twenty-first century, biology and the human sciences will become the primary driving forces for the development and application of new mathematics. So far, we have seen some applications of mathematics in these fields, some quite substantial. But that has involved old mathematics, developed for other purposes. What we have not yet seen to any great extent are new mathematics and new branches of mathematics developed specifically in response to the needs of those disciplines. In my view, that is where we will see much of the mathematical action in the coming decades. I suspect that some of that new mathematics will look quite different from most of today's mathematics. But I really don't have much idea what it will look like."

http://www.spiked-online.com/index.php?/surveys/2024_article/1310/

2 Philosophical situation of Gunther's place-valued logics

2.1 Parsons: Gödel-Gunther Correspondence

Charles Parsons

Günther argues that from this point of view one must distinguish two negations, one of which is expressed in the statement that the subject is not its object, another in the statement that the subject (as ego) is not the Thou. Günther seems to be driven toward many-valued logic by the fact that he doesn't consider an alternative to a truth- functional interpretation of propositional logic, and at least a third truth-value is needed in order to make the distinction between the two negations.

How, then, does he interpret his "truth"-values? At this point he does something that is from a logician's point of view crazy, because the values seem not to be truth-values at all. He uses the designations I, R and D, which he reads as "irreflexive", "reflexive" and "double-reflexive". In other words, they represent stages of reflection coming out of the analysis we have discussed. He even says in one place that all the values are "true" (1953, p. 48). The concepts of truth and falsity should "disappear without remainder" from the sort of logic he is constructing because they exclude a genuine third.[18] It seems that he has simply changed the subject, as a result of taking the relation of the I to what is not I as the paradigm of all negation.

Günther is, however, a somewhat slippier target, and I don't think I have grasped his thought at this point. He says that our thought is in a way necessarily two-valued. What the three-valued logic does is allow for the fact that two-valued thought can occur at different levels of reflection. How he conceives this is not at all clear to me. But he does say something about how it works in propositional logic. He singles out pairs of values and notes that one might treat that pair as truth and falsity, and certain functions might behave like, say, conjunction when just these two values are considered, perhaps behaving differently when the third value is taken into account. He saw the fact that two-valued structures can occur in different places in a three- or more-valued system as analogous to the place-value feature of Arabic or binary notation for numbers:

> *A many-valued logic is now nothing but a system that allows us to give to our single "actual" logic different place-values in the system of consciousness of such a kind that each place-value is connected with a different semantic meaning of the two-valued calculus that thus repeats itself. Such a many-valued system allows us thus to read off the structural interrelation of the different two-valued stages of consciousness.*

This remark would suggest that the two truth-values retain their status as genuine truth-values and that the values of the many-valued system have a quite different role. It is not clear how this would be reconciled with Günther's claim that his constructions constitute a genuine revision of logic.

If the "truth" values express levels of reflection, what would be the significance of being a theorem of the calculus? Such calculi are formulated in order to characterize conceptions of valid logical inference or logical truth. It's not clear that Günther's scheme has any place for these notions. That the concept of truth is not at center stage for him is indicated also by the fact that when he writes about intuitionism, although it is clear to him that in some sense the conception of existence is different from that in classical mathematics, he never remarks on the fact that there is a more underlying difference about truth. Charles Parsons

Kurt Gödel

I also miss any explanation at all of what your three truth-values actually mean in contradistinction to the "true" and "false" of classical logic. One can of course not demand an exact definition, certainly not within the framework of classical two-valued logic, but still [one can demand] an explanation in the same sense in which one can make the fundamental concepts of two-valued logic perspicuous (in spite of their undefinability).

An analysis of the sense of your truth-values seems to be the cardinal point which you should tackle in order to become comprehensible to your readers and to carry out further the construction of a logic corresponding to your ideas.

Gotthard Günther

That is, the topic of this logic is no longer Being conceived "ontologically", but the difference between Being (objectivity) and reflection, where this difference is itself conceived as a logical problem for reflection. The criterion that distinguishes a logic of Being from a logic of Reflection is the law of the excluded middle. For what is excluded in the tertium non datur is the logical possibility of reflecting once again, out-side (two-valued) logic which combines the two, upon the relation between what is objectively thought and the process of thought (mathematically: construction).

After reading your letter several times it occurred to me that my interpretation of a genuine value-triad (true-undetermined-false is not a genuine triad!) as a system of values in which the level of reflection of a concept (and not its true-false correspondence with factual data) is determined, creates difficulties for you, because you ignore my interpretation of the hermeneutic structure of a three-valued system of logic.

htttp://www.thinkartlab.com/pkl/archive/Gunther-Godel_german_english.pdf
http://www.vordenker.de/ggphilosophy/gg_briefwechsel-goedel_ger.pdf

http://www.thinkartlab.com/pkl/lola/Godel_Games/Godel_Games.htm
http://www.thinkartlab.com/pkl/lola/Godel_Games-short.pdf

2.2 Historical situations of Gunther's place-valued logics

In the air, at this time, was not programming, but the challenges of *many-valued* logics, all sorts of *antinomies* (called wrongly paradoxes, now), the *linguistic turn*, and obviously a total denial of anything dialectical. Not only as a result of the linguistic turn but much more because of the cold war. In this situation, German philosophy lost nearly everything of importance. Happily enough, German thinking came back by Japanese thinkers, by the French (post)structuralist movement, etc. But also the fact, that Gunther was involved in the development of cybernetics at the Biological Computer Laboratory (BCL) didn't help much to overcome the denial strategy towards Gunther's thoughts by the home grown victims of US hegemony and Soviet Russian dominance.

Gunther tried to mediate between USA, West-Germany and even East-Berlin. But this went wrong from the very beginning: His edition of US science fictions in 1952 with names like Campell, Asimov and Williamson had to be taken from the shelves.

The historical documents are accessible and of surprising actuality. But now, we are in another age of stupefaction (Verdummung).

Gunther, Transzendentalphilosophische Grundlagen der Kybernetik, 1965, Audio-CD, suppose 2000
http://www.vordenker.de/rk/rk_comp_meta.htm
http://de.wikipedia.org/wiki/Gotthard_Günther

Stellenwertlogik

Gunther's *Stellenwertlogik* was in some sense a German enterprise. It was the heroic attempt to formalize Hegel's theory of reflection as Gunther himself worked out of the metaphysical mist of Hegelianism in the 30s. His contributions had been two-fold: discovery of the rational logical structure of Hegel's Logic and the acceptance of the new mathematical logic movement, then called Logistik, as the adequate approach to logic. A decision which was opposed to nearly all philosophical trends at the time, except of Logical Positivism and some Thomists. At this time, the conceptual approach was dominating the scriptural work of formalization. This pre-BCL work was supported by a long-term but very small grant from the Bollingen Foundation, New York.

http://www.vordenker.de/ggphilosophy/gg_logik-sein-reflexion.pdf

Gotthard Günther, Selbstdarstellung im Spiegel Amerikas. In: L.J. Pongratz (Hrsg.), Philosophie in Selbstdarstellungen Bd. II, Meiner: Hamburg 1975, 1-76

2.2.1 Problems with the Fremdwerte. The situation before the BCL

Reflexionsüberschuss, *Reflexionsmuster*, *Fremdwerte (noise)*, *Quindezimalität*.
Hints from German friends and reports to Kurt Gödel.
Günther to Gödel, from letters not included in Parsons' Correspondence.

Januar 15. 1960
In der Anlage erlaube ich mir Ihnen das Manuskript einer kleinen Arbeit zu senden, die ich kürzlich fertig gestellt habe. Als ich vor etwa zwei Jahren meine Stellenwerttheorie veröffentlichte, erhielt ich von mehreren deutschen Kollegen Briefe, in denen ich darauf hingewiesen wurde, dass diese meine Theorie schon Wertserien wie z.B. die folgende nicht einbezöge: [*is missing, but must inclued sequences with more than 2 values, r.k.*]
Mein heutiger Artikel ist wenigstes eine teilweise Antwort darauf. Die volle Theorie der "Fremdwerte" wird im Detail im zweiten Band meiner nicht-Aristotelischen Logik abgehandelt werden.

Jan. 18. 1958
In der Arbeit *"Die Aristotelische Logik des Seins und die nicht-Aristotelische Logik der Reflexion"* benutze ich für Konjunktion und Disjunktion nur Wertfolgen von dem Typ: 1222, 2333, 1333 oder 1112 usw. D.h. immer nur Wertfolgen, die ausschließlich 2 verschiedene Werte enthalten. Das Verfahren ist richtig, sobald ein bestimmter Teilaspekt der dreiwertigen Logik in Frage kommt. Es versagt aber, wenn man sich die Frage nach dem logischen Sinn des Übergangs von der Dreiwertigkeit zur generellen Mehrwertigkeit stellt. Jetzt erhalten, wie ich nun weiß, solche "konjunktiven" und "disjunktiven" Wertserien wie 1223 oder 2133 oder 1233, resp. 1123 oder 1312 einen bestimmten logischen Sinn.

30. XII. 1960
Ich hatte schon seit dem Spätsommer die Absicht, mich wieder einmal zu melden, aber die Umstände schoben diese gute Absicht immer wieder hinaus. Ich glaube es ist mir im Sommer eine Entdeckung von erheblicher Tragweite geglückt. Dieselbe wirkt sich in einer Generalisierung meines Stellenwertsystems aus.
Dies Generalisierung beruht auf der Feststellung, dass in allen mehrwertigen Systemen nur die beschränkte Anzahl von 15 strukturelle voneinander verschiedenen vierstelligen Wertfolgen auftreten kann. Ich interpretiere unter diesen Umständen in meiner generalisierten Theorie die mehrwertigen Systeme nicht mehr als ein Stellenwertsystem der klassischen aussagenlogischen Konstanten sondern eben als eine Ordnung dieser invarianten Strukturen, die übrigens als Sub-System die so genannten klassischen Wahrheitsfunktionen, wie Konjunktion, Disjunktion, Implikation, usw. enthalten. http://www.thinkartlab.com/pkl/archive/GUNTHER-GODEL/GUNTHER-GODEL.htm

This letter hints to a further step in the development of a trans-classic logic. After Gunther included the "Fremdwerte" and discovered the "quindecimal" structure of his "Reflexionsmotive" he understands his logic now not so much as a place-valued system of classic logic but as an order-system of the 15 invariant basic patterns where the value-topic moved into the background and lost its dominance.

2.2.2 Bollingen foundation, ski patroller, late job seeker

Before Gunther joined the BCL at the age of 61, a step which was initiated by Warren McCulloch, he has given lectures about his newly discovered "*Generalis-erte Stellenwerttheorie*", the paper mentioned in the letter (15. Jan 1960) to Gödel, at different scientific institutions:

– RAND Corporation, Santa Monica, California
– Illinois Institute of Technology, Chicago
– University of Illinois, Urbana

Gunther published his results, about a generalized place-valued system, also for priority reasons – "aus Prioritäts- und anderen Gründen"–, quickly in the German avant-garde magazine of Cybernetics *"Grundlagenstudien aus Kybernetik und Geisteswissenschaft"* in 1960.

Ein Vorbericht über die generalisierte Stellenwerttheorie der mehrwertigen Logik
Grundlagenstudien aus Kybernetik und Geisteswissenschaft, 1960, Bd. 1, p. 99-101

Not worth mentioning, that nobody understood it or was interested at all. Not by the German cyberneticians, nor the Americans, maybe because it was written in German. At least, this discovery was crucial to him and to the development of Second-Order Cybernetics: he got the job Warren wanted for him.

Gunther's job situation before the BCL

Gunther was very much involved in skiing, professionally and private.

From a CV, free translation, r.k.

"After about 8 years of work as a Research Fellow at the Bollingen Foundation I was invited in the summer term 1961 to the State University of Chicago, Urbana, Illinois. My ternure expires at the 1. September 1968. A prolongation for 1969 had been permitted."

His grant at Bolllingen Foundation was extremely small, according to Heinz von Foerster, it was about USD 2000.- per anno. Thus, he had to combine his passion of skiing with a butter and bread job.

The salary at the BCL was about USD 21000 plus travel expenses, etc.

Invitation to the BCL

Heinz von Foerster about the story of the invitation of Gunther to the BCL, (Warren McCulloch and Rowena Swanson).

> Heinz von Foerster: "Na jedenfalls, der *McCulloch* war mein Schutzpatron und eines Tages – nein, nein: eines Nachts, er hat nie am Tag angerufen! -, eines Nachts um zwei ruft er mich an in Illinois – wann war das wohl? das muß so um '60 herum gewesen sein, '59 oder '61, das weiß ich jetzt nicht mehr genau – er ruft mich an, sagt: Heinz, ich bin hier in Richmond, Georgia, ich hab hier einen Mann getroffen, den kein Mensch versteht, außer du, und warum ladest du den nicht ein? Und da ich hab gesagt: Ja, was ist das für ein Mensch? - Ja, ein Logiker, der heißt Gotthard Günther, und lad' ihn doch ein zu deinem Seminar, oder was immer du hast! - Naja, wenn der Warren McCulloch mir sagt, ich soll ihn einladen, lad' ich diesen Gotthard Günther ein! Und wie der Warren schon angekündigt hat, den versteht kein Mensch – aber ich, Heinz, werde ihn wohl verstehen. So kam der Gotthard Günther."
> […]
> "McCulloch hat einen Vortrag gehalten, wie er mir erzählt hat: Dann am Ende des Vortrags kam so ein Mensch mit Brille und so komisch verzwickt, und der hat mich die besten Sachen gefragt, die ich je gefragt worden bin. Und da hab ich gefragt: 'Was machen Sie?' - Da hat er gesagt: 'Ich mach etwas Licht ..' - und da hab ich dich angerufen, Heinz, mach was für den Menschen!"
> […]
> "Und den guten Gotthard Günther einer Stiftung zu verkaufen mit seiner Idee war nicht leicht! Na, jedenfalls hat sich eine Frau gekümmert, die im *Air Force Office of Scientific Research* gearbeitet hat, eine *Rowena Swanson*, die uns auch geholfen hat – den Ernst von Glasersfeld zum Beispiel hat die Rowena Swanson unterstützt, weil die auch etwas unorthodoxe Arbeit geleistet hat, nicht im Chomskyschen Sinn, sondern ganz praktisch. Rowena Swanson, die hat einen Sinn für interessante und etwas *oddy* Leute gehabt, und hat meine Arbeit unterstützt. - Und die hat sich in den Gotthard Günther verliebt, also ich meine nicht erotisch, sondern: *Gott sei Dank, da ist einer, der redet solche Sachen, die kein Mensch versteht!*, also so etwa. Und, nachdem sie selber jüdisch war und das Problem kannte, nicht wahr, von Gotthard in Deutschland undsoweiter, war sie ein eiserner supporter vom Gotthard Günther."
> http://www.vordenker.de/hvf/kl_gg_hvf_interview.pdf

To learn more about the friendship of Gunther and McCulloch read *"Number and Logos"*. http://www.vordenker.de/numlog/numlog1.htm

It may be intriguing to read on Gunther's texts after 1965, still mark the "sponsored by the *Air Force Office of Scientific Research"* acknowledgment. His last proposal was about: *Decision Making Machines*, 1970 July 31. Obviously, Rowena found an exemplarily solution out of the retirement disasters.

> *„On the other hand, a machine, capable of genuine decision-making, would be a system gifted with the power of self-generation of choices, and the acting in a decisional manner upon its self-created alternatives. (…) A machine which has such a capacity could either accept or reject the total conceptual range within which a given input is logically and mathematically located."* Günther, Decision Making Machines, 1970

BCL, The Complete Publication of the Biological Computer Laboratory, (eds. Wilson, von Foerster), Illinois Blueprint Corp., Peoria, Ill 61603, 1976

2.2.3 Gunther's Soviet Studies

I always was wondering how Gunther was involved in studies of the development of cybernetics in the Soviet Union while at the BCL. Rowena Swanson was much involved in information retrieval and the use of computer technology. Obviously, Rowena was studying this topic of cybernetics in the USSR herself.

• Swanson, Rowena W. Cybernetics in Europe and the U.S.S.R.: Activities, Plans, and Impressions, AFOSR 66-0579, 1966 March. NBS# 6624483.

Some of Gunther's contributions:

– Idealismus, Materialismus und Kybernetik, in: Das Bewusstsein der Maschien, pp. 89-166, 1963
– Kybernetik und Dialektik – der Materialismus von Marx und Lenin, 1964
http://www.vordenker.de/ggphilosophy/gg_vortrag_koeln.pdf
– Cybernetics and the Dialectical Materialism of Marx and Lenin, 1964, Cologne
http://www.vordenker.de/ggphilosophy/gg_lecture-koeln.pdf
– A Study of the Development in Dialectic Theory in Marxist Countries and their Significance for the USA, 13pp., 1970 (proposal)
Later: http://www.vordenker.de/ggphilosophy/gg_maschine-seele-weltgeschichte.pdf
http://userpage.fu-berlin.de/~gerbrehm/gg_vortrag1.pdf

In this context Gunther was several times invited by the Academy of Science in Berlin, former GDR, for lectures about the development of Cybernetics in the USA. But his lectures had been mainly about a new understanding of dialectical materialism istead of cybernetics. In parallel, I invited him for lectures to the Free University of West-Berlin about the theory of polycontexturality.
Gunther has written a profound oeuvre on philosophy of history before and after his involvement with the BCL.
Before, he has written the opus:
Die Amerikanische Apokalypse. Ideen zu einer Geschichtsmetaphysik der westlichen Hemisphäre. Kurt Klagenfurt (Ed.), Profil 2002
After, e.g., *Maschine, Seele und Weltgeschichte*, 1980

Written during the Cold War, Gunther came up with shocking thesis that dialectical materialism is superior to Western idealism, and that Cybernetics should turn to materialist dialectics, and dialectics should learn from polycontextural logic.

"At this time - I invited Günther to the Free University of West Berlin and accompanied him to his lectures at the Academy of Science - we had a crucial point in common: both of us had to pass the mysterious Checkpoint Charley; now part of a museum. By passing this place of technological secrets Gotthard told me that he is a *"Hyäne des Pentagon"* (or that the other side told him this). I didn´t really understand, probably because I was hearing something sounding more like Princeton then Pentagon."
Rudolf Kaehr, Computation and Metaphysics
in: ARIFMOMETR, An Archaeology of Computing in Russia
Georg Trogemann, Alexander Nitussov, Wolfgang Ernst (Eds.), Vieweg 2001
http://www.khm.de/~alexandern/
http://www.vordenker.de/rk/rk_comp_meta.htm

2.2.4 Fremdwerte as logical Analogwerte

Before digitalism has overtaken technology and ideology, thinkers like John von Neumannn had been aware that this wouldn’t be the way they would like to live and think.

In respect to neurobiology von Neumann writes:

"Heute wissen wir, dass im neuronalen System sich abspielende Vorgänge mit beliebiger Häufigkeit von digitaler *zu* analoger *Struktur und von der letzteren zurück zur ersteren wechseln können."*

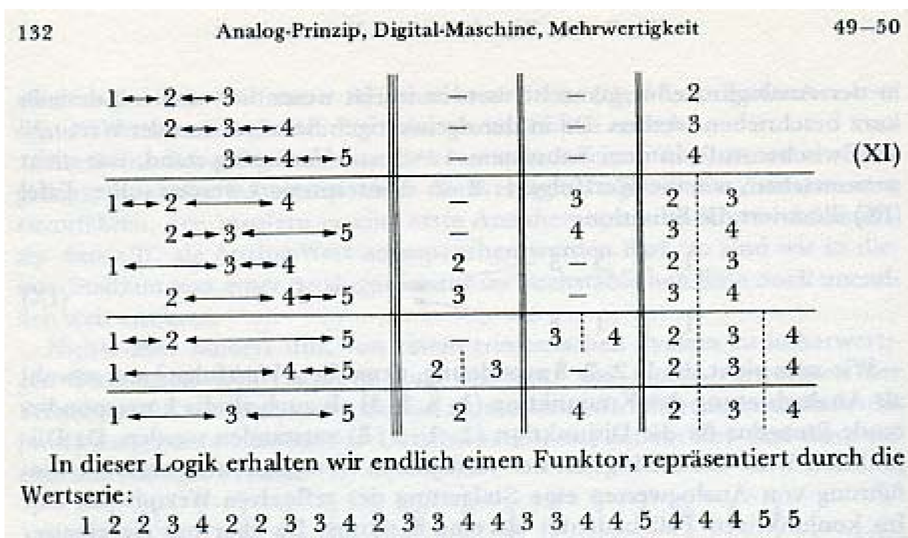
And in respect to automata theory:

"We are very far from possessing a theory of automata which deserves that name… Everybody who has worked in formal logic will confirm that it is one of the technically most refractory parts of mathematics. The reason for this is that it deals with rigid, all-or-none concepts, …"

Unnecessary to tell von Neumann something about multi-valued logics, neurocybernetics, cellular automata, etc. he was one of the fathers of all that, today, trendy stuff.

Gunther was much involved into such discussions of a mediation of digital and analog conceptual and technical structures. Hence, it is no surprise he tried to give an answer in the framework of his place-valued logic. The Fremdwerte, which didn’t have much domestication before found a job to mediate, logically, between digital structures, playing the part of analogy-makers. The hope was to deliver a theory of "Analog-structures in multi-valued Digital-systems".

But even the Fremdwerte didn’t have a well defined status of "strangeness". Next to the Fremdwerte of different degrees of strangeness another kind of Fremdwerte was allowed in the system. There had been suddenly "Pseudo-Fremdwerte" which had to be separated from the real Fremdwerte of all degrees. A complicated administration of real and unreal asylum seekers was the result.



132 Analog-Prinzip, Digital-Maschine, Mehrwertigkeit 49–50

1↔2↔3	—	—	2	
2↔3↔4	—	—	3	
3↔4↔5	—	—	4	(XI)
1↔2↔4	—	3	2 ⋮ 3	
2↔3↔5	—	4	3 ⋮ 4	
1↔3↔4	2	—	2 ⋮ 3	
2↔4↔5	3	—	3 ⋮ 4	
1↔2↔5	—	3 ⋮ 4	2 ⋮ 3 ⋮ 4	
1↔4↔5	2 ⋮ 3	—	2 ⋮ 3 ⋮ 4	
1↔3↔5	2	4	2 ⋮ 3 ⋮ 4	

In dieser Logik erhalten wir endlich einen Funktor, repräsentiert durch die Wertserie:

1 2 2 3 4 2 2 3 3 4 2 3 3 4 4 3 3 4 4 5 4 4 4 5 5

Again, the logical Fremdwerte are in no sense something like intermediary values in the sense of classical multi-valued logics. They are genuine logical values, but behave as intruders from other logical systems. Direct neighbors or collateral relatives, kith and kin or simply trouble-makers (Fremdwerte as noise).

Again, on the importance of the Fremdwerte:

"Unsere bisherige Deutung des logischen Phänomens der Mehrwertigkeit muss also durch eine zweite Stufe unserer Theory ergänzt werden, die sich mit einer spezifischen Eigenschaft generell n-wertiger Systeme befasst, einer Eigenschaft, die wir hier als das Auftreten von *Fremdwerten* in den zwei-wertigen Sub-Systemen der mehrwertigen Strukturen charakterisieren wollen." Analog-Prinzip, p. 45
Analog-Prinzip, Digital-Maschine und Mehrwertigkeit.
Grundlagenstudien aus Kybernetik und Geisteswissenschaft, 1960, Bd. 1, p. 41-50.
(not yet digitalized)

It will turn out, much later, that the Fremdwerte are not only strangers but the indications of interactionality between different logical systems. What was missing in Gunther's theory of Reflexionsstufen, levels of reflexion, was the disturbing fact, that distribution is enabling interaction. There seemed to be a conflict between the concept of distributed rationality, distributed over different centers of reflection and the theory of logical reflection at a single place of distribution. Distributed rationality, today multi-agent systems, is naturally asking, in addition to reflectionality, for interaction between agents.

To resume the complex interpretational chain of many-valuedness:
– Reflexions*ü*berschuss
– Fremdwerte
– Transjunction with rejection values (noise)

"Die Transjunktion entspricht generell jenem metaphysischen Tatbestand, den wir in früheren Veröffentlichungen als "Reflexionsüberschuss" bezeichnet haben". Günther 1962

Place-valued logics of classical Multi-valued Logics

$X \overset{3}{\rightarrow}_{Ł_3} Y$	T	a	F	b	F
T	T	a	F	b	F
a	T	T	a	b	b
F	T	T	T	F	F
b	T	T	T	T	b
F	T	T	T	T	T

b) Additives Beispiel :

$\overset{1}{\vee}_{Ł_3}$	T	a	F
T	T	T	T
a	T	a	a
F	T	a	F

$\overset{2}{\vee}_{Ł_3}$	F	b	F
F	F	F	F
b	F	b	b
F	F	b	F

$\overset{3}{\vee}_{Ł_3}$	T	a	b	F
T	T	T	T	T
a	T	a	a	a
b	T	a	b	b
F	T	a	b	F

$\vee^3_{Ł_3}$	T	a	F	b	F
T	T	T	T	T	T
a	T	a	a	a	a
F	T	a	F	F	F
b	T	a	F	b	b
F	T	a	F	b	F

Das Entsprechende gilt für die additive Vermittlung der Vollkunjunktion

$$p \overset{3}{\wedge}_{Ł_3} q$$

These tables are not only giving an example of a mediation of classical multi-valued logics in a place-valued logical system but are also demonstrating quite clear the difference between such an attempt and the mediation of analog and digital structures in a place-valued logic with Fremdwerte mentioned before.

2.2.5 Place-valued logics as logics of distributed rationality

We are used to think that reflection is a subjective matter happening inside the mind of a singular ego. Levels of reflection or meditation, up and down, intra-subjectively.

But with that we have forgotten that one of the major revolution in Gunther's thinking was to break with such a homogeneity of subjectivity and rationality. Subjectivity in philosophical tradition always was ego-centred. Gunther introduced the concept of subjectivity as distributed between I-subjectivity and Thou-subjectivity.

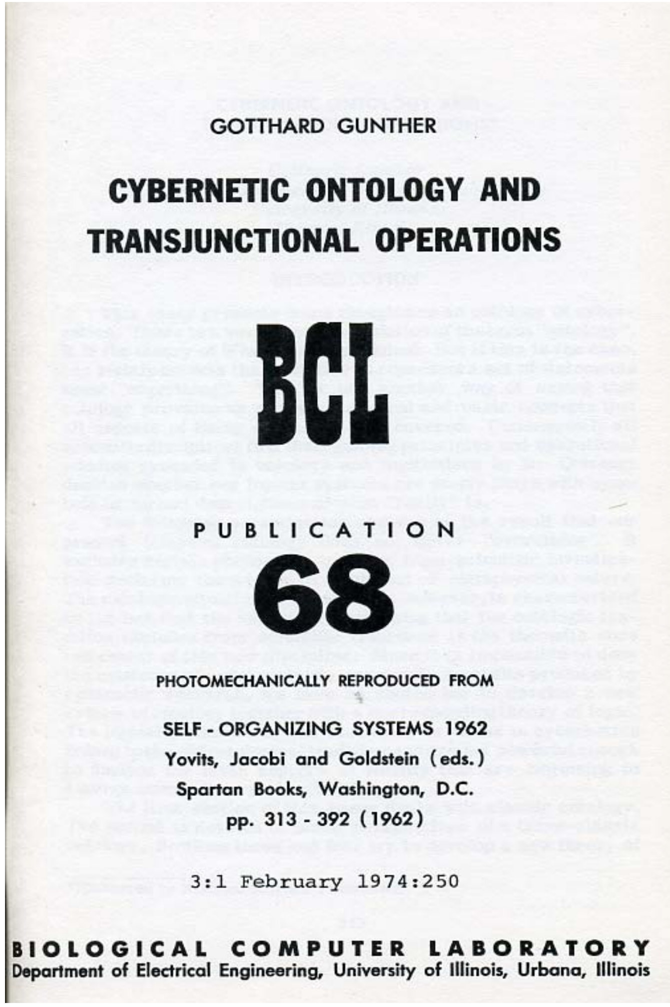
"A non-Aristotelian or trans-classical logic is a system of distributed rationality." Such a sentence is beyond any kind of egological idealism. It is a clear statement of a dialectical materialist conception of rationality. The logic of masses, conceived as a mass of autonomous subject-centres. *"In any m-valued logic our classical system is distributed over places."* Such places are not placed inside a subjective mind but in the world.

"A non-Aristotelian logic, however, takes into account the fact that subjectivity is ontologically distributed over a plurality of subject-centers. And since each of them is entitled to be the subject of logic human rationality must also be represented in a distributed form."

As a summary I mention some citations from 1962:

> Since 1953 this author has tried to make a start in this direction with a series of publications all of which attempt to deal with the proposition that the so far uncontested classic definition of logic should be abandoned in favor of a broader one. As philosophical maxims for this new transclassic logic we suggest:
> the dichotomy of form and matter does not hold in n-valued systems where n>2.
> the concept of 'object' is amphibolic[8] when n>2.
> the disjunction truth/falsity applies as value designation if and only if n=2.
>
> In the first volume of his *"Idee und Grundriss einer nicht-Aristotelischen Logik"* (1959) this author has endeavored to outline the historic antecedents and to develop – on a purely philosophic basis – the systematic concept of a field of genuine transclassic rationality.
>
> There are abundant historic antedecents in Kant (his Transzendentale Dialektik) Fichte, Hegel and Schelling, and since they all converge in that enigmatic product which Hegel calls "Logik" it seemed advisable to concentrate on him. However, that should not be construed as an attempt to vindicate the "spekulative Logik" in the eyes of modern symbolic Logic or even to amalgamate the two. This is clearly impossible. On the other hand: there can be no doubt that the Deutsche Idealismus has discovered a new systematic problem for Logic! It is the phenomenon of self-reflection. Kant, Fichte, Hegel and Schelling have stoutly maintained that this phenomenon, although "logical", is not capable of formalization.
>
> It is the main thesis of *"Idee und Grundriss..."* that the datum of self-reflection (consciousness) is fully amenable to formalization.
>
> To sum it up: A non-Aristotelian or trans-classical logic is a system of distributed rationality.
> Our traditional (two-valued) logic presents human rationality in a non-distributed form. This means: the tradition recognizes only one single universal subject as the carrier of logical operations. A non-Aristotelian logic, however, takes into account the fact that subjectivity is ontologically distributed over a plurality of subject-centers. And since each of them is entitled to be the subject of logic human rationality must also be represented in a distributed form. The means to do this is to interpret many-valued structures as place-value systems of our two-valued logic. In any m-valued logic our classical system is distributed over places.
>
> Gotthard Gunther, *The Tradition of Logic and the Concept of a Trans-Classical Rationality*, 1962, http://www.vordenker.de/ggphilosophy/gg_tradition-of-logic.pdf
> Materialism: http://www.vordenker.de/ggphilosophy/gg_theorie-mehrwert-logik.pdf

2.2.6 Transjunctions: Place-valued logics and morphogrammatics at the BCL


GOTTHARD GUNTHER

CYBERNETIC ONTOLOGY AND TRANSJUNCTIONAL OPERATIONS

BCL

PUBLICATION

68

PHOTOMECHANICALLY REPRODUCED FROM

SELF - ORGANIZING SYSTEMS 1962

Yovits, Jacobi and Goldstein (eds.)

Spartan Books, Washington, D.C.

pp. 313 - 392 (1962)

3:1 February 1974:250

BIOLOGICAL COMPUTER LABORATORY

Department of Electrical Engineering, University of Illinois, Urbana, Illinois

At the BCL the American approach came into the game. But, as usual, more or less all the Americans at the BCL had been Europeans. With the help of Heinz von Foerster, Ross Ashby, Dieter Schadach and the Chinese assistant Hsieh Na, Gunther was forced to change strategy.

After Kurt Gödel supported Gunther's approach, the new mentor was Warren McCulloch.

"In 1960 Günther met Warren S. McCulloch and a deep friendship began which was very stimulating for Günther´s further research studies. In 1961 he became a research professor at the Biological Computer Laboratory (BCL) at the University of Illinois, Urbana, where he worked until 1972." http://www.asc-cybernetics.org/foundations/cyberneticians.htm

New Strategy: First are the combinatorial facts of formalisms, then comes the philosophical interpretations.

What happened was the acceptance of formulas for formal reasons which had been excluded before for reasons of interpretation. This step was prepared by the very seminal paper "Generalisierte Stellenwerttheorie" inspired by some comments of German friends. The concept "Fremdwerte" got assimilated to the concept of transjunctional operations based on the pre-logical theory of morphogrammatics. Morphograms have *"no logical meaning"*. Cybernetic, p. 346 The pattern of "Fremdwerte" was baptized "*transjunctions*".

From the "Reflexionsmuster" with "Fremdwerte" to the 15 morphograms of morphogrammatics. At the BCL the seminal theory of Fremdwerte and the 15 basic-patterns got a strong push into combinatoral analysis and the development of morphogrammatics and morphogram-based place-valued logic. The main results of this endeavour, Fremdwerte as transjunctional rejection and as noise, are presented in "Cybernetic Ontology".

http://www.vordenker.de/ggphilosophy/gg_cyb_ontology.pdf
http://www.ece.uiuc.edu/pubs/bcl/

2.2.7 Relevancy: The observer's observation

About the epistemological background of the combinatorial analyses and the beginnings of *Second-Order Cybernetics* as based on Gunther's philosophy of distributed and mediated observers involving additionally to truth-semantics notions of relevance and meaning in place-valued logical systems.

> "McCulloch and Pitts have proved, inn their theory of formal neural networks, that anything (any function or activity) that can be described *"completely and unambiguously"* in a finite number of words is always realizable by a suitable finite formal neural network. [...]
> Yet, as von Neumann properly pointed out, two problems still remain:
> The *first* is whether such a network will fit into the physical limitations of the organism in question. The *second* problem, more interesting to us, is whether every existing mode of behavior can really be put *completely* and *unambiguously* into a *finite* number of words. [...]
>
> Deficiency of Classic Aristotelian Logic. To describe the world in terms of precisely two values, i.e., to analyze the world structure by using two-valued logic, every event must be stated in one of the following forms: "*it is so*" or "*it is not so*".
> Such a description has often been criticized as unable to handle the following problems: (a) the time structure of the universe and (b) the relation between the describer (subject) and his description (object). [...]
>
> "Ignoring the problem of *relevancy* between the premises and conclusion leads to the seeming absurdity of the rules."
>
> Thus, before applying the rules of deduction, a *relevancy measure* on the premises and their possible conclusions has to be made.
> Unfortunately, the problem of relevance has never treated in formal logic before.
>
> As we mentioned before, both the classic two-valued logic and the many-valued modal logic deal with the same question: *"Is the statement true or false?"*
> The truth or falsity of statements is considered in an *absolute* sense. That is, is S is a true statement about event A, then S is true no matter who is the observer of A.
>
> Furthermore, since S is true everybody must *accept* it as a true statement.
> Questions whether S is *meaningful* or not to a particular observer (or describer), whether S is relevant or not in a special situation, and the possibility of S being true in one case while false in another are not even considered inn the analysis of logics. [...]
>
> To revise the analysis, we should ask different questions, e.g.,
> *"What is the significance of the statement to me?"*
> The significance of a statement may be true or falsity, but may also be something else, e.g., relevancy, meaning.
>
> This question also brings up the relation between an *observer* and his observation.
>
> (After all, any description of the world, common or particular ones, is nothing but the *observer's observation*.)
>
> This question is indeed formulated in a multi-dimensional framework. The formalization of such a structure requires m values.
> Therefore, in answering such questions, one has to *analyze* its logical structure. It is possible to break the huge m-valued structure into several *para-structures* of lower-valuedness and after each substructure is properly treated, to *resynthesize*, and give an answer with a logical structure of the same order as that of the question.
> This paper deals mainly with the processes of analyzing and synthesizing such structures. Na, pp. 15/16 1964, [emphasis, r.k.]

2.2.8 The good news

Gruppe G_J	G_1					G_2									
k_J (Anzahl der Kenos im frame)	1 ○○					2 ○△									
r_J (max. Anzahl benutzter Kenos)	3 ○△□					4 ○△□★									
$d_J = \|G_J\|$	3					3									
Klasse C_{Jr}	C_{11}	C_{12}			C_{13}	C_{22}				C_{23}					C_{24}
Bezeichnung	α	β			γ	ξ				ρ					ϕ
$P_{Jr} = \|C_{Jr}\|$	1	3			1	4				5					1
morpho-grammatische Struktur	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○
	○	○	△	△	△	○	○	△	△	○	□	△	□	□	□
	○	△	○	△	□	○	△	○	△	□	○	□	△	□	★
	○	○	○	○	○	△	△	△	△	△	△	△	△	△	△
Bezeichnung	I	B	C	E	T_E	D	P	Q	K	T_D	T_Q	T_K	T_P	T	U

Good news combinatory table

G_i	$g\{3_i\}$	φ	Familie	σ	MP	R	M
G_1	1 ○○○	10	α^3	1	1	1	1
			$\alpha^2\beta$	9	1	2	2
			$\alpha^2\gamma$	3	1	3	3
			$\alpha\beta^2$	27	1+1 = 2	2	3
			$\alpha\beta\gamma$	18	2+1 = 3	3	4
			$\alpha\gamma^2$	3	2+4+1 = 7	3	5
			β^3	27	1+3+1 = 5	2	4
			$\beta^2\gamma$	27	4+5+1 = 10	3	5
			$\beta\gamma^2$	9	4+14+8+1 = 27	3	6
			γ^3	1	4+32+38+12+1 = 87	3	7
G_2	3 ○○△ ○△○ ○△△	18	$\alpha\xi^2$	16	1	2	2
			$\alpha\xi\rho$	40	1	3	3
			$\alpha\xi\phi$	8	1	4	4
			$\alpha\rho^2$	25	1+1 = 2	3	4
			$\alpha\rho\phi$	10	2+1 = 3	4	5
			$\alpha\phi^2$	1	2+4+1 = 7	4	6
			$\beta\xi^2$	48	1+1 = 2	2	3
			$\beta\xi\rho$	120	2+1 = 3	3	4
			$\beta\xi\phi$	24	3+1 = 4	4	5
			$\beta\rho^2$	75	2+4+1 = 7	3	5
			$\beta\rho\phi$	30	6+6+1 = 13	4	6
			$\beta\phi^2$	3	6+18+9+1 = 34	4	7
			$\gamma\xi^2$	16	2+1 = 3	3	4
			$\gamma\xi\rho$	40	2+4+1 = 7	3	5
			$\gamma\xi\phi$	8	6+6+1 = 13	4	6
			$\gamma\rho^2$	25	2+10+7+1 = 20	3	6
			$\gamma\rho\phi$	10	12+24+10+1 = 47	4	7
			$\gamma\phi^2$	1	12+60+54+14+1 = 141	4	8
g_3	1 ○△□	10	ξ^3	64	1	3	3
			$\xi^2\rho$	240	1+1 = 2	3	4
			$\xi^2\phi$	48	2+1 = 3	4	5
			$\xi\rho^2$	300	1+3+1 = 5	3	5
			$\xi\rho\phi$	120	4+5+1 = 10	4	6
			$\xi\phi^2$	120	4+14+8+1 = 27	4	7
			ρ^3	125	1+7+6+1 = 15	3	6
			$\rho^2\phi$	75	8+19+9+1 = 37	4	7
			$\rho\phi^2$	15	8+46+46+13+1 = 114	4	8
			ϕ^3	1	8+100+184+98+18+1 = 409	4	9

2.2.9 The bad news: non-resolvability

But this "good news"-analysis was based on a restricted and exemplary logical system, with 3 values and 2 variables.

Na is asking in her work, how to deal with the general case of m-valued, m>3 and n-ary, n>2 functions in balanced m=n, over-balanced m>n, and under-balanced m<n systems?

Function resolvability and System-decomposability

> "We can therefore conclude: Only over-balanced L(n, m)-systems possess system-decomposability with respect to their balanced and/or over-balanced sub-valued systems, which belong to the same n-order as L(n, m). These sub-valued systems are then subsystems of L(n, m). Others, balanced or under-balanced, have no subsystems at all.
> The theorem of system-decomposability can now be stated as follows:
> Theorem 3-1:
> **For any n (positive integer) any L(n, m)-system with m>n can always be decomposed into its L(n, s), where n<=s<m.**
> Theorem 3-2
> **Every-overbalanced system can ultimately be decomposed into its balanced subsystems.**
>
> Conclusion 3.5
> Definition 3-15
> **A system can be called resolvable if and only if every member of it is resolvable."**

Na's Equation 3.16a

$$N_s(n\ m) = \sum_{k=s+1}^{k_M} N(n, m; k) = \sum_{k=s+1}^{k_M} \frac{m!}{(m-k)!} S(n,k)$$

$$where\ k_M = \min(n, m)$$

Another example to be considered is L(3, 3). This is a balanced system. Therefore it should possess no subsystems at all. Let us try to decompose it into its sub-valued L(3, 2). Equation (3.16a gives)

$$N_2(3, 3) = \sum_{k=3}^{3} \frac{3!}{(3-k)!} S(3, k)$$
$$= 6 > 0$$

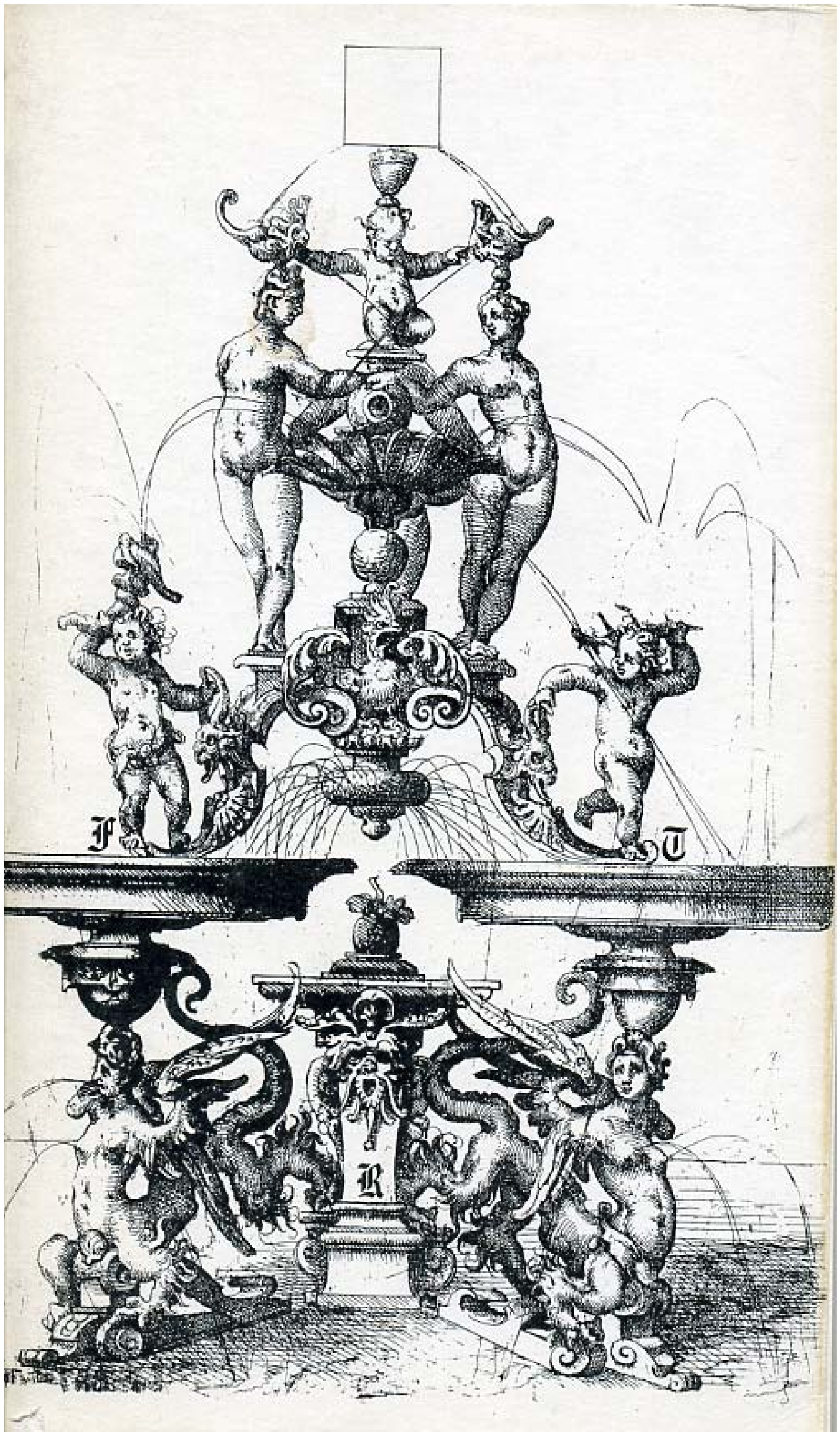
Obviously, for all val(var_1)≠val(var_2)≠val(var_3) in $G^{(3, 3)}$ a non-decomposable situation occurs. Thus, the system is not decomposable. and therefore it is non-resolvable.

An over-balanced system L(3, 4) has a system-decomposability into its balanced subsystems L(3, 3). *"Since L(3, 3) is not decomposable into L(3, 2), L(3, 2) cannot be a subsystem of L(3, 4). Otherwise transitivity of system-decomposability fails"*. Na

Hwa-Sung Hsieh Na (Shanghai), H. von Foerster, G. Gunther
On Structural Analysis of Many Valued Logics, April 1964, 131 pp.
Ph.D. Thesis, Dept. of Elec. Engr., University of Illinois; Urbana, 131 pp. (1964)
BCL TR No. 7.1 and BCL No. 106, Fiche No. 62/1 & 63

Reconstructions of parts of Na's combinatorial analysis by Thomas Mahler at:
http://www.thinkartlab.com/pkl/media/mg-book.pdf

Heinz von Foerster's view of Cybernetic Ontology



2.3 Truth-values and Many-valuedness in general

How was it possible that
the humbug of many logical values
persisted over the last fifty years?
—Roman Suszko, 1976.

Two's company:
"The humbug of many logical values"
Carlos Caleiro, Walter Carnielli,
Marcelo E. Coniglio and Jo˜ao Marcos

Abstract.
The Polish logician Roman Suszko has extensively pleaded in the 1970s for a restatement of the notion of many-valuedness. According to him, as he would often repeat, "there are but two logical values, true and false." As a matter of fact, a result by W´ojcicki-Lindenbaum shows that any Tarskian logic has a many-valued semantics, and results by Suszko-da Costa-Scott show that any many-valued semantics can be reduced to a two-valued one. So, why should one even consider using logics with more than two values? Because, we argue, one has to decide how to deal with bivalence and settle down the tradeoff between logical 2-valuedness and truth-functionality, from a pragmatical standpoint.

In logic research, too, there are all sorts of chauvinism and nationalism. Hence, all that was proven before by Helmut Thiele, Berlin, former GDR.

At this time it was a sort of an academic sport to proof the needless of many-valued logics. The unity of logic had to be defended. Bad enough, there was no chance to proof the uniqueness and exclussivity of existing logic. As a compromise, the term *"logoide formalismen"* was introduced, and appplied even to modal logic.

Paul F. Linke, Die Mehrwertigen Logiken und das Wahrheitsproblem. Review: Paul Bernays, Journal of Symbolic Logic, Vol. 17, No. 4 (Dec., 1952), pp. 276-277

2.4 An early answer to Parsons: "no new logic"

"Thus the three-valued calculus of symbolic logic becomes an *interpreted* system. Its interpretation is not, that it reveals the structure of a new non-Aristotelian logic. It is no new logic but a *system of transformations* by dint of which different logical viewpoints can be calculated and translated into, each other.

The three-valued calculus deals exclusively with the subjective differences between human beings as to their judgments of the surrounding world. What has been said with regard to the three-valued calculus applies – with proper generalization to any many-valued calculus of symbolic logic.

There are cases when the *displacement of rational principles* is undoubtedly much larger than between different human viewpoints. For instance: between human and animal intelligence." Gotthard Gunther 1953
http://www.vordenker.de/gunther_web/gg_logical-parallax.pdf

Turquette, Symbolic Logic
Die Philosophische Idee einer Nicht-Aristotelischen Logik. by Gotthard Gunther
The Logical Parallax. by Gotthard Gunther
Review author[s]: A. R. Turquette
Journal of Symbolic Logic, Vol. 19, No. 2 (Jun., 1954), p. 131

Like Gunther, Turquette and Barkely Rosser had been at the University of Illinois. There are unpublished replies to several critizisms (Turquette, Zinoviev, Schmitz).

2.5 Place-valued logics and many-valuedness

With *table III* of Cybernetic Ontology, Gunther gives a very suggestive hint how to make a difference between classical semantics of many-valued logics and his understanding of many-valuedness.

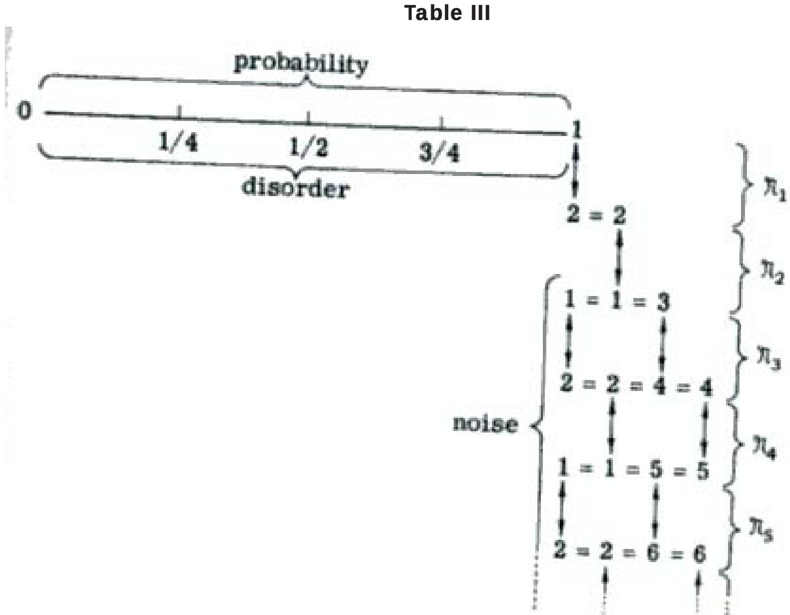
Additionally to this kind of distribution of 2-valuedness over different places to produce many-valuedness, Gunther introduced his morphograms as a value-independent base for a finite definition of all logical value-sequences.

His *"quindecimal place-valued logic system"* was his answer to the open questions of how to interpret many-valued semantics, i.e., the meaning of the single truth-values, and many-valued logical functions in general. In other words, what is the meaning a many-valued logic with, say 321 values? How can we identify in a reasonable way the meaning of the 321 values and how can we identify the astronomic amount of many-valued logical functions?

Obviously, such questions are presupposing that many-valued logics have to be considered as logic including all the philosophical and logical characteristics developed for classical logic and not as meaningless formal systems, useful perhaps for technical reasons only.

These had been the open questions of many-valued logics at the time.

Table III



But classic many-valued logic didn't attempt to solve this problem of representation of formulas out of a small set of elements. Classic many-valued logic was manly interested in modeling classic 2-valued situations, functions and semantics, into many-valuedness. Thus, a typical many-valued logic will have some negations, a conjunction, disjunction and an implication for all systems.

Because classical logics, 2-valued or many-valued, are monolitic systems they have not to deal in a systematic way with questions of decomposability and function resolvability. There, decomposition techniques are used for economic but not for systematic reasons.

The concept of a place-valued logic, Stellenwertlogik, appears as an antagonism in itself between the basic principles of logic and arithmetic, thus as self-contradictional.

2.5.1 Gunther's interpretation of many-valuedness

The single value-problem

Did Gunther succeed to give a solution for his question: What is the meaning of a single value in a multi-valued logical system?

First, he introduced 3 values with the three meanings:
Irreflexive: T
Reflexive: F
Double-reflexive: **F**
With that we have an interpretation for 3 values.

In a letter to Kurt Gödel he wrote:

> Die totale Reflexion umfasst drei differente Systeme:
> a) ein zweiwertiges System
> b) ein dreiwertiges System
> c) ein generell n-wertiges System, wobei n>4
>
> D.h. generelle Mehrwertigkeit schließt b) nicht ein! In anderen Worten: Es existiert reflexionstheoretisch nicht ein semantischer Übergang von klassischen a)-Denken zur Mehrwertigkeit überhaupt sondern zwei. Ich glaube ich kann jetzt absolut zuverlässig nachweisen, dass, während alle mehrwertigen Systeme, die der Bedingung n>4 genügen, strukturell homogen sind (und derart eine logische Einheit darstellen). Das dreiwertige System von dieser Homogenität ausgeschlossen ist. Es stellt ein logisches Zwischensystem dar! Seine logische Sonderrolle wird Ihnen sofort einleuchten, wenn ich sage: Dreiwertigkeit ist nur ein Stellenwertsystem für Seinsthematik, also für Zweiwertigkeit.
> Jan. 18. 1958 [1959

The result is, that only for systems with values equal or bigger 4 are of value. Probably, because only 4-valued systems are complete in respect to the quindecimality of the revised place-valued logic. But this is very similar to the statement of Lukasievicz.

In his paper *"Many-valued designation and a Hierarchy of first Order Ontologies"* (1968) but also in *"Minimalbedingungen..."* he introduces the distinction between positive/negative values and designation/non-designation. These distinctions are made from a global point of view. The single values are still quite under-specified: positive, $negative_1$, $negative_2$,..., $negative_n$.
http://www.vordenker.de/ggphilosophy/gg_many-val-desig-hierarch.pdf

I'm not sure, that this concept of an open negativity of values is answering the leading question about the logical meaning of single values in a general multi-valued system. It also seems to conflict with the other aim, to give a semantic interpretation to every single logical function of a place-valued logic.

On the base of a solution of the single-value problem a solution of the interpretation of all single logical functions, i.e., connectives, has some chance to be realized.

Cybernetic Ontology: How Gunther got rid of logical values

On the way from many-vlauedness to Reflexionsmuster Gunther got rid of the value-problem with his negation-invariant morphograms and more definitively with kenogrammatics. The value-problem was solved in rejecting it. A similar move happend in respect to the resolvability problem of functions, move to *holo-* and *morphgrams* (Na).

3 First version of place-valued logic

3.1 Semantics

In place-valued logic systems all truth-values are compatible. That is, for any two truth values in a binary function, a corresponding truth-value, as the result of the mapping exists. In other words, the functionality over the set of truth-values is closed. This makes a place-valued logical system a stable and closed system.

But this comfort is lost for systems with more than 2 variable, that is for systems with a higher complication than two.

Place-valued systems in Gunther's introduction are kind of a mix of local and global interpretations of different levels of the formal system. In fact, they are presupposing *mediation* and are not delivering a mechanism of logical mediation on all level of the tectonics of the formal system. Thus, place-valued logical systems in the sense of Gunther are failing to give a mediation of different logics.

Heinz von Foerster about Gotthard Gunther

hvf: He was fabulous man, a man who developed a kind of logic which looks, at first glance, as if it were a multivalued logic, but it isn't. Well, it is, and it is not. He called it a place-valued logic, and it is, I think, a very important contribution. Because it gets you out of that yes-no traps, the true-false trap. The essential point in Guenther's contribution is that he argues -- correctly, with a very good formalism -- that in order to take a proposition to be *true or false*, you have to have a *place* in which the proposition stands. That means, when you say "The sun is shining" you have to have a place where that proposition is to be put in, and only then you can say it is true or false. Furthermore, "The sun is shining" could be *rejected* as a proposition to be considered. That means, he introduced the notion of *rejection* instead of Boolean "true and false" and by that one has the means to consider a proposition as a whole as being *acceptable* or not.

A couple of years ago, the whole collected works of Gotthard Guenther came out, if you'd like to look at it. You should really know about Gotthard's work. It is very well understood and read in German, but very little in the American and English-speaking domain.
http://www.stanford.edu/group/SHR/4-2/text/interviewvonf.html

Gunther's opus is out of print since a longtime but collected and digitalized at:
http://www.vordenker.de/ggphilosophy/gg_bibliographie.htm

The strategy of place-valued logical systems is three-fold

First, take a mapping of the set of truth-values onto the set of truth-values.

Second, decompose the results into sub-systems, consisting of 2 values only.

Then identify the couples of values with their corresponding sub-system.

That is, break a total global function into its total local functions and give them an index.

$$Log^{(3,2)} : M^n \longrightarrow M :$$

$$\begin{bmatrix} log_1 : M^1 \bullet M^1 \longrightarrow M^1 \\ log_2 : M^2 \bullet M^2 \longrightarrow M^2 \\ log_3 : M^3 \bullet M^3 \longrightarrow M^3 \end{bmatrix}$$

$$for\ n = 2,\ M = \{1, 2, 3\},\ and$$

$$M^1 = \{1, 2\},\ M^2 = \{2, 3\},\ M^3 = \{1, 3\}$$

3.2 Decomposition problems for n>2

3.2.1 Non-decomposable constellations

$X\ Y\ Z$

$1\ 1\ 1$

$1\ 2\ 1 \xrightarrow[decomp]{} S_1\ (X, Y, Z)$

....

$1\ 2\ 3 \xrightarrow[decomp?]{} S_1, S_2, S_3\ ???$

....

$2\ 3\ 3 \xrightarrow[decomp]{} S_2\ (X, Y, Z)$

$$Log^{(3,3)} : M^n \longrightarrow M :$$

$$\begin{bmatrix} log_1 : M^1 \bullet M^1 \bullet M^1 \longrightarrow M^1 \\ log_2 : M^2 \bullet M^2 \bullet M^2 \longrightarrow M^2 \\ log_3 : M^3 \bullet M^3 \bullet M^3 \longrightarrow M^3 \end{bmatrix}$$

$$for\ n = 3,\ M = \{1, 2, 3\},\ and$$

$$M^1 = \{1, 2\},\ M^2 = \{2, 3\},\ M^3 = \{1, 3\}$$

$L^{(3,3)}$ is producing value-conflicts at places with 3 different values for 3 variables. That is, a decomposition into its place-valued functions is not possible.

That's way Gunther never used, officially, ternary functions. There is a manuscript in the Nachlass in Berlin about his efforts to solve the problem, "*Wurzelfunktionen*", written in a Hospital before he joined the BCL. A highly combinatory study is given by the female Chinese mathematician Hsieh Na then at the BCL as mentioned before as "bad news".

Why not start with ternary functions?

The idea to decompose functions into binary sub-functions is not taboo. It could be reasonable as well to start with non-decomposable ternary function and play the game of construction and decomposition for higher order place-valued logics based on 3-valued ternary units. But the game of non-decomposability would start again on next level.

3.2.2 Diagrams from the early beginnings (Cf. Sushi's Logics)

The following tables are simple examples from the very beginning of polycontextural logics, then called place-value systems, developed by Gotthard Gunther mainly at skiing in mountains of New Hampshire and then with bio-mathematical strength and the collaboration of Ross Ashby, Heinz von Foerster et al. in the early 60s at the famous pioneering BCL (Biological Computer Lab, Urbana, Ill, USA).

Place-value systems started in the late 50s as a new interpretation of multi-valued logics with the aim to give a semantic interpretation of all logical functions of m-valued logics.

First results: The composition/decomposition principle worked properly for unary and binary functions but not in general for n-ary connectives.

This, in the 60th, is not much, but it is more than the highly technical approach of today combining logics. Happily, the story went on and a general theory of mediation of formal systems of any kind is on the way to be developed. Thus, the example of combining semantic 2-valued logics is only a start and happens for didactical reasons only.

You can, if you want, switch from constructivist dialogical logic (Lorenzen, Game logics) to a combination of polylogics of any kind and any mixed copulation and you have not to be restricted by logical matrices. But it wouldn't be bad if there would exist at least a working logical semantics for combined logics on just such a simple base.

Example of a simple semantic mediation

Beispiel:

Variablen p	q	S_1 $\wedge_1$	S_2 $\wedge_2$	S_3 $\wedge_3$	Stellenwertjunktor $J = \wedge_1 \wedge_2 \wedge_3$
1	1	1	$\overline{VB_1}$	1	1
1	2	2			2
1	3			3	3
2	1	2			2
2	2	2	$\overline{VB_2}$ 2		2
2	3		3		3
3	1			3	3
3	2		3		3
3	3		3	$\overline{VB_3}$ 3	3

$J = \wedge_1 \wedge_2 \wedge_3$ in Matrixdarstellung:

J	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

This game of decomposition is based from the very beginning on the distinction between the global and the local. Globally you have a function with 3 values and two variables, locally you have decomposed this total 3-valued function in 3 two-valued (still total) functions.

Violated conditions of mediation

Beispiel: Die drei Junktoren j_1, j_2, j_3:

j_1	1	2
1	1	2
2	2	2

j_2	2	3
2	2	3
3	3	3

j_3	1	3
1	1	3
3	3	1

lassen sich nicht zu einem globalen Stellenwertjunktor zusammenfassen:

Variablen p	q	S_1 j_1	S_2 j_2	S_3 j_3	Stellenwertjunktor $J = j_1 j_2 j_3$
1	1	1	$\overline{VB_1}$	1	1
1	2	2			2
1	3			3	3
2	1	2			2
2	2	2	$\overline{VB_2}$ 2		2
2	3		3		3
3	1			3	3
3	2		3		3
3	3		3	$\overline{VB_3}$ 1	1,3

3.2.3 Combining logics and Place-valued logics

Place-valued topics of composition and decomposition are not only of historical interest. Similar problems occur in the new research field of Combining logics.

Semantics for combining logics is hard. (Cajello)

But it is hard in a double sense: hard conceptually, and hard from its combinatorics.

If we have a first idea about a semantic for fibred logics it turns out that category theory is not very helpful. What is suddenly needed is combinatorics to deal with very complex and complicated situations.

In a strict sense mediation of logics as in polycontextural logic is only a secondary application of a general mechanism of mediation ruled by the *proemial relationship*. Thus, mediation is properly applicable to consequence systems.

Contrary to the Combining Logics approach the distinction *local/global* is a basic architectonic concept of the whole formalism in polycontextural logics and is not to be reduced to modal logic constructs. This point is also clearly established by Pfalzgraf (1988) by his fibred/indexed distribution of logic systems. And, it was at the very beginning of Gunther's more conceptual and philosophical constructions of polycontextural logic. The transition from semantics to meontics, the distinction between negativity and non-designation are crucial examples.

Technical surveys:

A. Sernadas, C. Sernadas, and C. Caleiro. Fibring of logics as a categorial construction. Journal of Logic and Computation, 9(2):149-179, 1999.
http://www.cs.math.ist.utl.pt/ftp/pub/SernadasA/98-SSC-fiblog.pdf
and
Fibring of logics as a universal construction.
http://wslc.math.ist.utl.pt/ftp/pub/SernadasC/04-CCRS-fiblog23.pdf
http://www.cs.math.ist.utl.pt/cs/clc/fibring.html

Some Combinatorics

Semantics for combined logics is hard, but combinatorics of combined constellations even harder. It begins with the simple question: How many logical operations do we have for a $L^{(3,2)}$ logic and how can they be classified in different categories?

Number of logical functions for 2 variables and 3 values

$$\begin{aligned} N_J(3) &= \sum_{i=1}^{3} S(3^2, i) \times P(3^2, i) \\ &= 1 \times 3 + 255 \times 6 + 3025 \times 6 \\ &= 19683. \end{aligned}$$

Allgemein ist:

$$N_J(n) = \sum_{i=1}^{n} S(n^2, i) \times P(n^2, i) = n^{(n^2)}.$$

Place-value systems are a very restricted case of polycontextural logics. They are limited by the very concept of a function which involves all sorts of identity principles.

Sushi's Universal Logic Catalogue – The Ultimate Lambda Pow(d)ers

http://www.thinkartlab.com/pkl/media/SUSHIS_LOGICS.pdf

3.3 A first solution: Allgemeine Vermittlungstheorie

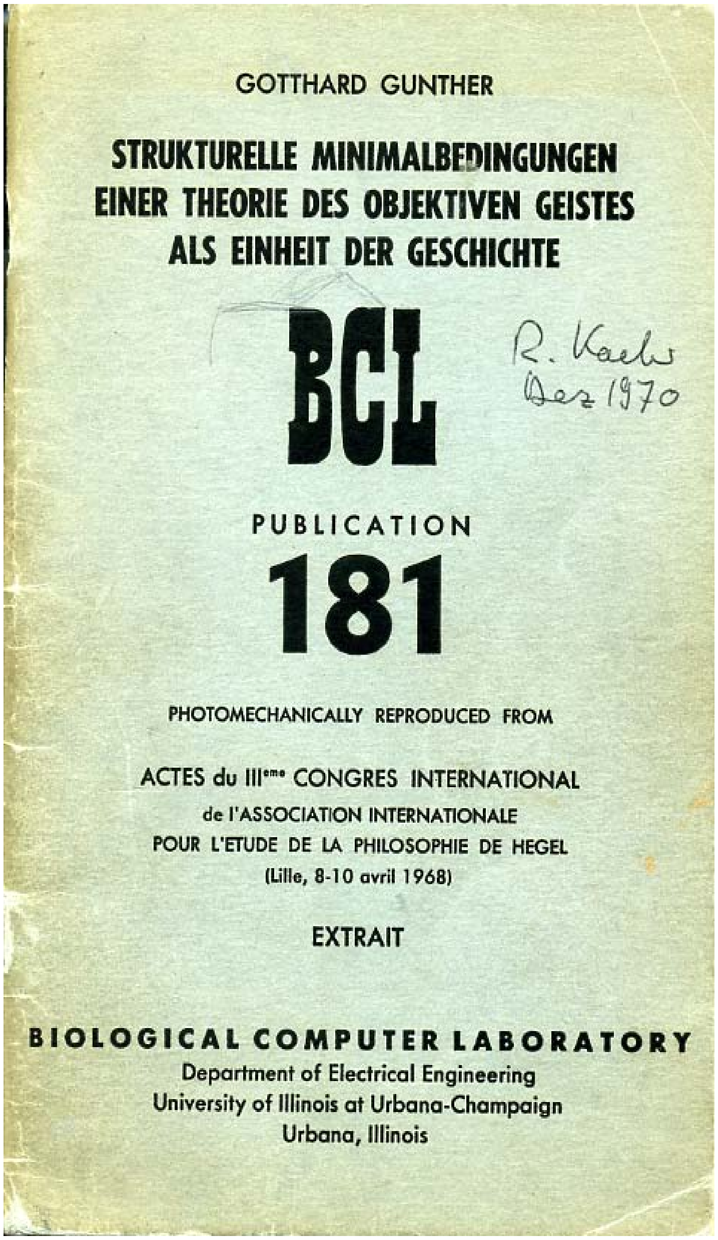
Gunther's silent revolution was written in a text at 1968 which was not only probably one of the few texts with a general political statement but also with a surprising generalization of his concept of logic. After the step from his place-valued logic to the generalized place-valued logic with its 15 Reflexionsmuster this step of a general theory of mediation was even more radical. The other subversion was the introduction of morphogrammatics. But the text is not mentioning the decomposition problems inherited from the past at all.

The Appendix can be read as an answer in two directions. One is a late answer to the decomposition problem, Na analyzed. The other is an answer to the politicized students who didn't understand why to deal with such abstract theories if the are speaking of dialectics, mediation and revolution.

This text was a revolution for the Guntherian approach to a logical theory of mediation. Until now, mediation was a topic of place-valued logics: the mediation between different logical systems, with or without Fremdwerte or transjunctions. Now it turns out that the concept of place-valued logics (Stellenwertlogik) as such has a complementary concept, the concept of *context-valued logics* (Kontextwertlogik). And further more, that both concepts, Stellenwert- and Kontextwertlogik, are mediated in a general Vermittlungstheorie.

This sketch of a theory never got a special attention. At least, after some years I understood it and my understanding resulted in a Diploma and a Ph.D. dissertation of my students in West-Berlin where it found some application.

In the search of a solution of the n-ary function-decomposition problem the concept developed in Gunther's *Appendix* has given enough hints to find a *first* solution of the problem by parametrization of functions.



The political statement has not lost its actuality:

"Ein zwingender Grund für die Einführung mehrwertig-analytischer Kombinatorik in die Interpretation der Hegelschen Reflexionsphilosophie und damit in die Theorie des objektiven Geistes ist damit gegeben, dass diese Theorie, da sie das ganze Geschichtsproblem in sich begreift, von einer so fantastischen strukturellen Komplexität ist, dass alle Intuition, die nicht von sicheren analytischen Mitteln geleitet wird, hier versagen muss.
Die Unmöglichkeit historische Katastrophen abzuwenden legt beredtes Zeugnis ab für die erschütternde Unfähigkeit des menschlichen Intellekts angesichts dieser Aufgabe.
Was benätigt ist, um diesen Zustand wenigstens zu lindern, ist eine enge Verbindung von exakten analytischen und hermeneutischen Methoden."
Gunther, Minimalbedingungen, p. 187/88

Table VI: over- , under-, and balanced systems

S^1 (p)	b
S^2 (p)	ü
S^3 (p)	ü

S^1 (p,q)	u
S^2 (p,q)	b v
S^3 (p,q)	ü v

S^1 (p,q,r)	u
S^2 (p,q,r)	u v
S^3 (p,q,r)	b v

A general theory of mediation has to consider the different aspects of complexity and complication, analyzed as under-, over- and balanced systems. The presumption still is that the whole theory has to be covered by the theory of total functions.

Context-logical decomposition of a 2-valued binary system

TAFEL VIII

S^2(p, q)	S^2(p,1)	S^2(p,2)	S^2(1,q)	S^2(2,q)
1 1	1		1	
2 1	2			1
1 2		1	2	
2 2		2		2

This little construction is formally not of special importance. Nevertheless it turned out that the whole drama from Hegel to Marx and other dialecticians never passed the formal structure of this very first structural possibility of context-valued logic. Non of the conceptually guided thinkers succeeded a complexity/complication structure higher. Very vague speculations happened with m=3 and n= 2. This has been demonstrated in dissertations about Fichte and Marx, and others. Which never passed the limits of extreme simplicity. The same happens today in AI and so called complexity theory.

Gunther's thesis that *intuition* is not prepared to deal with mediated systems of a reasonable complexity and complication is easily to verify.

Context-logical decomposition of a 2-valued ternary system

TAFEL IX

S^2(p, q, r)	S^2(p,q,1)	S^2(p,q,2)	S^2(p,1,r)	S^2(p,2,r)	S^2(1,q,r)	S^2(2,q,r)
1 1 1	1 1		1 1		1 1	
2 1 1	2 1		2 1			1 1
1 2 1	1 2			1 1	2 1	
2 2 1	2 2			2 1		2 1
1 1 2		1 1	1 2		1 2	
2 1 2		2 1	2 2			1 2
1 2 2		1 2		1 2	2 2	
2 2 2		2 2		2 2		2 2

Interestingly, this general theory of mediation is not mentioning anything about propositions or statements. What is presented in a very short sketch is a structural framework for logical systems of all sorts. Such a system is of *iterative* complexity because the focus is the n-arity and not on the m-valuedness. *Accretive* complexity is focussing the m-valuedness of the system.

Context-logical decomposition of a 3-valued ternary system

$S^3(p,q,r)$	$S^3(p,1,r)$	$S^3(p,2,r)$	$S^3(1,q,r)$	$S^3(2,q,r)$	$S^3(3,q,r)$	$S^3(p,q,1)$	$S^3(p,q,2)$	$S^3(p,q,3)$	$S^3(p,1,r)$
1 1 1	1 1			1 1			1 1		
2 1 1	2 1			2 1				1 1	
3 1 1	3 1			3 1					1 1
1 2 1	1 2				1 1		2 1		
2 2 1	2 2				2 1			2 1	
3 2 1	3 2				3 1				2 1
1 3 1	1 3					1 1	3 1		
2 3 1	2 3					2 1		3 1	
3 3 1	3 3					3 1			3 1
1 1 2		1 1		1 2			1 2		
2 1 2		2 1		2 2				1 2	
3 1 2		3 1		3 2					1 2
1 2 2		1 2			1 2		2 2		
2 2 2		2 2			2 2			2 2	
3 2 2		3 2			3 2				2 2
1 3 2		1 3				1 2	3 2		
2 3 2		2 3				2 2		3 2	
3 3 2		3 3				3 2			3 2
1 1 3			1 1	1 3			1 3		
2 1 3			2 1	2 3				1 3	
3 1 3			3 1	3 3					1 3
1 2 3			1 2		1 3		2 3		
2 2 3			2 2		2 3			2 3	
3 2 3			3 2		3 3				2 3
1 3 3			1 3			1 3	3 3		
2 3 3			2 3			2 3		3 3	
3 3 3			3 3			3 3			3 3

This table, finally, shows a mechanism to decompose 3-valued systems with ternary functions. Thus, the table presents a decomposition of the iterative components of a 3-valued accretive system from its balanced state to its under-balanced systems which are decomposable into 2-valued systems of a place-valued system with 2 variables.

I will omit the table of the full mediation of iterative and accretive logical systems, finally realizing the concept of a framework for a general logical theory of mediation (based on total mathematical functions).

Logical invariance: Context-valued logic

It seems to be clear that Gunther's *context-valued logic* is not a "logic of context" as it was introduced by Goddard / Routley, McCarthy and developed by many computer scientists. There is no domain of attributes which is contextualized but the basic logical function as such are involved in contextualizations. Complementary to the semantics or meontics of place-valued systems context-valued logics are defining a complementary notion of "truth": *context-invariant logical structures*. Such structures are independent from the variable from which their logical functors are contextualized.

Say, (p ooo q; r) = (p ooo r; q) = (q ooo r; p) shows context-invariance of (ooo).

3.4 A second solution of the decomposability problem

If we take the philosophical definition of place-value logics literally, we are not forced at all to model it with total functions over truth-value sets. It is enough to distribute and mediate 2-valued systems of whatever number of variables. It is surely also possible to construct a dissemination of logics *without* getting involved with truth-values at all. But this is another story! Therefore, the number of functions is reduced to the number of mediated n-ary m-valued functions, and this is much less than $m^{m/n}$.

$$Log^{(3,\,3)} : M^n \longrightarrow M :$$

$$\begin{bmatrix} log_1 : M^1 \bullet M^1 \bullet M^1 \longrightarrow M^1 \\ log_2 : M^2 \bullet M^2 \bullet M^2 \longrightarrow M^2 \\ log_3 : M^3 \bullet M^3 \bullet M^3 \longrightarrow M^3 \end{bmatrix} / \bmod VB_{var}$$

$$for\ \ n = 3,\ M = \{1, 2, 3\},\ and$$

$$M^1 = \{1, 2\},\ M^2 = \{2, 3\},\ M^3 = \{1, 3\}$$

$$VB_{var} = \forall var^i \forall val^i : val^1\left(var^1\right) \neq val^2\left(var^2\right) \neq val^3\left(var^3\right)$$

$$var = \{X, Y, Z\},\ val = M = \{1, 2, 3\}$$

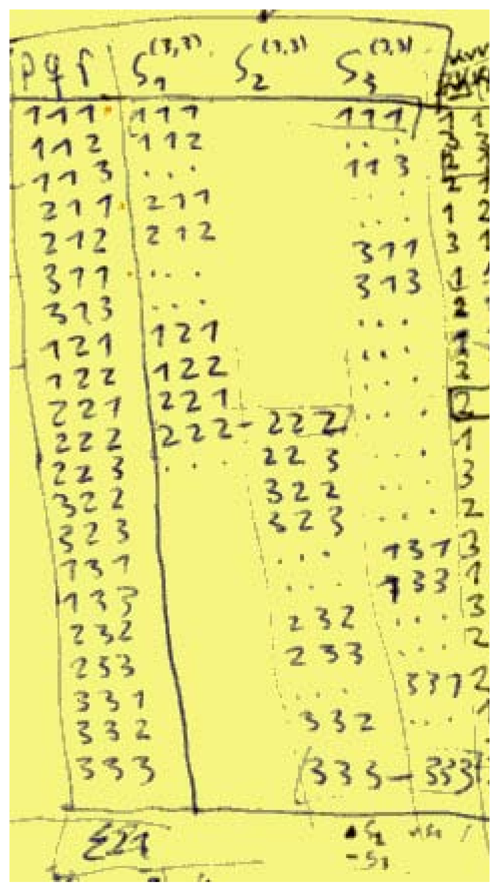
Mod VB_{var}, modulo conditions of mediation for variables, is cutting value-constellations for 3 variables and 3 values.
Such a cut is structural and therefore has to be done at the very beginning of the construction: the "input" variables, too.

$$M^n{}_{/\bmod VB_{var}} \longrightarrow M_{/\bmod VB_{var}}$$

$$(m = 3,\ n = 3) : \left[(X, Y, Z)_{/\bmod\ var}\right] \left[(X, Y, Z)_{/\bmod\ (1-2)},\ (X, Y, Z)_{/\bmod (2-3)},\ (X, Y, Z)_{/\bmod\ (1-3)}\right]$$

After having place-valued logics deliberated from their generous incarceration by global total logical functions, the development of general n-ary and m-valued place-valued logic has become a natural excersiceas I proposed in the late 80s.

Under the pre-conditions given by Gunther's approach to a place-valued system we can now properly develop logical systems of any complexity and complication. One important *pre-condition* is to decompose functions into their morphogrammatic components. That is, to understand the place-valued system as a *"quindecimal place-valued system"* of logical functions.

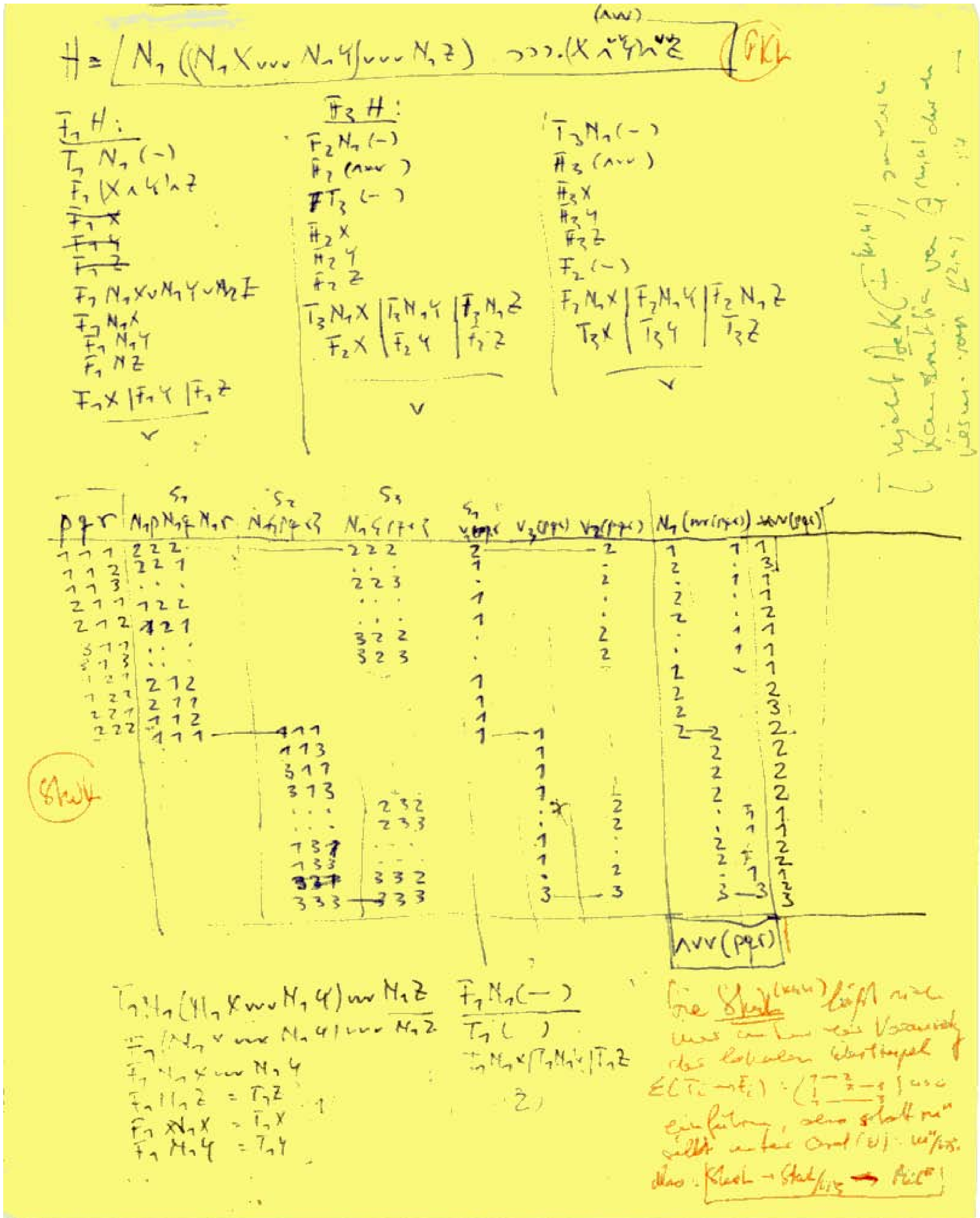


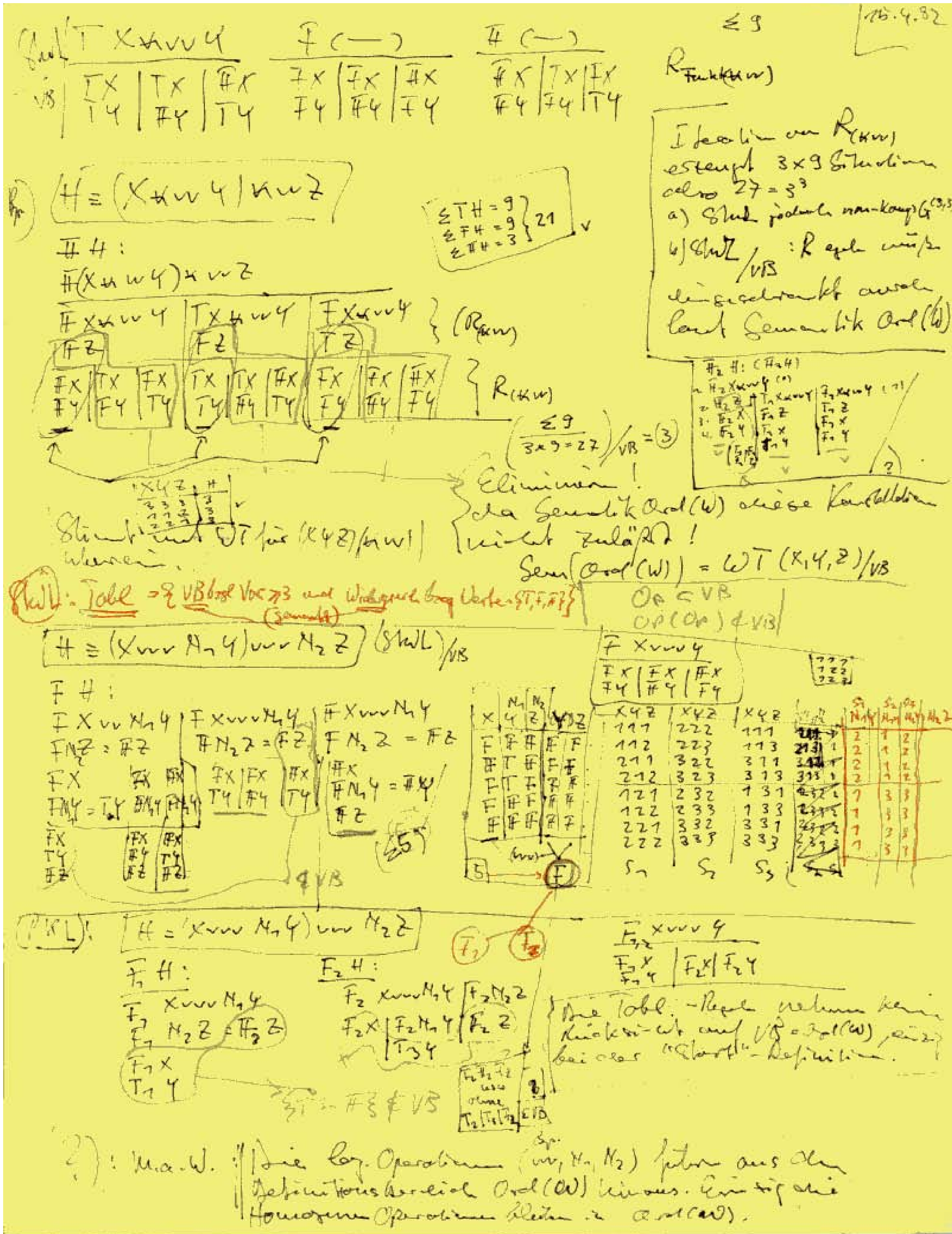
Why can we not be happy with our current solutions?

The main answer is: this place-valued logic is NOT yet giving a full mechanism of *"distribution and mediation",* i.e., *dissemination,* of logical systems but is still requiring and presupposing mediation by its logical functions on different structural levels.

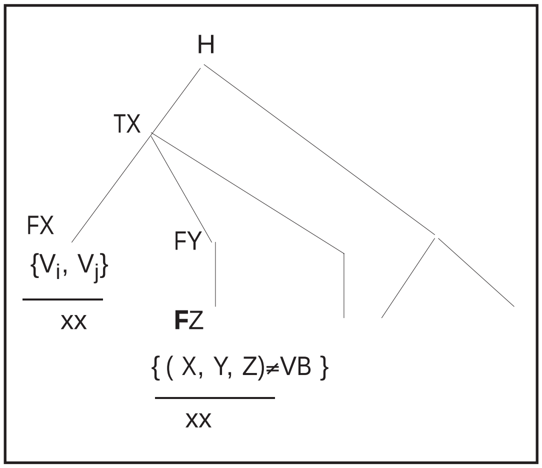
Nevertheless, such place-valued logics may be of interest for special tasks.

Logic scribbles (15.04.1992) from the Yellow Pages



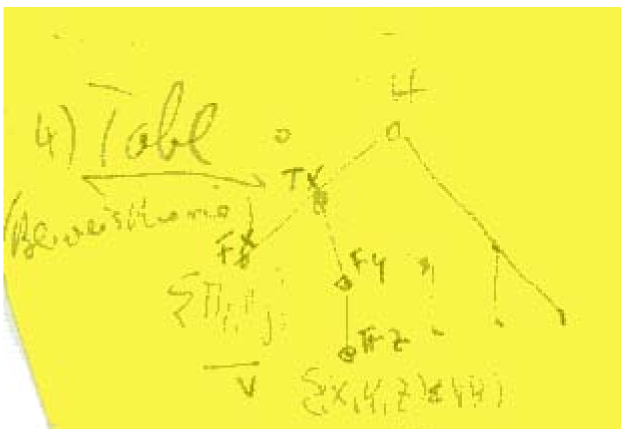


Tableaux tree for situations



The decomposition of formal H is producing on one branch the situations TX and FX. These two signatures for the sub-formula X belong to the set $\{V_i, V_j\}$, thus the situations are *compatible* and are producing a contradiction which is closing that branch.

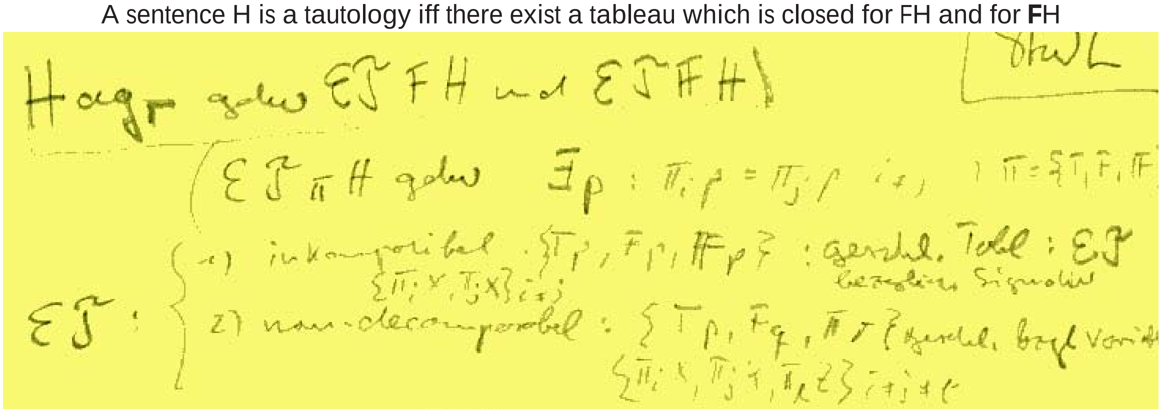
The situations TX, FY and **F**Z are belonging to the set of *non-decomposable* situations, $\{(X, Y, Z) \neq VB\}$, thus the branch is closed. Not because of a contradiction between compatible signatures but because of a violation of the conditions of mediation (VB) for 3 variables, i.e., TX, FY and **F**Z.



This distinction between compatible/incompatible and de-composable/non-decomposable is still defined inside the *stable* logical system, functionally ruled by global total functions.

A branch of a proof tree is closing if a contradiction on a compatible branch occurs or if a non-decomposable situation occurs on a branch of the tree. Thus, at least two criteria are ruling the situations of a proof tree. Such situations are not known in classic 2- or many-valued propositional logic.

A sentence H is a tautology iff there exist a tableau which is closed for FH and for **F**H



in all its branches depending on the two criteria of contradiction and non-decomposability. Non-decomposability, obviously, is independent of the signatures of the roots of the proof tree, i.e., FH and **F**H, simply because it is a structural property.

3.5 A third solution: Morphogrammatic rejection of the value-issue

As mentioned before, another resolution of the decomposition problem appeared in *Cybernetic Ontology* as a total *rejection* of the value-problems, its truth-functions and decomposability obstacles. Despite the overwhelming problems with decomposability and the resulting decision to change direction, Gunther was able to convince Rowena Swanson to continue the funding of his research in trans-classic logic.

The new research proposal focussed on kenogrammatic structures:

> The Kenogrammatic Structure of Many-Valued Logic (together with Dieter Schadach) in: Accomplishment Summary 1966/67; BCL 14-19 (1967); BCL-Report # 67.2; Fiche # 118/1

The research which followed had been, again, mainly combinatorial studies, now about kenogrammatic systems, set in theory of mappings, realized by Gunther's assistant Dieter Schadach by mainly two papers:

> *"A Classification of Mappings between finite Sets and some Application"*, BCL Report No. 2.2, Febr. 1, 1967, sponsored AF–OSR Grant 480 - 64
> *"A System of Equivalence Relations and Generalized Arithmetic"*, BCL Report 4.1, Aug. 1, 1967, sponsored AF - OSR Grant 480 - 64
> http://www.ballonoffconsulting.com/pdf/1987AppendixII.pdf

This attitude of rejecting the "value issue" was radicalized by Gunther's introduction to kenogrammatics presented together with Heinz von Foerster within his paper *"The Logical Structure of Evolution and Emanation"* at the New York Academy of Science in 1967.

> "Since the classic theory of rationality is indissolubly linked with the concept of value, first of all one has to show that the whole "value issue" covers the body of logic like a thin coat of paint. Scrape the paint off and you will discover an unsuspected system of structural forms and relations suggesting methods of thinking which surpass immeasurably all classic theories."
> http://www.vordenker.de/ggphilosophy/gg_logic_structure.pdf

The presentation at the New York Academy of Science, given together with Heinz von Foerster, surely was a serious compliment and great acknowledgement of the long-term work supported by the Directorate of Information Services at the AFOSR attended by Rowena Swanson.

But first, Hwa-Sung Hsieh Na mentioned the new direction of Gunther's trans-classical logic research in the summary of her paper *"On Structural Analysis of Many Valued Logic"* (1964) sponsored by AF Grant 8-63:

> "No mechanism is proposed to analyze relations among various balanced systems; nor for relations between balanced and the under-balanced systems.
> Professor Gunther's recent study on *protograms* and *heterograms*, perhaps, will give some suggestion in this direction." Na, p. 127

Another move was made by Gunther's new approach to a reflectional semantics of many-valued systems presented at the International Congress for Philosophy, Vienna 1968

> http://www.vordenker.de/ggphilosophy/gg_many-val-desig-hierarch.pdf

Also studies to the*"founding relation"*, later the *"proemial relationship"* started.

http://www.vordenker.de/ggphilosophy/gg_formal-logic-totality.pdf

The introduction of kenograms and morphograms inspired not only morphogrammatics (1963–) but also kenogrammatic based number theories. A new operator was introduced: the *arithmetic place-designator*.

The last proposal addressed to Rowena Swanson was *"Proposal for the Continuation of a Study of the Behavior of Natural Numbers in a Trans-classic System of Logic"*, May 26 1969

As the publication list shows, the proposal was accepted: Grant AF-AFOSR-70-1865 and Grant 68-1391.

Published as *"Natural Numbers in Trans-Classic Systems"*, Part I, II
in: Journal of Cybernetics, Vol. 1, 1971, No. 2, pp. 23-33 and No. 3, pp. 50-62
http://www.vordenker.de/ggphilosophy/gg_natural-numbers.pdf

But this was also the end of the story. *The Mansfield Amendment: RANN 1971*

http://www.gwu.edu/~umpleby/recent_papers/2003_Heinz_von_Foerster_and_Mansfield_Amendment.pdf
http://userpage.fu-berlin.de/~gerbrehm/gg_gehlen.pdf
In this letter to Arnold Gehlen Gunther writes his disappoinment and deep anger.

The next proposal from 15 October 1970 was denied. The attendance was suddenly not Rowena but Dr. Merle M. Andrew from the *Directorate of Mathematical and Information Science of the AFOSR* in Arlington, Virginia.

Again, we should remember, Gotthard Gunther was born 1900.

And the not funded research to a trans-classic theory of *Decision Making Machines* appeared, at least partly, as the very influential paper *"Cognition and Volition"*.

Albeit without the famous acknowledgment "sponsored by the *Air Force Office of Scientific Research".*

Cognition and Volition - A Contribution to a Theory of Subjectivity.
in: Cybernetics Technique in Brain Research and the Educational Process, 1971
Fall Conference of American Society for Cybernetics, Washington D.C., p. 119-135.
http://www.vordenker.de/ggphilosophy/c_and_v.pdf

Back in Germany, Gunther started his studies of *Negational Cycles* with all kinds of Hamilton paths and other combinatorial problems. He got some help by Gerhard Thomas (Berlin) and Alexander Andrew (University of Reading, UK). A. Andrew also programmed the *Table of Stirling Numbers* at the BCL. He was, probably, involved at this time with Ross Ashby. Philosophically, it was conceived as part of his new theory of *"negative languages"*. Logically, he was focussing on *unary* logical functions from a more or less global point of view. Hence, avoiding the tedious problems of general n-ary functions.

The whole story shows that there was no professional logician involved by contract to help the *philosopher* at the Electrical Engineering Department to develop his mathematical and logical theories. As my supervisor professor Dieter Rödding (*24.8.1937, +4.6.1984), then director of the world famous *Institut für Logik und Grundlagenforschung der Mathematik*, Münster, Westfahlen, Germany, told me (1969?), after he learned that I'm working on Gunther's ideas, just a few years after Gunther had given a lecture at the Academy of Science of Rheinland-Westfahlen and earned hard criticism: *"Das ist ein Projekt, das ein Einzelner nicht schaffen kann."*

3.6 State of the Art of Gunther's research at the leave from the BCL

Arrived in Hamburg, Germany, Gunther felt at the edge of his theory of polycontexturality and non-Aristotelian machines started during his time at the BCL.

Reflections from the external point of view taken by Gunther in Hamburg about the possibilities how the BCL could have been saved and a summary about his "final" understanding of polycontexturality is given in a letter to Heinz von Foerster (1972). Also a new argument against the reduction strategy of the defender of two-valuedness of logic appears: The Dagwood Sandwich Strategy. It can be seen as as a further development and explanation of the earlier *what-and-how* argument, i.e., it makes a difference *how* we describe/construct a systems: with two-valued logic, place-valued logics or morphogrammatics.

3.6.1 Results of the value-problem

symmetry/asymmetry
positive/negative
designation/non-designation
acceptance/rejection

3.6.2 Theory of reflection

Cybernetic ontology
reflectional chain of first-order ontologies/logics

3.6.3 Kenogrammatics and dialectical numbers

iteration/accretion
proto-, deutero- and trito-numbers
place-designator

3.6.4 Theory of Polycontexturality

contextures
proemial relation
kenogrammatics
bivalence/multi-valuedness
hierarchy/heterarchy

3.6.5 The Dagwood Sandwich

An idea about the relationship of man and machine in the light of a polycontextural distribution of Platonic Pyramids on Proto-structures is given.

"Das Verhältnis Mensch:Maschine is ein Verhältnis von Verbundkontexturen, die sich auf verschiedenen Allgemeinheitsstufen der Platonischen Pyramide befinden."

A more explicit presentation of the idea of mapping Platonic Pyramids onto Proto-Structures is given in: *Life as Polycontexturality* (1973).

http://vordenker.de/ggphilosophy/gg_life_as_polycontexturality.pdf

A bitter End and a new Beginning

Much after the *Dagwood Sandwich* the long-term and deep friendship with Heinz von Foerster was disturbed. Probably because Heinz didn't call or visit Gunther in Hamburg during his lecture at a Cybernetic Congress in Nürnberg, 1973. Personally, I didn't meet Heinz in Nürnberg, but Peter Hejl (West-Berlin) did – and the story of Humberto Maturana in Germany started. And with him the new trends of Radical Constructivism and Second-Order Cybernetics became influential. I met Heinz much later in St. Gallen, Switzerland. But it easily could have been another congress because there had been letters to Heinz until 1979. In one, 1978, I was called "a *crack pot* of astronomic dimensions but with great ability". I always liked this magic term. Especially in the use of my son Ossip. With Georg Spencer Brown's calculus Gunther was out of the trends.

The Dagwood Sandwich of Polycontexturality and Bivalence (1972)

Es kann zwar beim Abstieg noch Schwierigkeiten geben, aber ich glaube, die grösste Schwierigkeit liegt hinter mir. Sie bestand darin, eine genaue Formulierung für das Verhältnis von klassischer und trans-klassischer Logik zu finden, und einen endgültigen Einwand gegen die ewige Litanei zu machen: wir können das alles auch mit zweiwertiger Logik. Die Antwort besteht, wie Du aus dem Text sehen wirst, darin, dass sich die Platonische Pyramide der Zweiwertigkeit ~~mit einem Dagwood Sandwich verhalten~~ und die Struktur der Poly-Kontexturalität wie ein Dagwood Sandwich verhalten. Erst kommt die Platonisch-Aristotelische Zweiwertigkeit, dann kommt Mehrwertigkeit (die Poly-Kontexturalität impliziert). Die Mehrwertigkeit produziert wieder Zweiwertigkeit, nämlich die Dualität von Akzeptions- und Rejektionswerten. Diese neue Zweiwertigkeit produziert wieder neue Mehrwertigkeit, resp. Kontextur; in der tut sich dann wieder eine Zweiwertigkeit auf, und dann kommt wieder Kontextur und Mehrwertigkeit. Aber selbst Dagwood Bumstead muss bei dem Bau seiner giant sandwiches schliesslich bei einem Aussenlayer stehen bleiben. Und niemand kann es ihm verwehren, wenn das gerade Zweiwertigkeit ist. Aber die Wahl bedeutet eine semantische Entscheidung. Lege ich auf Zweiwertigkeit wert, bedeutet das, dass ich in der Pyramide nach oben steige, d.h. es kommt mir auf die Spitze, also höchste Allgemeinheit an. Bewege ich mich aber im mehrwertig-kontexturalen Raum, dann bin ich garnicht an der Spitze interessiert, sondern an der individuellen Vielfältigkeit der Basis. Diese beiden Tendenzen sind in einer trans-klassischen Logik unlösbar miteinander verflochten und lasse ich die eine oder andere Tendenz dominieren, so heisst das nur, dass ich im gegebenen Fall zur Lösung eines Problems mehr Allgemeinheit oder mehr logische Individualisierung brauche. Kontextur ist das logische Äquivalent von Individualität. Die Platonische Dichotomie ist das logische Äquivalent für Allgemeinheit. Du kannst aber auch sagen – so paradox es auch klingen mag – Kontextur ist das logische Äquivalent für Materialität und Qualität; das Platonisch-Aristotelische aber ist das Äquivalent für Form. Das Verhältnis von Form und Inhalt ist dialektisch und von da komm wir zum Dialog, aus dem ja Hegel bekanntlich seine elementare Theorie der Dialektik abgeleitet hat. Das Verhältnis Mensch: Maschine ist ein Verhältnis von Verbundkontexturen, die sich auf verschiedenen Allgemeinheitsstufen der Platonischen Pyramide befinden. Aber davon später mehr.

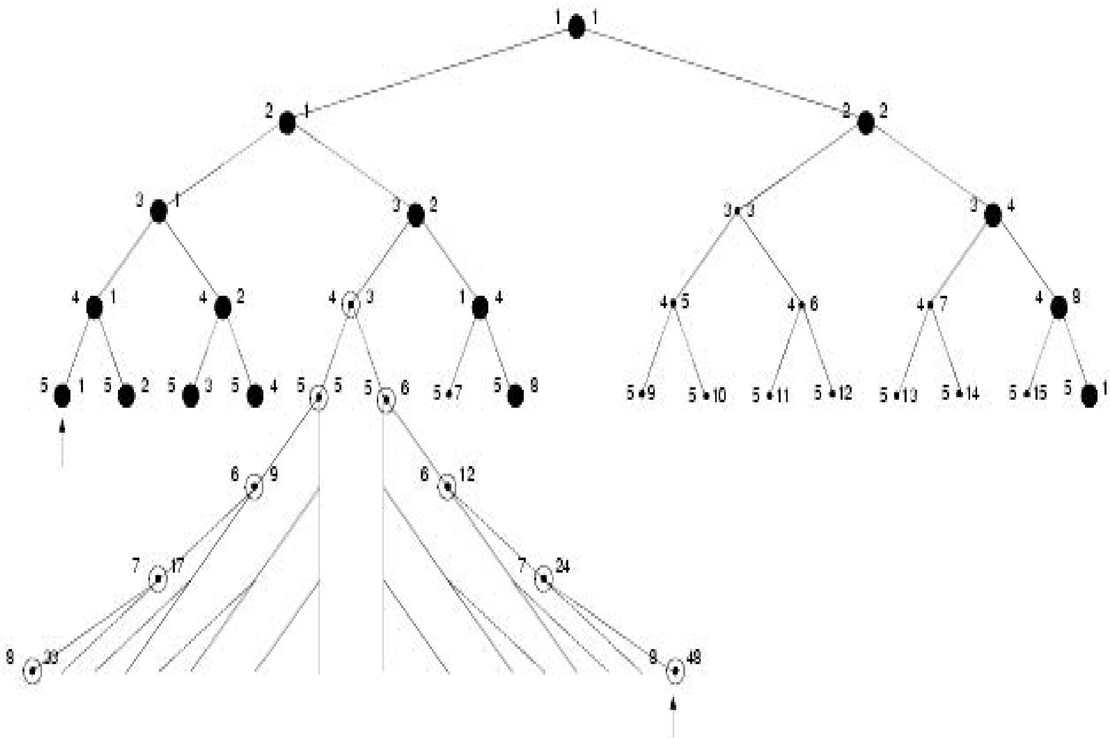
21. 8. 1972 http://userpage.fu-berlin.de/~gerbrehm/gg_foerster3.pdf

"Dass sich die Platonische Pyramide der Zweiwertigkeit und die Struktur der Poly-Kontexturalität wie ein Dagwood Sandwich verhalten. […] Aber selbst Dagwood Bumstaed muss bei dem Bau seiner giant sandwiches schliesslich bei einem Aussenlayer stehen bleiben." Gunther 1972

The Dagwood strategy of Platonic Pyramid, negation and compound contextures.

> "The process of gradually shaping individualities out of mere separate entities begins when a universal contexture joins other contextures in such a way that the result is what we shall call a *compound contexture*. A compound contexture does not originate if we just gather at our stipulated bottom of the pyramid a smaller or larger amount of elementary contextures. It is required that a compound contexture "closes" at least a single diairesis which holds between two elementary contextures. A compound contexture, even in its most elementary form, extends at least over three diairetic levels of the Platonic pyramid."
> http://www.thinkartlab.com/pkl/archive/GUNTHER-BOOK/NEGATI2.html

Mapping of a Platonic pyramid onto the protostructure of kenogrammatics (1972)



Anger and an idea how the BCL could have been saved: Gunther's non-Aristotelian Machine (1975)

Bitternis dabei, denn es bestand eine Möglichkeit, dass mit [redacted] etwas mehr Unterstützung das BCL evt. hätte gerettet werden können. Ich habe es damals nicht geahnt, aber ich sehe jetzt, dass ich mich im letzten Jahre in Urbana schon bis auf Haaresbreite dem Konzept einer nicht-aristotelischen Maschine genähert hatte. Ich weiss das jetzt, weil es mir, seit ich in Hamburg bin, geglückt ist zu entdecken, was eigentlich ein biologisches System qua nicht-aristotelische Maschine ist. Der Begriff entwickelte sich eigentlich ganz

errechnet habe. Hätten wir das 1970 oder 71 gehabt, hätten wir damit eine Reklame machen können, die das BCL wahrscheinlich gerettet hätte. Leider kann ich die Idee in einem Brief nicht ausreichend beschreiben, ohne vermutlich Missverständnisse zu produzieren. Im grossen und ganzen handelt es sich um

3.VII. 75 http://userpage.fu-berlin.de/~gerbrehm/gg_foerster3.pdf

3.7 Comparison between place-valued and polylogical systems

3.7.1 A transition from place-value systems to polycontextural logics

To produce polylogical systems out of this semantic considerations we simply have to take the local aspects of the distributed systems seriously. The easiest way to do this is to give the subsystems (variables, operators, values, etc.) a corresponding sub-system index. More mathematically, the local systems are distributed over the index-set of their fibred category (Pfalzgraf).

3.7.2 Comparison

Tableaux proof of formula H shows well enough the difference and similarity between the two approaches.

The tableaux rules in the polycontextural setting are reflecting strictly the distributed local logical functions, whereas the place-value tableaux are not reflecting the distributed logical functions but the truth-value matrix of the global functions. The syntax of the example for conjunctions is local, shown as a distribution of 3 conjunctions, but the tableaux rules are global, not representing the local conjunctions as such, but the global matrix of the conjunctions. This is true also for other, not tableaux based, formalisations.

The global aspects of polycontextural functions, its mediation rules, are not represented explicitly by the local tableaux but are written in the conditions of mediation of local tableaux, the *proemiality* of mediated sub-systems.

3.7.3 Open Problems with Gunther's place-valued logic, again

The problem with Gunther's place-valued logic, even with the correction to extend it to n-ary functions, lies in the fact, that half of the steps of the decomposition and mediating process is still hidden in the mental space of a human logician. In other words, the mediation is not realized as a calculus but as an *interpretation* of a half-mediated calculus as a mediated logic.

That is, in the syntax, a distribution of logical operators are written, say [&, &, v]. Then, the global value constellations are calculated by the rules of the distributed operators. As a result, a global function is produced. Then, this global function has to be interpreted along the structural rules of the decomposition into sub-systems. These results of decompositions have to be named by the 15 morphograms representing the structure of the logical functions of the sub-systems. With this detour to semantics and de-/composition, we are back at the level of the syntactical operators of the departure.

This manoeuvre of mediation can be formalized on the base of strict semantics of sub-system logical values,i.e., of indexed sets of truth-values and their mediation.

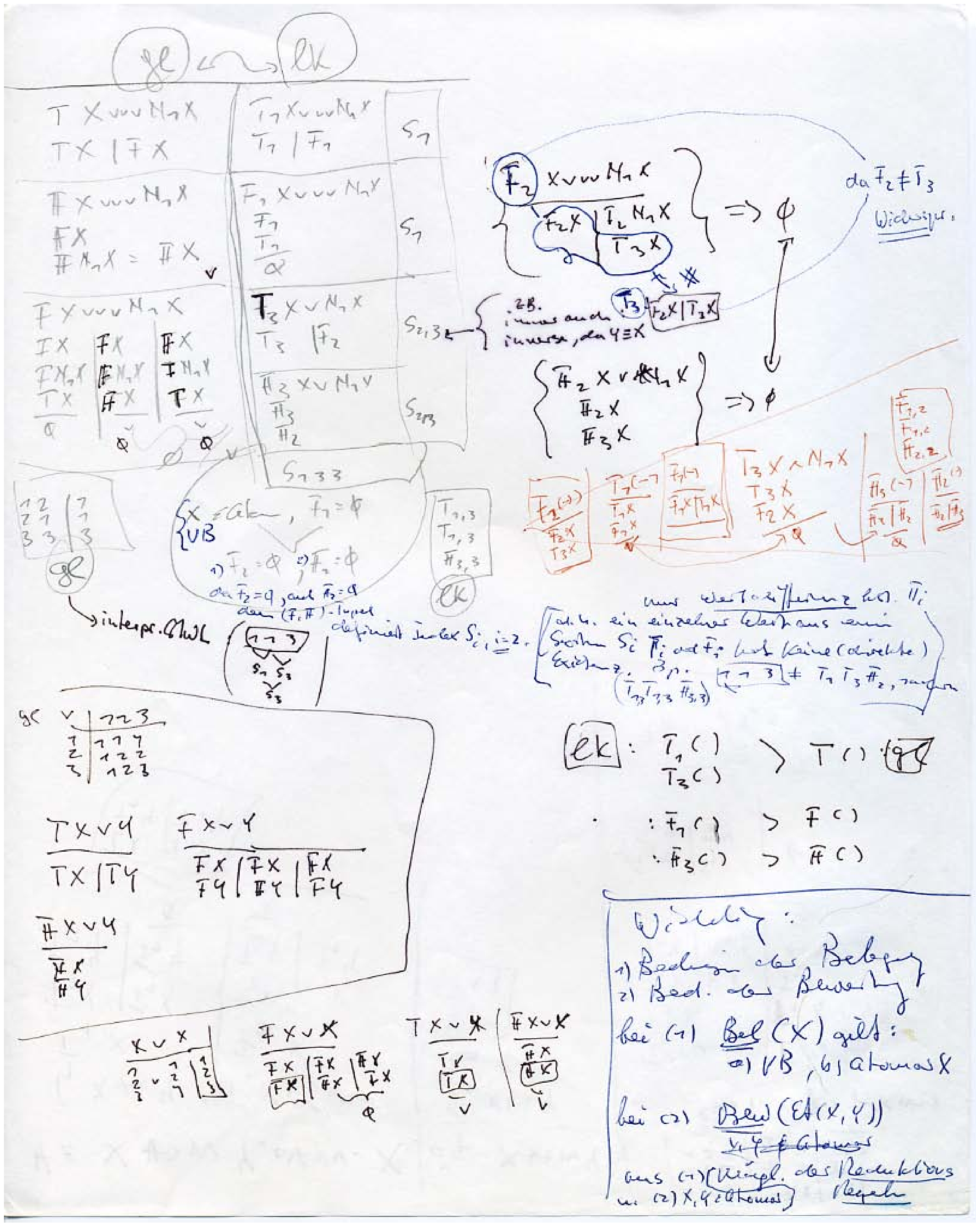
It is reasonable to draw a further distinction. In placed-valued logics, logical truth-values are elements of sets, say {T, F, **F**}. Each value is an independent unit. Thus, it is correct to write: $N_1(\mathbf{F}) = \mathbf{F}$.

In disseminated, polycontextural logics, such a function would be reasonable only as a special case of a frozen situation. Disseminated logics are, because their dissemination is fundamental, logics of *differences* and not of elements. Thus, their units are differences and not elements of a set. That is, disseminated values are structured, therefor a set, like {T, F, **F**}, has to be read as a system of differences with $[t_{1,3}, f_1/t_2, t_{1,3}]$. Thus, we should write: $N_1(\mathbf{F_{2,3}}) = \mathbf{F_{3,2}}$ instead of $N_1(\mathbf{F}) = \mathbf{F}$ omitting differences.

Obviously, the structure of the values involved is ruled by the *proemial relation*.

A further conflict is given by the fact, that the finiteness of the "quindecimal" system, i.e, the interpretability of functions to 15 basic morphograms is still contrasted with a potential infinity of different logical values. Polylogical systems consists of tuples of logical values (t_i, f_j) based on a potential infinite index set of complexity, i.e., $i, j \in N$.

The global/local game, again



Thus, classical topics, for general many-valued and place-valued logics, to give a "natural" interpretation to the single logical or truth-values of the system, are, at least from a local point of view, obsolete. This is not excluding the possibility of interpretations for the global logical situations.

Comparison between local and global tableaux

$$\begin{array}{c}t_1\ X \wedge\wedge\wedge Y\\ \hline t_1\ X\\ t_1\ Y\end{array}\qquad \begin{array}{c}f_1\ X \wedge\wedge\wedge Y\\ \hline f_1\ X \mid f_1\ Y\end{array}\qquad \begin{array}{c}t_2\ X \wedge\wedge\wedge Y\\ \hline t_2\ X\\ t_2\ Y\end{array}\qquad \begin{array}{c}f_2\ X \wedge\wedge\wedge Y\\ \hline f_2\ X \mid f_2\ Y\end{array}$$

$$\begin{array}{c}t_3\ X \wedge\wedge\wedge Y\\ \hline t_3\ X\\ t_3\ Y\end{array}\qquad \begin{array}{c}f_3\ X \wedge\wedge\wedge Y\\ \hline f_3\ X \mid f_3\ Y\end{array}\qquad \begin{array}{l}t_1,\ t_3 \longrightarrow T\\ f_1,\ t_2 \longrightarrow F\\ f_2,\ f_3 \longrightarrow \mathbf{F}\end{array}$$

$$\begin{array}{c}T\ X \wedge\wedge\wedge Y\\ \hline T\ X\\ T\ Y\end{array}\qquad \begin{array}{c|c|c}\multicolumn{3}{c}{F\ X \wedge\wedge\wedge Y}\\ \hline F\ X & T\ X & F\ X\\ T\ Y & F\ Y & F\ Y\end{array}\qquad \begin{array}{c}\mathsf{F}\ X \wedge\wedge\wedge Y\\ \hline \mathsf{F}\ X \mid \mathsf{F}\ Y\end{array}$$

$$\begin{array}{c|c|c}\multicolumn{3}{c}{t_{1,3}\ X <><><> Y}\\ \hline t_{1,3}\ X & t_2\ X & f_2 X\\ t_{1,3}\ Y & f_2\ Y & t_2\ Y\end{array}\qquad \begin{array}{c|c|c}\multicolumn{3}{c}{f_1/t_2\ X <><><> Y}\\ \hline f_1/t_2\ X & t_3\ X & f_3\ X\\ f_1/t_2\ Y & f_3\ Y & t_3\ Y\end{array}\qquad \begin{array}{c|c|c}\multicolumn{3}{c}{f_{2,3}\ X <><><> Y}\\ \hline t_1\ X & f_1\ X & f_{2,3}\ X\\ f_1\ Y & t_1\ Y & f_{2,3}\ Y\end{array}$$

$$\begin{array}{c|c|c}\multicolumn{3}{c}{T\ X <><><> Y}\\ \hline T\ X & F\ X & \mathsf{F}\ X\\ T\ Y & \mathsf{F}\ Y & F\ Y\end{array}\qquad \begin{array}{c|c|c}\multicolumn{3}{c}{F\ X <><><> Y}\\ \hline F\ X & T\ X & \mathsf{F}\ X\\ F\ Y & \mathsf{F}\ Y & T\ Y\end{array}\qquad \begin{array}{c|c|c}\multicolumn{3}{c}{\mathsf{F}\ X <><><> Y}\\ \hline T\ X & F\ X & \mathsf{F}\ X\\ F\ Y & T\ Y & \mathsf{F}\ Y\end{array}$$

The two examples of comparison show clearly the difference in structural information given by the local tableaux. This is of importance for the study of the property of unification (alpha-, beta- and gamma-, delta-categories). The global patterns are more or less useless for meta-logical considerations like unifications.

Polycontextural formulas and situations easily can be *reduced* to place-valued logical formulas and definitions.

Reduction from polycontextural to place-valued monoform conjunction

$(p \wedge\wedge\wedge q) \in PCL$

$p \backslash q$	$t_{1,3}$	f_1, t_2	$f_{2,3}$
$t_{1,3}$	$t_{1,3}$	f_1	f_3
f_1, t_2	f_1	f_1, t_2	f_2
$f_{2,3}$	f_3	f_2	$f_{2,3}$

$\xrightarrow{index-reduction}$

$(p \wedge\wedge\wedge q) \in PVL$

$p \backslash q$	T	F	F
T	T	F	F
F	F	F	F
F	F	F	F

Place-valued logics as a mix of local and global strategies

$$\left\langle \mathrm{op}^1 \mathrm{op}^2 \mathrm{op}^3 \right\rangle_{local} \left(X_{globa}\right) \xrightarrow{application} \left(\left(\mathrm{op}^1 \mathrm{op}^2 \mathrm{op}^3\right)\left(X_{globa}\right)\right)_{global}$$

$$\left\langle \left(\mathrm{op}^1 \left(\mathrm{op}^2 \left(\mathrm{op}^3\right)\right)\right) \right\rangle_{local} \left(X_{globa}\right) \xrightarrow{application} \left(\left(\mathrm{op}^1 \mathrm{op}^2 \mathrm{op}^3\right)\left(X_{globa}\right)\right)_{global}$$

3.7.4 Conjugation diagrams between place-valued and polycontextural settings

The following diagrams are using a notation for logical values which is, probably for reasons of presentation, written 1973, a mix of global notation {T, F, **F**} and local notation (t_i, f_j), with i, j ε N.

Structural conjugation and diagram for monoform conjunction

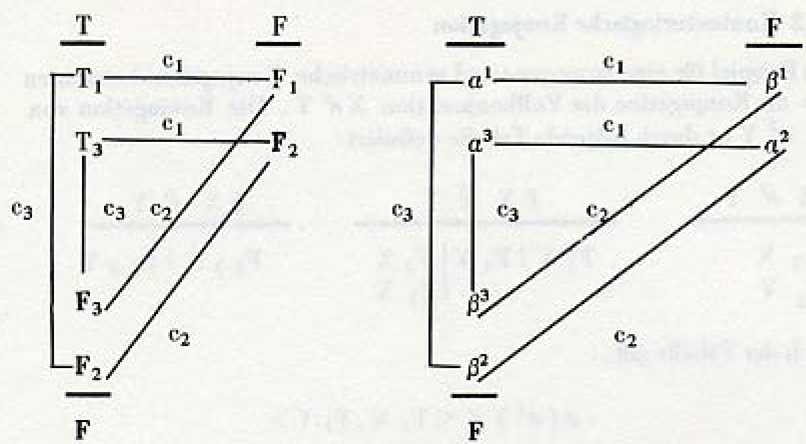
$TX \wedge^3 Y$	$FX \wedge^3 Y$	$FX \wedge^3 Y$
$T_{1,3}\ X$ $T_{1,3}\ Y$	$F_1 X \mid F_1 Y \mid F_2 X$ $\mid F_2\ X$	$F_{2,3} X \mid F_{2,3} Y$

Nach der Tabelle gilt :

$$\varphi(a^1) = \langle T_1 X, T_1 Y \rangle$$
$$\varphi(a^3) = \langle T_3 X, T_3 Y \rangle$$

Die Konjugationen :

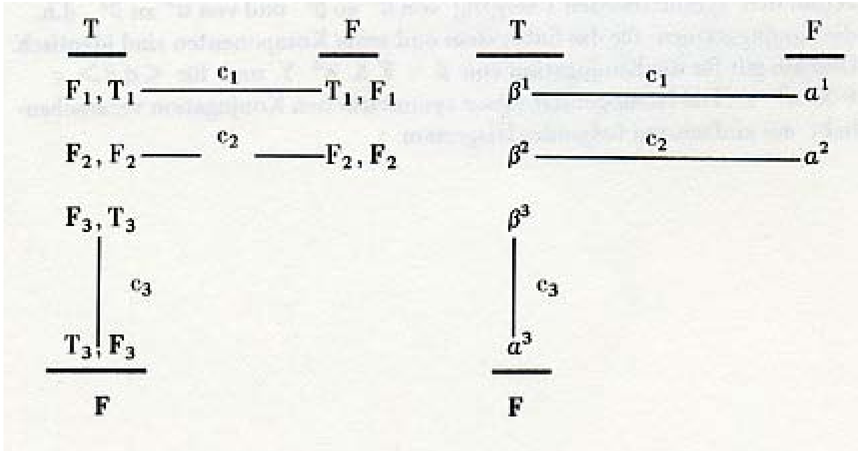
$$\mathrm{kon}^1(a^1) = F_1 X \ \wedge^3 Y = \beta^1$$
$$\mathrm{kon}^3(a^3) = F_3 X \ \wedge^3 Y = \beta^3$$
$$\mathrm{kon}^1(\varphi(a^1)) = \langle F_1 X, F_1 Y \rangle = \langle \beta^1_1, \beta^1_2 \rangle$$
$$\mathrm{kon}^3(\varphi(a^3)) = \langle F_3 X, F_3 Y \rangle = \langle \beta^3_1, \beta^3_2 \rangle$$



Zusammengefaßt : $c_i \ \pi \ c_i \ X \wedge^3 Y \ , \ i = 1, 2, 3$

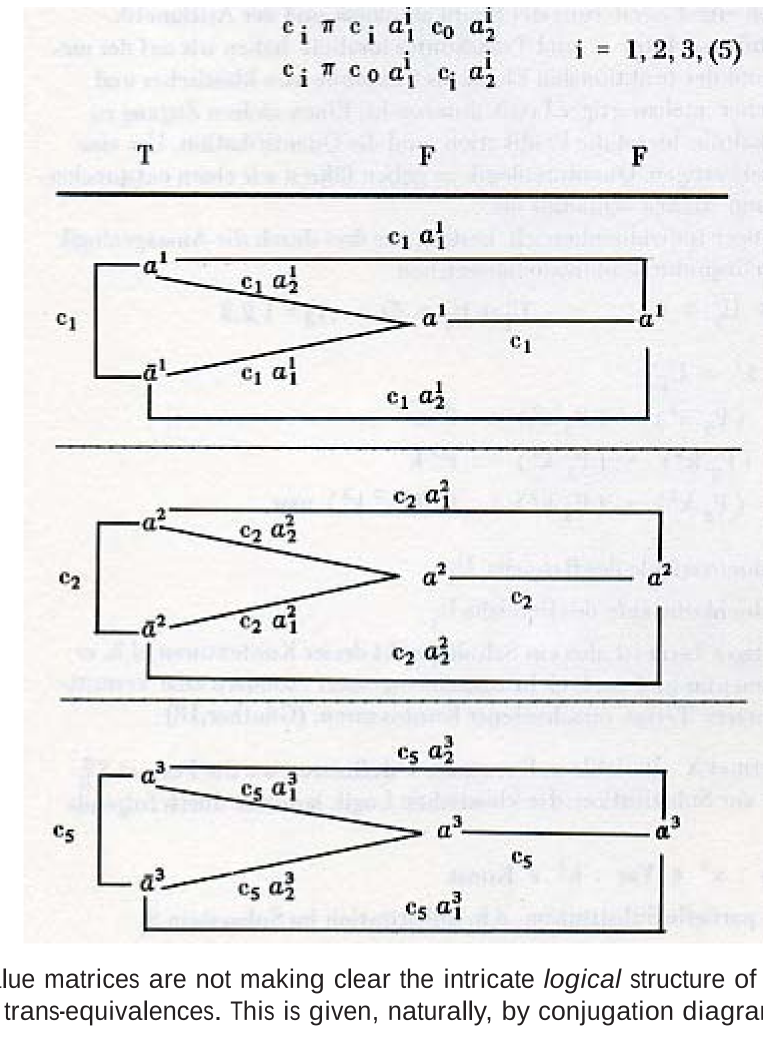
Such *conjugation diagrams*, as I first introduced them in *Materialien 1973–75*, in applying Smullyan's concept of Unification (1968) to polycontextural logics, are of great help to study the structure of complex logics. Not only proofs are reduced but insights into the logical structure of systems m≥ 3 are made accessible to analysis. They also proof the advantage of the "local" approach to polycontexturality.

Structural conjugation diagram for implication



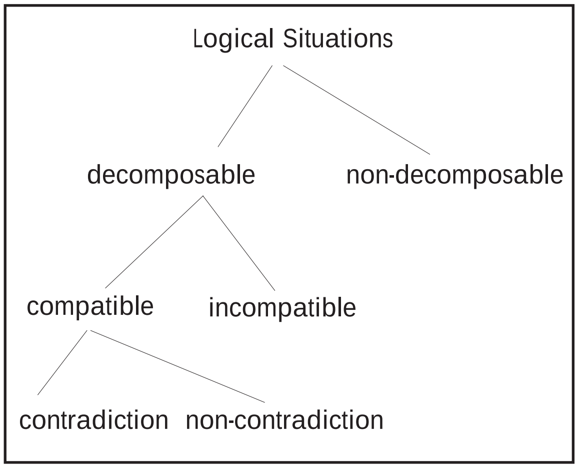
The internal logical structure of a class of implications is made transparent.

Micro-analysis of trans-equivalence

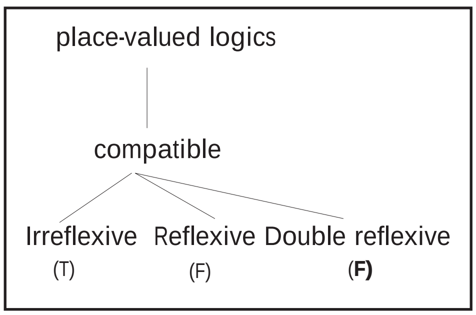


Truth-value matrices are not making clear the intricate *logical* structure of transjunctions and trans-equivalences. This is given, naturally, by conjugation diagrams.

Tree of structural situations



Because mediation is not presupposed in polycontextural logics, in contrast to place-valued logics, it is not guaranteed that value constellations are compatible and delivering a harmonious mediation.



In place-valued logics, values are always building a *harmonic* constellation based on their compatibility.
This is possible because the mechanism of mediation is presupposed and has not be realized by the place-valued system itself.
That is, all value constellations are accepted by the logical functions.
For n=1,2, the system is semantically closed.
Place-valued logical systems can be considered as logics where the sentences or proposition are not only having a truth-value but also an index of the place from where they are stated. Such a place, point of view, can be seen as an index of the *relevance* of the statement which then can be true or false.

Thus, place-valued logical propositions have a double characteristics, as truth-values and as index of relevance. But this is not realized in Gunther's notations.

Thus, we can say, in place-valued logics the interaction between points of view, marking the relevance of a statement, are harmonized in advance.

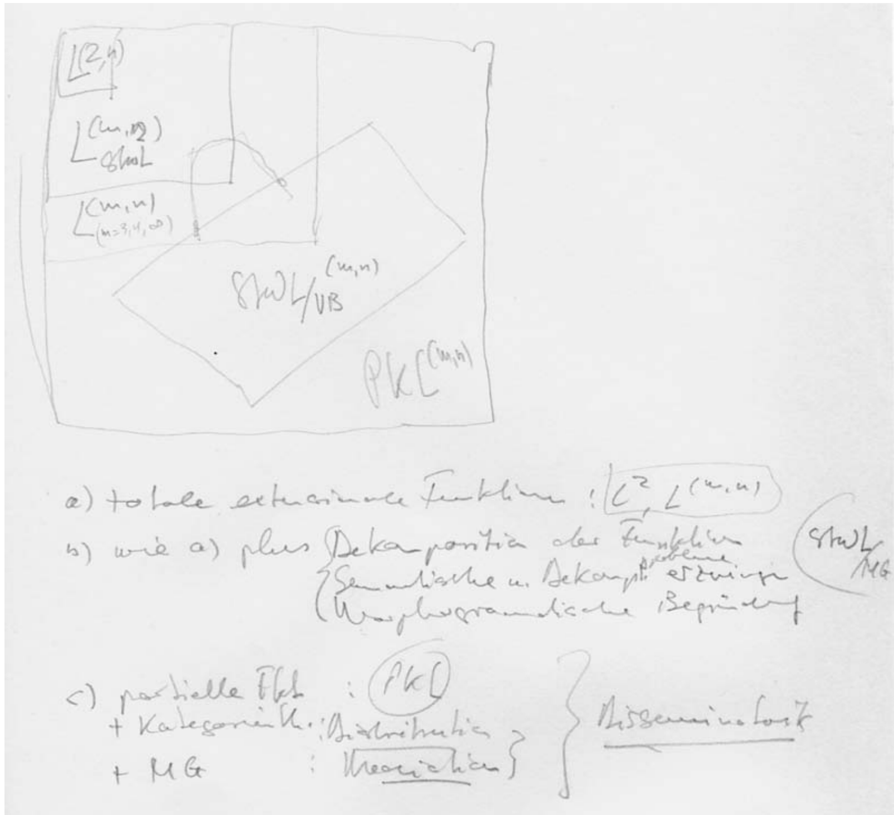
In contrast, in polycontextural logics the mediation of such points of view of relevance have to be established inside the formal system. Obviously, some constellations are full-filling the conditions of mediation and are therefore harmonized. But others are not. This is opening up considerations beyond the analogs of logical semantics. That is, we can ask of the degrees of successful or unsuccessful mediations. About the distance of non-mediated sub-systems.

The following definitions, constructions and formulas are strictly reduced to a place-valued logical system with only 2 variables and 3 truth-values. Generalizations to place-valued logics with more than 3 values are naturally possible.

Thus only a tiny fragment of place-valued logics is developed. Nevertheless, interesting topics can be studied which are preparing to understand genuine polycontextural logics.

It seems, that the *living tissue* is not a closed harmonized semantic space.

Expanding classic logic?



Which kind of place-valued logic formulas are valid in a polycontextural logic?

There are sorts of formulas which have coincidences in place-valued logics and poly-contextural logics. At least there is a one-to-one translation for them from one logic the other. After having introduced a solution for general n-ary functions this overlapping holds for n-ary formulas, too.

Harmonic formulas, which are not disturbing the mediation of their parts, can easily be translated from one to the other logic type. Mostly, distribution is produced by negations and transjunctions. Asymmetric use of negations is producing more or less always a distortion of the harmony of the formula. That is, the conditions of mediation are violated by the production of incompatible situations.

With the translation from poly-contextural to place-valued logics, important informations get lost. Everything about local configurations is lost in the global presentation.

It is not the place to give any proof about the described situation. The descriptions are based on experiences with the development of formulas. It describes too, the way out of the cage of global total functions to polycontextural logics.

Transjunctions are not representable by junctions in polylogic

A proof could easily be constructed with the help of the rules, listed below. It should be easy to proof that there is now way to define transjunctive formulas with the help of the distribution laws for disjunction and conjunction for polylogic, only.

That is, term developments in the conjunctive-disjunctive domain are not able to leave the domain. Such a domain is closed under the term reduction rules for disjunction and conjunction.

3.8 Term rules for junction and transjunctions

The term calculus for poly-contextural logics was first introduced in the work "Tableaux Beweiser" by Bashford/Kaehr 1992 as a first attempt to deal with the question of meta-rules.

http://www.thinkartlab.com/pkl/lola/VERSIONT/

Term Rules

$$R_0: \quad \frac{t_1 \; et \; (t_2 \; or \; t_3)}{(t_1 \; et \; t_2) \; or \; (t_1 \; et \; t_3)}$$

$$\frac{(t_1 \; or \; t_2) \; et \; t_3}{(t_1 \; et \; t_3) \; or \; (t_2 \; et \; t_3)}$$

$$R1: \quad \frac{(t \; simul \; ta) \odot (t' \; simul \; t'a)}{(t \odot t') \; simul \; (ta \odot t'a)}$$

$$R2: \quad \frac{t \; et \; (t' \; simul \; t'a)}{(t \; et \; t') \; simul \; ta}$$

$$\frac{(t \; simul \; ta) \; et \; t'}{(t \; et \; t') \; simul \; ta}$$

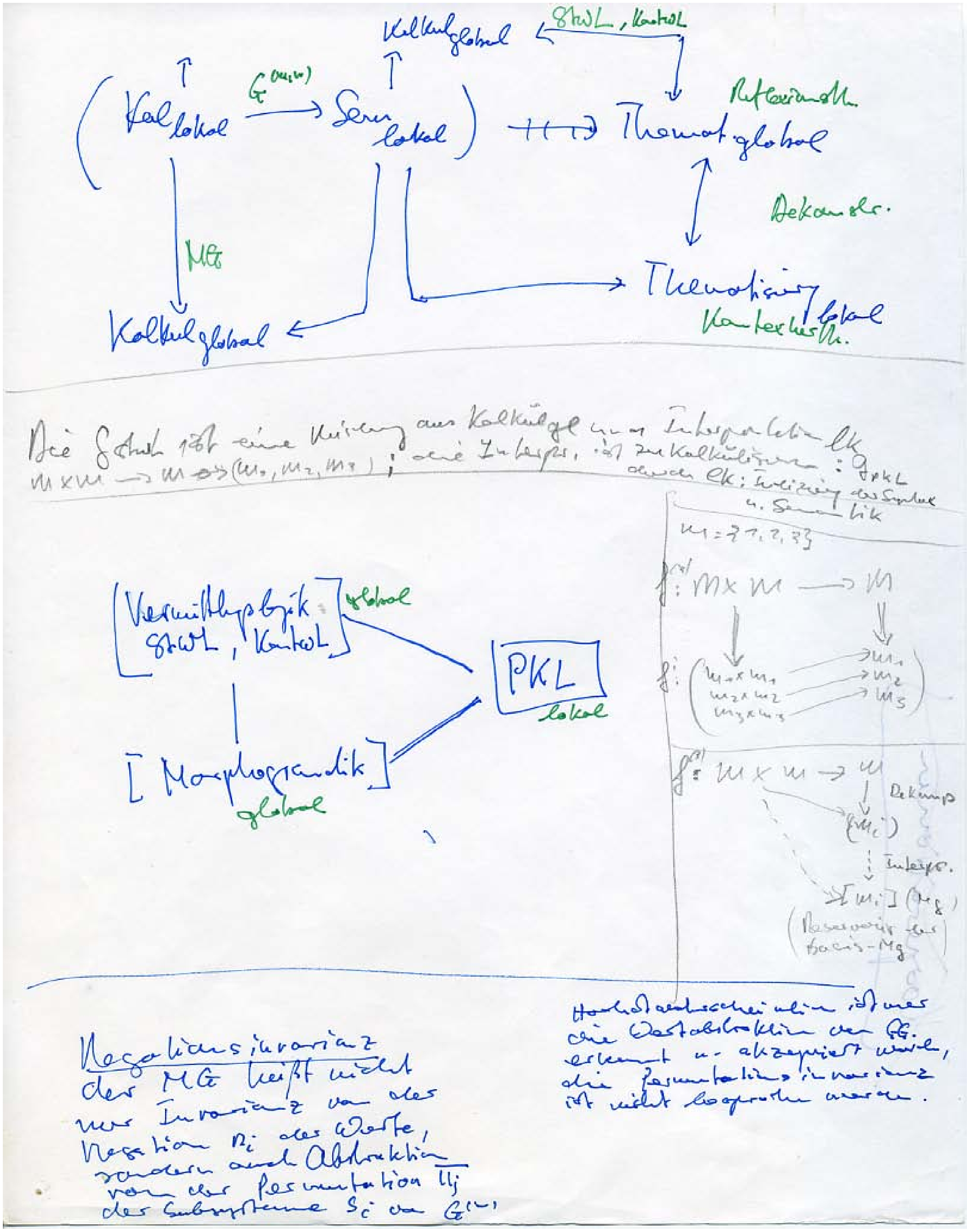
$$R3: \quad \frac{(\{t\} \; simul \; ta) \; or \; (\{t'\} \; simul \; ta')}{(t \; or \; t') \; simul \; (ta \; or \; t'a)}$$

$$R4: \quad \frac{\{t\} \; or \; (\{t'\} \; simul \; t'a)}{(t \; or \; t') \; simul \; t'a}$$

$$\frac{(\{t\} \; simul \; ta) \; or \; \{t'\}}{(t \; or \; t') \; simul \; ta}$$

$$R5: \quad \frac{(t \; simul \; ta) \; simul \; t'a}{t \; simul \; (ta \; et \; t'a)}$$

Gunther's place-valued logic is a mix of a global calculus and a local interpretation



3.9 Logics disseminated over a polycontextural matrix

After having introduced a stringent mechanism to give sub-systems an index, we have to take this information into account and represent it on a matrix adequate to the described structure. This is a transition from a linear placed-valued system to a polycontextural system and further to a tabular polycontextural system. A tabular polycontextural system is a system of dissemination of formal systems over a matrix. From here, there are "natural" steps to go to n-categorial polycontextural systems.

$$\frac{t_1\; X \;\oplus\wedge\wedge\; Y}{\begin{array}{c} t_1\; X \\ t_1\; Y \end{array}} \qquad \frac{f_1\; X \;\oplus\wedge\wedge\; Y}{\begin{array}{c} f_1\; X \\ f_1\; Y \end{array}}$$

$$\frac{t_2\; X \;\oplus\wedge\wedge\; Y}{\begin{array}{c|c} t_2\; X & f_1\; X \\ t_2\; Y & f_1\; Y \end{array}} \qquad \frac{f_2\; X \;\oplus\wedge\wedge\; Y}{f_2\; X \mid f_2\; Y \;\Bigg\|\; \begin{array}{c|c} f_1\; X & t_1\; X \\ t_1\; Y & f_1\; Y \end{array}}$$

$$\frac{t_3\; X \;\oplus\wedge\wedge\; Y}{\begin{array}{c||c} t_3\; X & t_1\; X \\ t_3\; Y & t_1\; Y \end{array}} \qquad \frac{f_3\; X \;\oplus\wedge\wedge\; Y}{f_3\; X \mid f_3\; Y \;\Bigg\|\; \begin{array}{c|c} f_1\; X & t_1\; X \\ t_1\; Y & f_1\; Y \end{array}}$$

PM	$O1$	$O2$	$O3$
$M1$	$log1$	$log1$	$log1$
$M2$	$\varnothing$	$log2$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$log3$

PM	$O1$	$O2$	$O3$
$M1$	$trans$	$trans$	$trans$
$M2$	$\varnothing$	and	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	and

$$(\oplus\wedge\wedge) : L^{(3)} \;*\; L^{(3)} \longrightarrow L^{(3)} : \left[L_1,\, (L_2 \,||\, L_1), \left(L_3 \,||\, L_1\right)\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 \;*\; L_1 \xrightarrow[transjunct\ <>]{} L_1 : \begin{cases} f_1 \;*\; t_1,\; t_1 \;*\; f_1 \;\to\; f_2,\; f_3 \\ t_1 \;*\; t_1 \;\to\; t_1,\; t_3 \\ f_1 * f_1 \;\to\; f_1,\; t_2 \end{cases} \\ Log_2 : L_2 \;*\; L_2 \xrightarrow[conjunction]{} L_2 \;||\; L_1 \\ Log_3 : L_3 \;*\; L_3 \xrightarrow[conjunction]{} L_3 \,||\, L_1 \end{array}\right.$$

PolyLogics. Towards a formalization of polycontextural Logics.
http://www.thinkartlab.com/pkl/lola/PolyLogics.pdf

Decomposition problems in logical systems, elsewhere

Very different decomposition techniques had been developed by Jon Muzio:
http://www.cs.uvic.ca/~jmuzio/publications75-81.html

Recently, Jochen Pfalzgraf has published new techniques based on a fibre bundle interpretation of polycontextural logics.
http://www.rac.es/ficheros/doc/00158.pdf
http://www.cosy.sbg.ac.at/~jpfalz/publications.html

4 Sketch of a place-valued logical system $G^{(3,2)}$

Place-valued logic $G^{(3,2)}_{neg, impl}$:

A. (Some) Definitions

1. Monoform junctions and transjunctions in negation plus implication

$p \vee\vee\vee q := \left(p \supset^1\supset^1\supset^3 q\right) \supset^1\supset^1\supset^3 q$: *disjunction*

; *short* : $\supset^1\supset^1\supset^3 = \supset\supset\supset$

$p \wedge\wedge\wedge q := N_5\left(N_5 p \vee\vee\vee N_5 q\right)$: *conjunction*

$p \odot\odot\odot q := \left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_2 \left(q \supset\supset\supset p\right)$: *trans – equivalence*

$p \oplus\oplus\oplus q := \left(p \odot\odot\odot q\right) \odot\odot\odot p$: *transjunction*

$p \equiv\equiv\equiv q := \left(p \supset\supset\supset q\right) \wedge\wedge\wedge \left(q \supset\supset\supset p\right)$: *disj. equivalence*

$p \rightarrow\rightarrow\rightarrow q := N_1\left(N_1\left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_3\left(p \supset\supset\supset q\right)\right)$: *conj. implication*

$p \equiv\equiv\equiv q := \left(p \rightarrow\rightarrow\rightarrow q\right) \wedge\wedge\wedge \left(q \rightarrow\rightarrow\rightarrow p\right)$: *conj. equivalence*

$p \otimes\otimes\otimes q := \left(p \vee\vee\vee q\right) \oplus\oplus\oplus p$: *repl. – trans. – disjunction*

DeMorgan for conjunctions / disjunctions

$p \vee\wedge\wedge q := N_1\left(N_1 p \vee\vee\vee N_1 q\right)$

$p \wedge\vee\wedge q := N_2\left(N_2 p \vee\vee\vee N_2 q\right)$

$p \wedge\vee\vee q := N_4\left(N_3 p \vee\vee\vee N_3 q\right)$

$p \vee\wedge\vee q := N_3\left(N_4 p \vee\vee\vee N_4 q\right)$

$p \wedge\wedge\wedge q := N_5\left(N_5 p \vee\vee\vee N_5 q\right)$

$p \wedge\wedge\vee q := \left(p \wedge\vee\vee q\right) \wedge\wedge\wedge \left(p \vee\wedge\vee q\right)$

$p \vee\vee\wedge q := \left(p \vee\wedge\wedge q\right) \vee\vee\vee \left(p \wedge\vee\wedge q\right)$

2. Partial transjunctions

$p \wedge\oplus\oplus q := \left(p \wedge\wedge\wedge q\right) \vee\vee\vee \left(N_1 p \wedge\wedge\wedge N_5 q\right) \vee\vee\vee \left(N_5 p \wedge\wedge\wedge N_1 q\right)$

$p \wedge\vee\oplus q := \left(p \vee\vee\vee q\right) \wedge\wedge\wedge \left(N_1 p \vee\vee\vee q\right) \wedge\wedge\wedge \left(p \vee\vee\vee N_1 q\right)$

$p \vee\wedge\oplus q := \left(p \wedge\wedge\wedge q\right) \vee\vee\vee \left(N_2 p \wedge\wedge\wedge q\right) \vee\vee\vee \left(p \vee\vee\vee N_2 q\right)$

3. Monoform transjunction in negation plus implication and conjunction, only.

$$p \oplus\oplus\oplus q := \left\langle \begin{matrix} \left[\left(\left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_2 \left(q \supset\supset\supset p\right)\right) \supset\supset\supset p\right] \\ \wedge\wedge\wedge \\ N_2\left[p \supset\supset\supset \left(\left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_2 \left(q \supset\supset\supset p\right)\right)\right] \end{matrix} \right\rangle$$

Definitions for Negations

$N_3\ X^{(3)} := N_1\left(N_2\ X^{(3)}\right)$

$N_4\ X^{(3)} := N_2\ (\ N_1\ X^{(3)}\ )$

$N_5\ X^{(3)} := N_1\left(N_2\left(N_1\ X^{(3)}\right)\right) := N_2\left(N_1\left(N_2\ X^{(3)}\right)\right)$

3. Monoform transjunction in negation plus implication and conjunction, only.

$$p \oplus\oplus\oplus q \coloneqq \left\langle \begin{array}{l} \left[\left(\left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_2 \left(q \supset\supset\supset p\right)\right) \supset\supset\supset p\right] \\ \wedge\wedge\wedge \\ N_2 \left[p \supset\supset\supset \left(\left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_2 \left(q \supset\supset\supset p\right)\right)\right] \end{array} \right\rangle$$

4. Monoform transjunction in negation plus implication, conjunction and disjunction, only.

$$p \oplus\oplus\oplus q \coloneqq \left\langle \begin{array}{l} N_5 \left[\left(N_5 \left(p \supset\supset\supset q\right) \vee\vee\vee N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right) \supset\supset\supset p\right] \\ \wedge\wedge\wedge \\ N_2 \left[N_5 \left(N_5 \left(p \supset\supset\supset q\right) \vee\vee\vee N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right)\right] \end{array} \right\rangle$$

5. Monoform transjunction in negation plus implication and disjunction, only.

$$p \oplus\oplus\oplus q \coloneqq N_5 \left\langle \begin{array}{l} N_5 \left\langle N_5 \left[\left(N_5 \left(p \supset\supset\supset q\right) \vee\vee\vee N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right) \supset\supset\supset p\right]\right\rangle \\ \vee\vee\vee \\ N_5 \left\langle N_2 \left[N_5 \left(N_5 \left(p \supset\supset\supset q\right) \vee\vee\vee N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right)\right]\right\rangle \end{array} \right\rangle$$

6. Monoform transjunction in negation plus implication and disjunction, final step.

$$p \oplus\oplus\oplus q \coloneqq N_5 \left\langle \begin{array}{l} \left\langle \begin{array}{l} \left\langle N_5 \left\langle N_5 \left[\left(\left(N_5 \left(p \supset\supset\supset q\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right) \supset\supset\supset p\right]\right\rangle\right\rangle \\ \supset\supset\supset \\ \left\langle N_5 \left\langle N_2 \left[N_5 \left(N_5 \left(p \supset\supset\supset q\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right)\right]\right\rangle\right\rangle \end{array} \right\rangle \\ \supset\supset\supset \\ \left\langle N_5 \left\langle N_2 \left[N_5 \left(N_5 \left(p \supset\supset\supset q\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right)\right]\right\rangle\right\rangle \end{array} \right\rangle$$

7. Monoform transjunction in negation plus implication and disjunction, bracket cascads.

$$p \oplus\oplus\oplus q \coloneqq N_5 \left\langle \begin{array}{l} \left\langle \begin{array}{l} \left\langle N_5 \left\langle N_5 \left[\left(\begin{array}{l} \left(N_5 \left(p \supset\supset\supset q\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right)\right) \\ \supset\supset\supset \\ N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right) \end{array}\right) \supset\supset\supset p\right]\right\rangle\right\rangle \\ \supset\supset\supset \\ \left\langle N_5 \left\langle N_2 \left[N_5 \left(\begin{array}{l} N_5 \left(p \supset\supset\supset q\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right) \\ \supset\supset\supset \\ N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right) \end{array}\right)\right]\right\rangle\right\rangle \end{array} \right\rangle \\ \supset\supset\supset \\ \left\langle N_5 \left\langle N_2 \left[N_5 \left(\begin{array}{l} N_5 \left(p \supset\supset\supset q\right) \supset\supset\supset N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right) \\ \supset\supset\supset \\ N_5 \left(N_2 \left(q \supset\supset\supset p\right)\right) \end{array}\right)\right]\right\rangle\right\rangle \end{array} \right\rangle$$

Definition of mono-form transjunction by implication and negation only.

4.1 Visual training

Again, monoform transjunction in negation plus implication and conjunction, only.

$$p \oplus\oplus\oplus q := \left\langle \begin{array}{l} \left[\left(\left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_2 \left(q \supset\supset\supset p\right)\right) \supset\supset\supset p\right] \\ \wedge\wedge\wedge \\ N_2\left[p \supset\supset\supset \left(\left(p \supset\supset\supset q\right) \wedge\wedge\wedge N_2 \left(q \supset\supset\supset p\right)\right)\right] \end{array} \right\rangle$$

Different visualization seem to offer a better insight into the structure of the formula

$$p \oplus\oplus\oplus q := \left\langle \begin{array}{l} \left[\begin{pmatrix} \left(p \supset\supset\supset q\right) \\ \wedge\wedge\wedge \\ N_2\left(q \supset\supset\supset p\right) \end{pmatrix} \supset\supset\supset p\right] \\ \wedge\wedge\wedge \\ N_2\left[p \supset\supset\supset \begin{pmatrix} \left(p \supset\supset\supset q\right) \\ \wedge\wedge\wedge \\ N_2\left(q \supset\supset\supset p\right) \end{pmatrix}\right] \end{array} \right\rangle$$

than the more linear version. For reasons of space, the strict linear presentation has to be omitted. The bracket game can be played one round further.

$$p \oplus\oplus\oplus q := \left\langle \begin{array}{l} \left[\begin{array}{l} \begin{pmatrix} \left(p \supset\supset\supset q\right) \\ \wedge\wedge\wedge \\ N_2\left(q \supset\supset\supset p\right) \end{pmatrix} \\ \supset\supset\supset p \end{array}\right] \\ \wedge\wedge\wedge \\ N_2\left[\begin{array}{l} p \supset\supset\supset \\ \begin{pmatrix} \left(p \supset\supset\supset q\right) \\ \wedge\wedge\wedge \\ N_2\left(q \supset\supset\supset p\right) \end{pmatrix} \end{array}\right] \end{array} \right\rangle \quad \text{or} \quad \left\langle \begin{array}{l} \left[\begin{array}{l} \begin{pmatrix} \left(p \supset\supset\supset q\right) \\ \wedge\wedge\wedge \\ N_2\left(q \supset\supset\supset p\right) \end{pmatrix} \\ \supset\supset\supset p \end{array}\right] \\ \wedge\wedge\wedge \\ N_2\left[\begin{array}{l} p \supset\supset\supset \\ \begin{pmatrix} N_2\left(q \supset\supset\supset p\right) \\ \wedge\wedge\wedge \\ \left(p \supset\supset\supset q\right) \end{pmatrix} \end{array}\right] \end{array} \right\rangle$$

$$\textit{structure short}: \; p \oplus\oplus\oplus q \equiv \left\langle\left[(X) \supset\supset\supset p\right] \wedge\wedge\wedge N_2\left[p \supset\supset\supset (X)\right]\right\rangle$$

The different presentations of the same formula are making quite clear the transformation of asymmetric implications into a symmetric result as a monoform transjunction.

Such games of presentation are of importance to learn how to deal with highly complicated formulas in a general setting. After some training, it is not only reasonable but necessary to delegate the job of dealing with complex and complicated formulas to a theorem prover, like LOLA.

LOLA is a semi-automatic theorem prover which is delivering proof trees and term developments for proofs and is written in ML. LOLA can flexibly be configured for different logical systems.

http://www.thinkartlab.com/pkl/lola/LOLA.pdf

4.2 Tableaux rules for the place-valued logic $G^{(3,\,2)}$

Essential tableaux rules for negations and implicational orders for $PVL^{(3)}$.

Place – valued semantics of implication $(\supset^1\supset^1\supset^3)$

$p \supset^1\supset^1\supset^3 q$

$p \setminus q$	1	2	3
1	1	2	3
2	1	1	2
3	1	1	1

Place – valued semantics of implication $(\supset^1\supset^3\supset^3)$

$p \supset^1\supset^3\supset^3 q$

$p \setminus q$	1	2	3
1	1	2	3
2	1	1	3
3	1	1	1

of negations N_1, N_2

p	N_1	N_2
1	2	1
2	1	3
3	3	2

Tableaux rules for signed formulas

Tableaux rules for monoform implication$^{(1,\,1,\,3)}$

$$\frac{T\; X \supset^1\supset^1\supset^3 Y}{T\,X \;\Big|\; F\,Y \;\Big|\; \begin{matrix} F\,X \\ F\,Y \end{matrix}} \qquad \frac{F\; X \supset^1\supset^1\supset^3 Y}{\begin{matrix} T\,X \\ F\,Y \end{matrix} \;\Big|\; \begin{matrix} F\,X \\ \mathsf{F}\,Y \end{matrix}} \qquad \frac{\mathsf{F}\; X \supset^1\supset^1\supset^3 Y}{\begin{matrix} T\,X \\ \mathsf{F}\,Y \end{matrix}}$$

Tableaux rules for monoform implication$^{(1,\,3,\,3)}$

$$\frac{T\; X \supset^1\supset^3\supset^3 Y}{T\,X \;\Big|\; F\,Y \;\Big|\; \begin{matrix} F\,X \\ F\,Y \end{matrix}} \qquad \frac{F\; X \supset^1\supset^3\supset^3 Y}{\begin{matrix} T\,X \\ F\,Y \end{matrix}} \qquad \frac{\mathsf{F}\; X \supset^1\supset^3\supset^3 Y}{\begin{matrix} F\,X \\ \mathsf{F}\,Y \end{matrix} \;\Big|\; \begin{matrix} T\,X \\ \mathsf{F}\,Y \end{matrix}}$$

It should be mentioned, again, that with a proper distribution of the basic logical functions over their places there is no systematic need to distinguish between, say, disjunctive implications and conjunctive ones. It may use as a notational abbreviation without any strict logical consequences. Thus, inn this setting, there is only one disjunction but distributed over different structural places. This wording holds for both, the place-valued and the polycontextural approach to polylogical systems.

Tableaux for negations

$$\frac{T\,(\neg_1\, X^{(3)})}{F\, X^{(3)}} \qquad \frac{F\,(\neg_1\, X^{(3)})}{T\, X^{(3)}} \qquad \frac{\mathsf{F}\,(\neg_1\, X^{(3)})}{\mathsf{F}\, X^{(3)}}$$

$$\frac{T\,(\neg_2\, X^{(3)})}{T\, X^{(3)}} \qquad \frac{F\,(\neg_2\, X^{(3)})}{\mathsf{F}\, X^{(3)}} \qquad \frac{\mathsf{F}\,(\neg_2\, X^{(3)})}{F\, X^{(3)}}$$

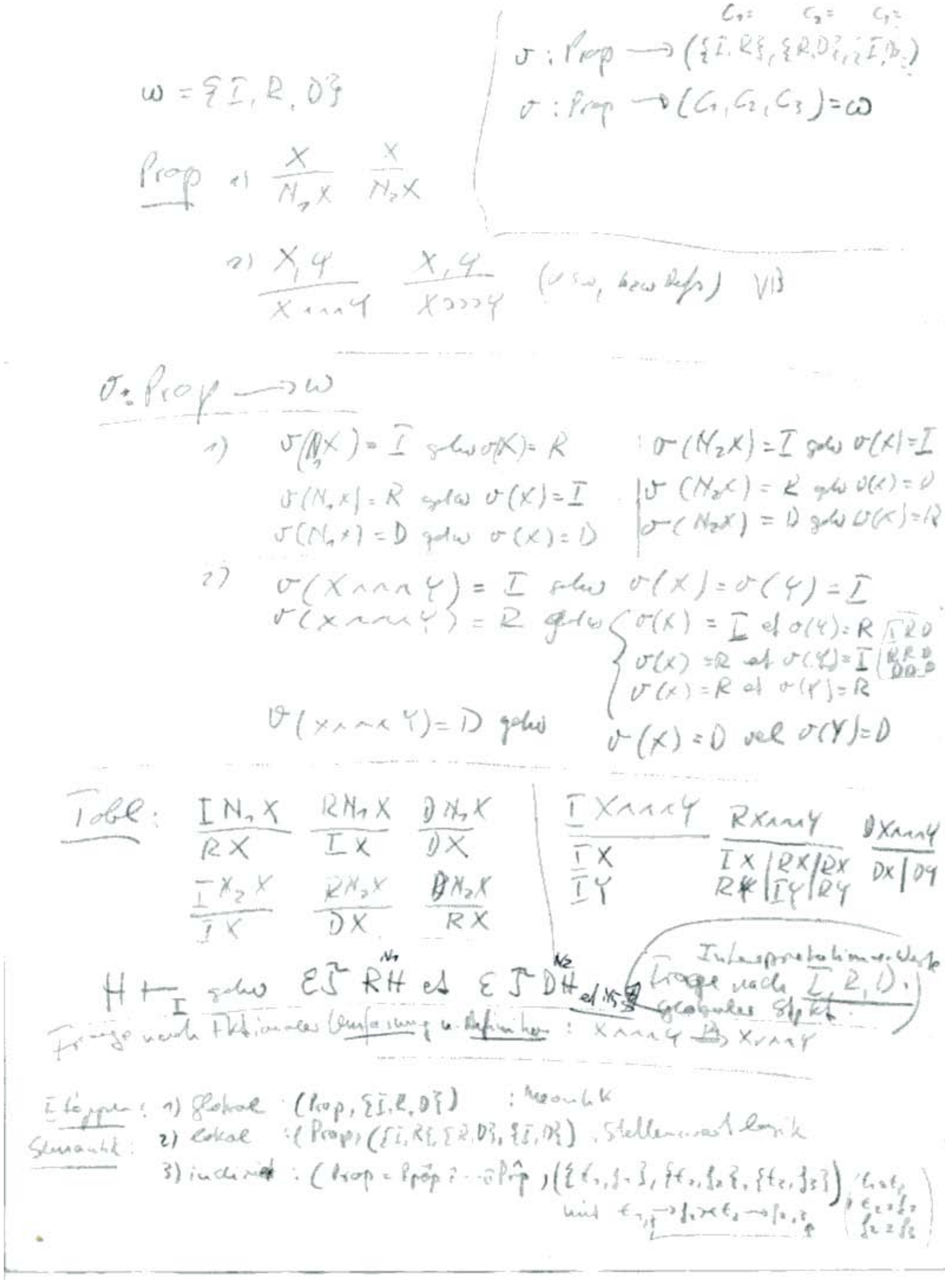
$$\neg_3\, X^{(3)} := \neg_1\,(\neg_2\, X^{(3)})$$
$$\neg_4\, X^{(3)} := \neg_2\,(\neg_1\, X^{(3)})$$
$$\neg_5\, X^{(3)} := \neg_1\,(\neg_2\,(\neg_1\, X^{(3)})) := \neg_2\,(\neg_1\,(\neg_2\, X^{(3)}))$$

Tableaux for conjunction and transjunction with conjunction

$$\frac{T\; X \wedge\wedge\wedge Y}{\begin{matrix} T\,X \\ T\,Y \end{matrix}} \qquad \frac{F\; X \wedge\wedge\wedge Y}{\begin{matrix} F\,X \\ T\,Y \end{matrix} \;\Big|\; \begin{matrix} T\,X \\ F\,Y \end{matrix} \;\Big|\; \begin{matrix} F\,X \\ F\,Y \end{matrix}} \qquad \frac{\mathsf{F}\; X \wedge\wedge\wedge Y}{\mathsf{F}\,X \;|\; \mathsf{F}\,Y}$$

$$\frac{T\; X \oplus\wedge\wedge Y}{\begin{matrix} T\,X \\ T\,Y \end{matrix}} \qquad \frac{F\; X \oplus\wedge\wedge Y}{\begin{matrix} F\,X \\ F\,Y \end{matrix}} \qquad \frac{\mathsf{F}\; X \oplus\wedge\wedge Y}{\begin{matrix} F\,X \\ T\,Y \end{matrix} \;\Big|\; \begin{matrix} T\,X \\ F\,Y \end{matrix} \;\Big|\; \mathsf{F}\,X \;|\; \mathsf{F}\,Y}$$

Semantics of Gunther's (I, R, D)-logic


$\omega = \{I, R, D\}$

$\sigma : Prop \to (\{I, R\}, \{R, D\}, \{I, D\})$

$\sigma : Prop \to (C_1, C_2, C_3) = \omega$

Prop 1) $\frac{X}{N_1 X}$ $\frac{X}{N_2 X}$

2) $\frac{X, Y}{X \wedge\wedge\wedge Y}$ $\frac{X, Y}{X \vee\vee\vee Y}$ (usw, bzw Defs) VB

$\sigma : Prop \to \omega$

1) $\sigma(N_1 X) = I$ gdw $\sigma(X) = R$ | $\sigma(N_2 X) = I$ gdw $\sigma(X) = I$

$\sigma(N_1 X) = R$ gdw $\sigma(X) = I$ | $\sigma(N_2 X) = R$ gdw $\sigma(X) = D$

$\sigma(N_1 X) = D$ gdw $\sigma(X) = D$ | $\sigma(N_2 X) = D$ gdw $\sigma(X) = R$

2) $\sigma(X \wedge\wedge\wedge Y) = I$ gdw $\sigma(X) = \sigma(Y) = I$

$\sigma(X \wedge\wedge\wedge Y) = R$ gdw $\sigma(X) = I$ et $\sigma(Y) = R$; $\sigma(X) = R$ et $\sigma(Y) = I$; $\sigma(X) = R$ et $\sigma(Y) = R$

$\sigma(X \wedge\wedge\wedge Y) = D$ gdw $\sigma(X) = D$ vel $\sigma(Y) = D$

Tabl: $\frac{I N_1 X}{R X}$ $\frac{R N_1 X}{I X}$ $\frac{D N_1 X}{D X}$ | $\frac{I X \wedge\wedge\wedge Y}{I X, I Y}$ $\frac{R X \wedge\wedge\wedge Y}{I X, R Y \mid R X, I Y \mid R X, R Y}$ $\frac{D X \wedge\wedge\wedge Y}{D X \mid D Y}$

$\frac{I N_2 X}{I X}$ $\frac{R N_2 X}{D X}$ $\frac{D N_2 X}{R X}$

$H \vdash_I$ gdw $\varepsilon \mathfrak{I}^{N_1} R H$ et $\varepsilon \mathfrak{I}^{N_2} D H$ ...

(Interpretationsweise: Frage nach I, R, D. globales Objekt)

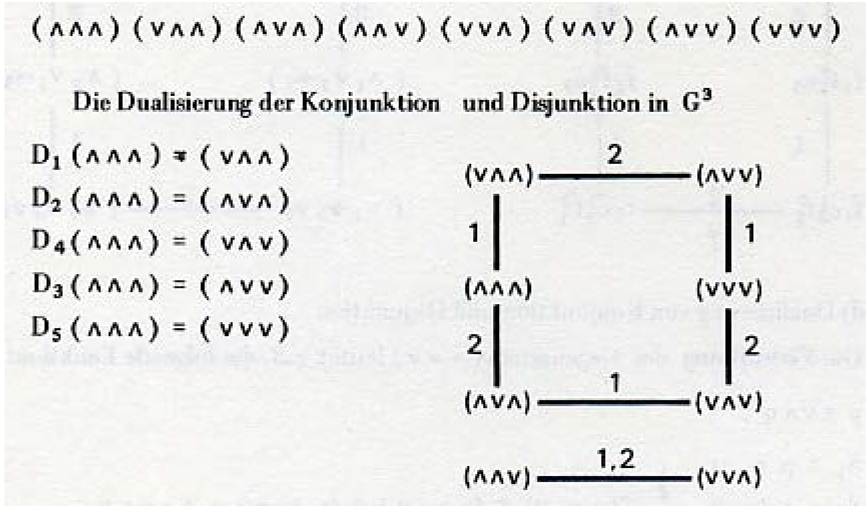
Frage nach Funktionaler Umfassung u. Definition: $X \wedge\wedge\wedge Y$ ... $X \vee\vee\vee Y$

Etappen: 1) global (Prop, {I, R, D}) : monolithisch

Semantik: 2) lokal : (Prop, ({I, R}, {R, D}, {I, D})) Stellenwertlogik

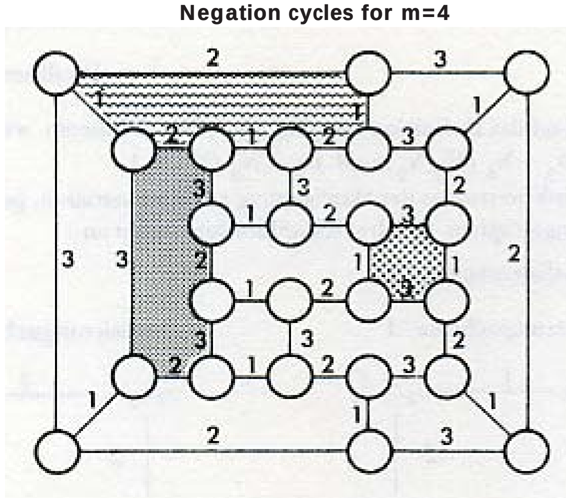
3) indiziert : (Prop = $Prop^1 \cdots Prop^3$, ({t1, f1}, {t2, f2}, {t3, f3})) mit $t_1 \to f_1$, ...

Dualization of 3-valued mono-form disjunction and conjunction



These are the abbreviations of the formulas Gunther was using to demonstrate his place-valued logic. Not only there is a nice system of transitive compounds of junctions but also a separation of the dualization system into transitive and intransitive junctions. This approach is easily extended to more complex systems with m≥4.

Negation cycles for m=4



Negation cycles are basic for the study of complex dualization systems.

4.3 A list of hand-made place-valued logical formulas (from 1973-75)

The reader might enjoy to check those 3-valued place-valued logical formulas, hopefully with the help of a slightly adapted classical theorem prover.

These formulas are formulas *"on demand"* by the request of a professor for analytical philosophy at one of the universities in West-Berlin who missed some amount of formulas in the original paper. The structural informations didn't satisfy his academic desires. Probably, I produced them over night by hand. They are hand-proven and hand-set. Nevertheless, they are not only a scientific joke but of some ingenuity, too. But, they are as they are. No guarantee included!

Main definitions, again, for notational adoption

Definitionen monoformer Funktionen

p ∨∨∨ q := (p ⊃⊃⊃ q) ⊃⊃⊃ q	Disjunktion
p ∧∧∧ q := N_5(N_5p ∨∨∨ N_5q)	Konjunktion
p ⌶⌶⌶ q := (p ⊃⊃⊃ q) ∧∧∧ N_2(q ⊃⊃⊃ p)	Transäquivalenz
p ⋈⋈⋈ q := (p ⌶⌶⌶ q) ⌶⌶⌶ p	Transjunktion
p ○○○ q := (p ∨∨∨ q) ⋈⋈⋈ p	repl.–transj.–Disjunktion
p ⊕⊕⊕ q := q ○○○ p	impl.–trans.–Disjunktion
p ☆☆☆ q := N_5(N_5p ○○○ N_5q)	impl.–trans.–Konjunktion
p □□□ q := q ☆☆☆ p	repl.–trans.–Konjunktion
p === q := (p ⊃⊃⊃ q) ∧∧∧ (q ⊃⊃⊃ p)	disj. Äquivalenz
p →→→ q := N_1(N_1(p ⊃⊃⊃ q) ∧∧∧ N_3(p ⊃⊃⊃ q)))	konj. Implikation
p ≡≡≡ q := (p →→→ q) ∧∧∧ (q →→→ p)	konj. Äquivalenz

Some amazing formulas for implications

Gesetze der Implikation

Reflexivität

⇒p ⊃⊃⊃ p ; ⇒p ⊃⊃→ p ; ⇒p →→⊃ p ; ⇒p →→→ p

Absorptionsgesetze

p ⊃⊃⊃ (p ⊃⊃⊃ q) ⇔ p ⊃⊤⊃ q

(p ⊃⊃⊃ q) ⊃⊃⊃ q ⇔ p ┘☆┘ q

⇒q ⊃⊃⊃ (p ⊃⊃⊃ q)

p →→→ (p →→→ q) ⇔ p →→→ q

(p →→→ q) →→→ p ⇔ p (Peirce)

⇒p →→→ (p →→→ q)

p →→→ q ⇒ p ⊃⊃⊃ q

Paradoxien der Implikation

p ⇒ q ⊃⊃⊃ p

N_5p ⇒ p ⊃⊃⊃ q

p ⇒ q →→→ p

N_5p ⇒ p →→→ q

N_1p ⊃⊃⊃ (p ⊃⊃⊃ q) ⇔ p⊤⊃⊃ q

N_1p ⊃⊃⊃ (p ⊃⊃⊃ q) ⇔ p ⊃⊃⊃ (N_1p ⊃⊃⊃ q)

N_2p ⊃⊃⊃ (p ⊃⊃⊃ q) ⇔ p ⊃⊃⊃ (N_2p ⊃⊃⊃ q)

Kontextlogische Elimination der Paradoxien

p ; q ⇒ q ⊃⊃⊃ p

N_5p ; q ⇒ p ⊃⊃⊃ q

p ; q ⇒ q →→→ p

N_5p ; q ⇒ p →→→ q

As we see, it is possible to define transjunctions with the help of junctions. But it will not be possible to define junctions out of transjunctions because they are self-dual. In other words, a system like [N_1, N_2, (&&&)] is functionally complete for a place-valued logic with m=3, n=2.

Some people need formulas, not structures, here they are

Fundamentale Theoreme der drei–kontexturalen Aussagenlogik

Gesetze der Negation	Gesetze der Modaloperatoren
$N_i N_i p \Leftrightarrow p$, $i = 1,2,5$	$L_1\, p \supset\supset \supset p \supset\supset\supset M_1 p$
$N_3 N_4 p \Leftrightarrow N_4 N_3\, p$	$L_2\, p \supset\supset\supset p \supset\supset\supset M_2\, p$
$N_1 N_2 N_1 p \Leftrightarrow N_2 N_1 N_2 p$	$L_3\, p \supset\supset\supset p \supset\supset\supset M_3\, p$
$N_{1\cdot 2\cdot 1\cdot 2\cdot 1\cdot 2}\, p \Leftrightarrow p$	$L_1\, p \supset\supset\supset M_2 p$; $M_2 p \supset\supset\supset M_1 p$
$N_{2\cdot 1\cdot 2\cdot 1\cdot 2\cdot 1}\, p \Leftrightarrow p$	$M_2 p \supset\supset\supset M_3 p$; $L_1\, p \supset\supset\supset N_2\, p$
$N_i N_i N_i\, p \Leftrightarrow N_i\, p$, $i = 1, 2, 5$	$L_1 L_1 p \Leftrightarrow L_1 p$; $M_1 M_1 p \Leftrightarrow M_1 p$

$M_1 L_1 p \Leftrightarrow L_1\, p$; $L_1 M_1 p \Leftrightarrow M_1 p$; $M_1 M_3 p \Leftrightarrow M_3 M_1 p$; $L_1 L_2 p \Leftrightarrow L_2 L_1 p$

$L_2 L_3 p \Leftrightarrow L_3 L_2 p$; $L_1 L_2 p \Leftrightarrow L_2 L_3 p$; $M_1 N_2 p \Leftrightarrow M_1 N_4 p$; $N_1 M_1 N_1 p \Leftrightarrow N_1 M_1 p$

$M_2 p \Leftrightarrow N_2\, L_1 p$; $L_1 p \Leftrightarrow N_1 L_2 N_4 p$; $L_1 p \Leftrightarrow N_5\, M_1\, N_5 p$; $N_2 M_2 p \Leftrightarrow L_1 p$

Gesetze der Konjunktion und der Disjunktion

Idempotenz

$p \wedge\wedge\wedge p \Leftrightarrow p$; $p \wedge\vee\wedge p \Leftrightarrow p$; $p \vee\vee\wedge\, p \Leftrightarrow p$; $p \vee\vee\vee p \Leftrightarrow p$

Kommutativität

$p \wedge\wedge\wedge q \Leftrightarrow q \wedge\wedge\wedge p$; $p \wedge\vee\wedge q \Leftrightarrow q \wedge\vee\wedge p$; $p \vee\vee\vee q \Leftrightarrow q \vee\vee\vee p$

Absorptionsgesetze

$p \wedge\wedge\wedge (p \vee\vee\vee q) \Leftrightarrow p$; $p \wedge\wedge\vee (p \vee\vee \wedge q) \Leftrightarrow p$; $p \vee\vee\vee (p \wedge\wedge\wedge q) \Leftrightarrow p$

$p \wedge\wedge\wedge (p \wedge\wedge\wedge q) \Leftrightarrow p \wedge\wedge\wedge q$; $p \vee\vee\vee (p \vee\vee\vee q) \Leftrightarrow p \vee\vee\vee q$

$p \wedge\wedge\wedge (q \vee\vee\vee N_1 q \vee\vee\vee N_5 q) \Leftrightarrow p$; $p \vee\vee\vee (q \wedge\wedge\wedge N_2 q \wedge\wedge\wedge N_5 q) \Leftrightarrow p$

$p \wedge\wedge\wedge (q \vee\vee\vee N_1 q) \Leftrightarrow p \lrcorner \wedge\wedge q$; $p \vee\vee\vee (q \wedge\wedge\wedge N_1 q) \Leftrightarrow p \llcorner \vee \circ q$

$p \wedge\wedge\wedge (q \vee\vee\vee N_2 q) \Leftrightarrow p \wedge \lrcorner \star q$; $p \times\times\times (q \times\times\times N_2 q) \Leftrightarrow p \vee \llcorner \vee q$

$p \wedge\wedge\wedge (q \vee\vee\vee N_5 q) \Leftrightarrow p \wedge \lrcorner\lrcorner q$; $p \vee\vee\vee (q \wedge\wedge\wedge N_5 q) \Leftrightarrow p \llcorner \vee \llcorner q$

$p \vee\vee\vee (p \wedge\wedge\vee q) \Leftrightarrow p \lrcorner\lrcorner \vee q$; $p \vee\vee\wedge (p \wedge\wedge\wedge q) \Leftrightarrow p \lrcorner\lrcorner \wedge q$

$p \vee\vee\wedge (p \wedge\wedge\vee q) \Leftrightarrow p$; $p \wedge\wedge\vee (p \vee\vee\wedge q) \Leftrightarrow p$

De Morgansche Gesetze

$p \wedge\wedge\wedge q \Leftrightarrow N_5(N_5 p \vee\vee\vee N_5 q)$; $p \vee\vee\vee q \Leftrightarrow N_5(N_5 p \wedge\wedge\wedge N_5 q)$

$p \wedge\wedge\wedge q \Leftrightarrow N_1(N_1 p \vee\wedge\wedge N_1 q)$; $p \vee\vee\vee q \Leftrightarrow N_1(N_1 p \wedge\vee\vee N_1 q)$

$p \wedge\wedge\wedge q \Leftrightarrow N_2(N_2 p \wedge\vee\wedge N_2 q)$; $p \vee\vee\vee q \Leftrightarrow N_2(N_2 p \vee\wedge\vee N_2 q)$

Komplementierungen

$N_1(N_1 p \wedge\wedge\wedge q) \Leftrightarrow p \subset \wedge\wedge q$ $\qquad N_1 p \vee\vee\vee q \Leftrightarrow p \supset \circ \oplus q$

$N_2(N_2 p \wedge\wedge\wedge q) \Leftrightarrow p \star \subset \Box q$ $\qquad N_2 p \vee\vee\vee q \Leftrightarrow p \vee \supset \vee q$

$N_5(N_5 p \wedge\wedge\wedge q) \Leftrightarrow p \dot{\lrcorner} \llcorner \subset q$ $\qquad N_5 p \vee\vee\vee q \Leftrightarrow p \llcorner \lrcorner \supset q$

$N_5 p \vee\vee\vee q \Leftrightarrow N_5(p \wedge\wedge\wedge N_5 q)$ $\qquad p \vee\vee\vee N_5 q \Leftrightarrow N_5(N_5 p \wedge\wedge\wedge q)$

Quatrum non datur

$p \wedge\wedge\wedge N_2 p \wedge\wedge\wedge N_5 p \Leftrightarrow q \wedge\wedge\wedge N_2 q \wedge\wedge\wedge N_5 q$

$p \vee\vee\vee N_1 p \vee\vee\vee N_5 p \Leftrightarrow q \vee\vee\vee N_1 q \vee\vee\vee N_5 q$

Wahrheitsbedingungen

$p \wedge\wedge\wedge q \Leftrightarrow (p \vee\vee\vee q) \wedge\wedge\wedge (p \vee\vee\vee N_5 q) \wedge\wedge\wedge (N_5 p \vee\vee\vee q) \wedge\wedge\wedge$
$\qquad \wedge\wedge\wedge (N_2 p \vee\vee\vee q) \wedge\wedge\wedge (p \vee\vee\vee N_2 q)$

$p \wedge\vee\wedge q \Leftrightarrow (p \vee\vee\vee q) \wedge\wedge\wedge (N_5 p \vee\vee\vee q) \wedge\wedge\wedge (p \vee\vee\vee N_5 q)$

$p \vee\wedge\vee q \Leftrightarrow (p \vee\vee\vee q) \wedge\wedge\wedge (N_2 p \vee\vee\vee q) \wedge\wedge\wedge (p \vee\vee\vee N_2 q)$

$p \vee\wedge\wedge q \Leftrightarrow (p \wedge\wedge\wedge q) \vee\vee\vee (N_1 p \wedge\wedge\wedge q) \vee\vee\vee (p \wedge\wedge\wedge N_1 q)$

$p \wedge\vee\vee q \Leftrightarrow (p \wedge\wedge\wedge q) \vee\vee\vee (N_5 p \wedge\wedge\wedge q) \vee\vee\vee (p \wedge\wedge\wedge N_5 q)$

$p \vee\vee\wedge q \Leftrightarrow (p \vee\vee\vee q) \wedge\llcorner\llcorner (N_5 p \vee\vee\vee q) \wedge\llcorner\llcorner (p \vee\vee\vee N_5 q)$

$(p \vee\vee\vee q) \wedge\wedge\wedge (N_5 p \vee\vee\vee N_5 q) \Leftrightarrow (p \wedge\wedge\wedge N_5 q) \vee\vee\vee (N_5 p \wedge\wedge\wedge q)$

Distributionsgesetze

$p \wedge\wedge\wedge (p \vee\vee\vee q) \Leftrightarrow p \vee\vee\vee (p \wedge\wedge\wedge q)$

$p \wedge\wedge\vee (p \vee\vee\wedge q) \Leftrightarrow p \vee\vee\wedge (p \wedge\wedge\vee q)$

$p \vee\wedge\vee (p \wedge\vee\wedge q) \Leftrightarrow p \wedge\vee\wedge (p \vee\wedge\vee q)$

$p \wedge_3^3 (q \vee_3^3 r; p); r \Leftrightarrow (p \wedge_3^3 q; r) \vee_3^3 (p \wedge_3^3 r; q); r$

Absorptionsgesetze

$(p \wedge\wedge\wedge (q \vee\vee\vee N_5 q)) \wedge\wedge\wedge q \Leftrightarrow p \wedge\wedge\wedge q \; ; \; (p \wedge\wedge\wedge (q \vee\vee\vee N_3 q)) \vee\vee\vee p \Leftrightarrow p$

$(p \wedge\wedge\wedge (q \vee\vee\vee N_1 q)) \wedge\wedge\wedge q \Leftrightarrow p \wedge\wedge\wedge q \; ; \; (p \wedge\wedge\wedge (q \vee\vee\vee N_5 q)) \vee\vee\vee p \Leftrightarrow p$

$(p \wedge\wedge\wedge (q \vee\vee\vee N_3 q)) \wedge\wedge\wedge q \Leftrightarrow p \wedge\wedge\wedge q$

Definitionen

$tp \Leftrightarrow p \vee\vee\vee N_1 p \vee\vee\vee N_5 p \; ; \; fp \Leftrightarrow (p \vee\vee\vee N_2 p) \wedge\vee\wedge (p \wedge\wedge\wedge N_1 p)$

$f'p \Leftrightarrow p \wedge\wedge\wedge N_2 p \wedge\wedge\wedge N_5 p \; ; \; M_1 p \Leftrightarrow p \vee\vee\vee N_1 p \; ; \; M_2 p \Leftrightarrow p \vee\vee\vee N_2 p$

$M_3 p \Leftrightarrow p \vee\vee\vee N_5 p \; ; \; L_1 p \Leftrightarrow p \wedge\wedge\wedge N_2 p \; ; \; L_2 p \Leftrightarrow p \wedge\wedge\wedge N_5 p$

$N_1(p \wedge\wedge\wedge N_1 q) \vee\vee\vee (N_5 p \vee\vee\vee q) \Leftrightarrow p \supset\supset\supset q$

Gesetze der Kontraposition

$p \supset\supset\supset q \Leftrightarrow N_5 q \supset\supset\supset N_5 p$ $\quad$ $p \to\to\to q \Leftrightarrow N_5 q \to\to\to N_5 p$

$N_5 p \supset\supset\supset q \Leftrightarrow N_5 q \supset\supset\supset p$ $\quad$ $N_5 p \to\to\to q \Leftrightarrow N_5 q \to\to\to p$

$p \supset\supset\supset N_5 q \Leftrightarrow q \supset\supset\supset N_5 p$ $\quad$ $p \to\to\to N_5 q \Leftrightarrow q \to\to\to N_5 p$

$p \subset\subset\subset q \Leftrightarrow N_5 p \supset\supset\supset N_5 q$ $\quad$ $p \leftarrow\leftarrow\leftarrow q \Leftrightarrow N_5 p \leftarrow\leftarrow\leftarrow N_5 q$

$p \subset\supset\supset q \Leftrightarrow N_1 p \supset\supset\supset N_1 q$ $\quad$ $p \leftarrow\to\to q \Leftrightarrow N_1 p \to\to\to N_1 q$

$p \supset\subset\supset q \Leftrightarrow N_2 p \supset\supset\supset N_2 q$ $\quad$ $p \to\leftarrow\to q \Leftrightarrow N_2 p \to\to\to N_2 q$

$p \subset\supset\subset q \Leftrightarrow N_3 p \supset\supset\supset N_3 q$ $\quad$ $p \leftarrow\to\leftarrow q \Leftrightarrow N_3 p \to\to\to N_3 q$

$p \supset\subset\subset q \Leftrightarrow N_4 p \supset\supset\supset N_4 q$ $\quad$ $p \to\leftarrow\leftarrow q \Leftrightarrow N_4 p \to\to\to N_4 q$

reductio ad absurdum

$p \supset\supset\supset N_5 p \Leftrightarrow N_5 p \vee\vee\vee N_1 p$ $\quad$ $N_5 p \to\to\to p \Leftrightarrow N_1 p \vee\vee\vee p$

$N_1 p \supset\supset\supset p \Leftrightarrow p \vee\vee\vee N_1 p$ $\quad$ $N_1 p \to\to\to p \Rightarrow p \vee\vee\vee N_1 p$

$N_2 p \supset\supset\supset p \Rightarrow p \vee\vee\vee N_5 p$ $\quad$ $N_2 p \to\to\to p \Rightarrow p \vee\vee\vee N_5 p$

$p \supset\supset\supset N_1 p \Leftrightarrow N_1 p \vee\vee\vee N_5 p$ $\quad$ $N_5(p \supset\supset\supset q) \Rightarrow p \wedge\wedge\wedge N_5 q$

$p \supset\supset\supset N_2 p \Leftrightarrow p \vee\vee\vee N_5 p$

Modus tol lendo tollens u.a.

$p \Rightarrow (q \supset\supset\supset N_5 p) \supset\supset\supset N_5 q$ $\quad$ $p \Rightarrow (p \to\to\to q) \to\to\to q$

$p \Rightarrow (p \supset\supset\supset q) \supset\supset\supset q$ $\quad$ $p, p \to\to\to q \Rightarrow q$

$p \Rightarrow N_5 p \supset\supset\supset q$ $\quad$ $p \Rightarrow N_5 p \to\to\to q \vee\vee\vee p$

Gesetze der Äquivalenz

Reflexivität

$\Rightarrow p === p \; ; \Rightarrow p ==\equiv p$ $\quad$ $\Rightarrow p \equiv == p \; ; \Rightarrow p \equiv\equiv\equiv p$

Kommutativität

$p === q \Leftrightarrow q === p$ $\quad$ $p \equiv\equiv\equiv q \Leftrightarrow q \equiv\equiv\equiv p$

Wahrheitsbedingungen und Definitionen

$p === q \Leftrightarrow (p \wedge\wedge\wedge q) \vee\vee\vee (N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee (N_5 p \wedge\wedge\wedge N_5 q)$

$p === q \Leftrightarrow (p \supset\supset\supset q) \wedge\wedge\wedge (q \supset\supset\supset p)$

$p \equiv\equiv\equiv q \Leftrightarrow (p \supset\supset\supset q) \vee\wedge\wedge N_2(p \supset\supset\supset q)$

Inversion , Widerlegung, Kontraposition

$p === q \Leftrightarrow N_5 p === N_5 q$ $\quad$ $p \not\equiv\equiv\equiv q \Leftrightarrow N_1 p \equiv\equiv\equiv N_1 q$

$N_2(p === q) \Leftrightarrow N_2 p \equiv\equiv\equiv q$

$N_1(p===q)\Leftrightarrow p\succ\prec\succ\prec=q$ $\qquad N_1(p\equiv\equiv\equiv q)\Leftrightarrow p\succ\prec\equiv\equiv q$

$N_5(p===q)\Rightarrow N_5p===q$ $\qquad p\equiv\equiv\equiv N_2p\Leftrightarrow p$ $\qquad N_2p\Leftrightarrow N_2(p={}^3N_2p)$

$N_1p===q\Leftrightarrow p\succ\prec^{\oplus}\circ q$ $\qquad N_1p\equiv\equiv\equiv q\Leftrightarrow N_1(p=\bowtie\bowtie q)$

$N_2p===q\Leftrightarrow p\,\square\succ\prec\,\star q$ $\qquad N_2p\equiv\equiv\equiv q\Leftrightarrow N_2(p\,{}^{\oplus}\succ\prec^{\star}q)$

$N_5p===q\Leftrightarrow p===N_5q$ $\qquad N_5p\equiv\equiv\equiv q\Rightarrow N_2(p\equiv\equiv\equiv N_5q)$

$p===N_1p\Leftrightarrow(p\wedge\wedge\wedge N_1p)\vee\vee\vee N_5p$; $N_1(p\equiv\equiv\equiv p)\Leftrightarrow fp$

$p===N_1p\Leftrightarrow N_1(p===p)\vee\vee\vee N_5p$; $N_2(p\equiv\equiv\equiv p)\Leftrightarrow tp$

$p===N_2p\Leftrightarrow fp\vee\vee\vee p$ $\qquad p===N_4p\Leftrightarrow p\wedge\wedge\wedge N_1p$

$p===N_3p\Leftrightarrow N_3(p\wedge\wedge\wedge N_2p)$ $\qquad p===(p\wedge\wedge\wedge N_5p),\Leftrightarrow N_5p$

$p===(p\wedge\wedge\wedge N_1p)\Leftrightarrow N_5(p\wedge\wedge\wedge N_1p)$

$p===(p\wedge\wedge\wedge N_2p)\Leftrightarrow p\vee\vee\vee N_5p$

$N_1(p===p)\Leftrightarrow N_1(p\vee\vee\vee N_1p\vee\vee\vee N_5p)$

$p===(p\wedge\wedge\wedge N_2p\wedge\wedge\wedge N_5p)\Leftrightarrow N_5p\Leftrightarrow p===f'p$

$p===(p\vee\vee\vee N_1p\vee\vee\vee N_5p)\Leftrightarrow p\vee\vee\vee N_5p\Leftrightarrow p===tp$

$p===N_1(N_2p\wedge\wedge\wedge N_5p)\Leftrightarrow f'p$

Gesetze der Transjunktion und der Transäquivalenz

Idempotenz, Reflexivität

$p\bowtie\bowtie\bowtie p\Leftrightarrow p$; $\qquad$; $p\between\between\between p\Leftrightarrow q\between\between\between q$

Kommutativität, Anti–Kommutativität

$p\bowtie\bowtie\bowtie q\Leftrightarrow p\bowtie\bowtie\bowtie q$ $\qquad p\between\between\between q\Leftrightarrow N_2(q\between\between\between p)$

Absorptionsgesetze

$(p\bowtie\bowtie\bowtie q)\bowtie\bowtie\bowtie q\Leftrightarrow p$ $\qquad (p\between\between\between q)\between\between\between q\Leftrightarrow p$

$(p\bowtie\bowtie\bowtie q)\bowtie\bowtie\bowtie p\Leftrightarrow q$ $\qquad (p\between\between\between q)\between\between\between p\Leftrightarrow p\bowtie\bowtie\bowtie q$

$p\bowtie\bowtie\bowtie(q\bowtie\bowtie\bowtie N_2q)\Leftrightarrow N_2p$ $\qquad p\between\between\between(p\between\between\between q)\Leftrightarrow N_2(p\bowtie\bowtie\bowtie q)$

$p\bowtie\bowtie\bowtie(q\bowtie\bowtie\bowtie N_5q)\Leftrightarrow N_5p$ $\qquad (p\bowtie\bowtie\between q)\bowtie\bowtie\between q\Leftrightarrow p$

$p\bowtie\bowtie\bowtie(q\bowtie\bowtie\bowtie N_1q)\Leftrightarrow N_1p$ $\qquad (p\bowtie\bowtie\between q)\bowtie\bowtie\between p\Leftrightarrow p\llcorner\llcorner\bowtie q$

$p\bowtie\bowtie\bowtie(q\bowtie\bowtie\bowtie N_1q\bowtie\bowtie\bowtie N_5q)\Leftrightarrow N_3(p\bowtie\bowtie\bowtie q)$

$p\bowtie\bowtie\bowtie(q\bowtie\bowtie\bowtie N_2q\bowtie\bowtie\bowtie N_5q)\Leftrightarrow N_4(p\bowtie\bowtie\bowtie q)$

$(p\bowtie\bowtie\bowtie q)\bowtie\bowtie\bowtie(p\llcorner\llcorner\lrcorner q)\Leftrightarrow p\lrcorner\lrcorner\llcorner q$

$(p\bowtie\bowtie\bowtie q)\bowtie\bowtie\bowtie(p\lrcorner\lrcorner\llcorner q)\Leftrightarrow p\llcorner\llcorner\lrcorner q$

$(p\bowtie\bowtie\bowtie q)\bowtie\bowtie\bowtie(p\between\between\between q)\Leftrightarrow N_2p$

$(p\bowtie\bowtie\bowtie q)\between\between\between(p\between\between\between q)\Leftrightarrow N_2q$

Distributionsgesetze

(p ⅩⅩⅩ q) ⅩⅩⅩ q ⇔ (p ⅠⅠⅠ q) ⅠⅠⅠ q
q ⅠⅠⅠ (p ⅠⅠⅠ q) ⇔ N_2(q ⅩⅩⅩ (p ⅩⅩⅩ q)
((p ⅩⅩⅩ q) ⅩⅩⅩ q) ⅩⅩⅩ p ⇔ p ⅩⅩⅩ (q ⅩⅩⅩ q)

Negations– , Modal-Gesetze

N_i (p ⅩⅩⅩ q) ⇔ N_i p ⅩⅩⅩ N_i q   i = 1,2,3
M_1(p ⅩⅩⅩ q) ⇔ (M_1p ⅩⅩⅩ M_1q) ⅩⅩⅩ (p = ∧ ∧ q)
M_1p ⅩⅩⅩ M_1q ⇔ M_1(p ⅩⅩⅩ q) ⅩⅩⅩ (p ∧∧∧ q)
p ⅩⅩⅩ N_1p ⇔ fp ; p ⅩⅩⅩ N_2p ⇔ tp ; p ⅩⅩⅩ N_5p ⇔ fp
(p ⅩⅩⅩ N_1p) ⅩⅩⅩ p ⇔ N_1p ; (p ⅩⅩⅩ N_1p) ⅩⅩⅩ N_1p ⇔ p

Definitionen , Wahrheitsbedingungen

(p ∧ ∨ ∧ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p ∧ Ⅹ ∧ q
(p ∧ ∧ ∨ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p ∧∧Ⅹ q
(p ∨ ∧ ∧ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p Ⅹ∧∧ q
(p ∨ ∨ ∧ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p ⅩⅩ∧ q
(p ∨ ∧ ∨ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p Ⅹ∧Ⅹ q
(p ∨ ∧ ∧ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p ∧ⅩⅩ q
(p ∧∧∧ q) ⅩⅩⅩ (p ∨∨∨ q) ⇔ p ⅩⅩⅩ q
(p ∨∨∨ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p ⅩⅩⅩ q
(p ⅩⅩⅩ q) ⅩⅩⅩ (p ⅩⅩⅩ q) ⇔ p ⅩⅩⅩ q
(p ⅩⅩⅩ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p ∨∨∨ q
(p ⅩⅩⅩ q) ⅩⅩⅩ (p ∨∨∨ q) ⇔ p ∧∧∧ q
(p ∧∧∧ q) ⅩⅩⅩ (p ∧∧∧ q) ⇔ p ∧∧∧ q
(p ∨∨∨ q) ⅩⅩⅩ (p ∨∨∨ q) ⇔ p ∨∨∨ q
(p ○○○ q) ⅩⅩⅩ (p □□□ q) ⇔ p ⅩⅩⅩ q
(p ○○○ q) ⅩⅩⅩ (p ⊕⊕⊕ q) ⇔ p ∧∧∧ q
(p ⊃⊃⊃ q) ⅩⅩⅩ (p ⊂⊂⊂ q) ⇔ p === q
(p ⊃⊃⊃ q) ⅩⅩⅩ (p □□□ q) ⇔ N_2(p □□□ q)
(p ○○○ q) ⅩⅩⅩ (p ☆☆☆ q) ⇔ q
p ∧ Ⅹ Ⅹ q ⇔ (p ∧∧∧ q) ∨∨∨ (N_1p ∧∧∧ N_5q) ∨∨∨ (N_5p ∧∧∧ N_1q)
p ∧ ∨ Ⅹ q ⇔ (p ∨∨∨ q) ∧∧∧ (N_1p ∨∨∨ q) ∧∧∧ (p ∨∨∨ N_1q)
p ∧ ∨ Ⅹ q ⇔ (p ∧∧∧ q) ∨∨∨ (N_2p ∧∧∧ q) ∨∨∨ (p ∧∧∧ N_2q)

Gesetze der implikativ– und replikativ– transjunktiven Disjunktion und Konjunktion

Idempotenz

p ○○○ p ⇔ p ; p ☆☆☆ p ⇔ p ; p ⊕⊕⊕ p ⇔ p ; p □□□ p ⇔ p
p ○□○ p ⇔ p ; p ☆○☆ p ⇔ p ; p ⊕☆☆ p ⇔ p ; p □○□ p ⇔ p
p ○☆⊕ p ⇔ p ; p ○⊕□ p ⇔ p ; p ⊕□☆ p ⇔ p ; p □○☆ p ⇔ p

Kommutativität

p ○○○ q ⇔ q ⊕⊕⊕ p ; p ☆☆☆ q ⇔ q □□□ p ; p ○○□ q ⇔ q ○○☆ p
p ○○⊕ q ⇔ q ⊕⊕○ p ; p ☆□□ q ⇔ q □☆☆ p ; p ○⊕□ q ⇔ q ⊕○☆ p

De Morgansche Gesetze

N_5(p ○○○ q) ⇔ N_5p ☆☆☆ N_5q N_5(p ⊕⊕⊕ q) ⇔ N_5p □□□ N_5q
N_1(p ○○○ q) ⇔ N_1p ☆○○ N_1q N_1(p ⊕⊕⊕ q) ⇔ N_1p □⊕⊕ N_1q
N_2(p ○○○ q) ⇔ N_2p ○☆○ N_2q N_2(p ⊕⊕⊕ q) ⇔ N_2p ⊕□⊕ N_2q

Absorptionsgesetze der replikativ–transjunktiven Disjunktion

(p ○○○ q) ○○○ p ⇔ p ∨○○ q ; p ○○○ (p ○○○ q) ⇔ p ∟○○ q
p ○○○ (p ○○○ q) ⇔ p ∟∨∨ q ; p ○○○ (p ○○○ q) ⇔ p ⊕☆⊕ q
p ○○○ (p □□□ q) ⇔ p ○∨∨ q ; p ○○○ (p ☆☆☆ q) ⇔ p ∨┘┘ q
p ○○○ (p ⊕⊕⊕ q) ⇔ p ○∟∨ q ; q ○○○ (p □□□ q) ⇔ p ∟∧∟ q
q ○○○ (p ☆☆☆ q) ⇔ p ∟∨∨ q ; q ○○○ (p ⊕⊕⊕ q) ⇔ p ∟∨∨ q
(p ○○○ q) □□□ p ⇔ p ∨┘┘ q ; (p ○○○ q) □□□ q ⇔ p ∧∟∧ q
(p ○○○ q) ☆☆☆ p ⇔ p ┘∨∨ q ; (p ○○○ q) ☆☆☆ q ⇔ p ⊕☆⊕ q
(p ○○○ q) ⊕⊕⊕ p ⇔ p ○∨∨ q ; (p ○○○ q) ⊕⊕⊕ q ⇔ p ⧗☆⧗ q
p ○○○ (p ⊃⊃⊃ q) ⇔ N_2p ; p ○○○ (p ⧗⧗⧗ q) ⇔ N_2(p ○□○ q)
p ○○○ (p ∧∧∧ q) ⇔ p ; p ○○○ (p ∨∨∨ q) ⇔ p ○○○ q
p ○○○ (p →→→ q) ⇔ N_2(p [illegible] q); p ○○○ (q ○○○ N_1q) ⇔ p ○┘○ q
p ○○○ (q ○○○ N_2q) ⇔ p ○⧗┘ q ; p ○○○ (q ○○○ N_5q) ⇔ p ○┘⊂ q

Abschwächung

(p ☆☆☆ q ⇒ (p ⊕⊕⊕ q) ; (p □□□ q) ⇒ (p ○○○ q)

Some interesting DeMorgan formulas for transjunctions are included. There are even formulas defining junctions out of a kind of transjunctions. But they are not self-dual transjunctions but so called disjunctive transjunction implying some junctional elements.

Dreiwertige Stellenwertlogik in Implikation und Negation 43h

Distributionsgesetze

$p \circ\circ\circ (p \star\star\star q) \Leftrightarrow p \vee\wedge\wedge (p \vee\vee\vee q)$

$p \circ\circ\circ (p \circ\circ\circ q) \Leftrightarrow q \circ\circ\circ (p \oplus\oplus\oplus q)$

$(p \circ\circ\circ (p \circ\circ\circ q)) \vee\vee\vee p \Leftrightarrow (p \circ\circ\circ q) \circ\circ\circ p$

$(p \circ\circ\circ q) \circ\circ\circ p \Leftrightarrow ((p \circ\circ\circ q) \circ\circ\circ q) \vee\vee\vee p$

$p \circ\circ\circ (p \square\square\square) \Leftrightarrow (p \square\square\square (p \circ\circ\circ q)) \vee\vee\vee p$

$(p \circ\circ\circ (p \star\star\star q)) \vee\vee\vee p \Leftrightarrow p \vee\vee\vee q$

$(p \circ\circ\circ q) \square\square\square (p \square\square\square q) \Leftrightarrow p \square\square\square q$

$(p \circ\circ\circ q) \star\star\star (p \square\square\square q) \Leftrightarrow p \mathsf{XXX} q$

$(p \circ\circ\circ q) \oplus\oplus\oplus (p \square\square\square q) \Leftrightarrow p \mathsf{XXX} q$

$(p \circ\circ\circ q) \square\square\square (p \oplus\oplus\oplus q) \Leftrightarrow p \star \llcorner \star q$

Gesetze der differenzierten totalen Transjunktion

$p\, u_2\, p \Leftrightarrow p$	Idempotenz
$(p\, u_2\, q)\, u_2\, p \Leftrightarrow q$	1. Absorption
$(p\, u_2\, q)\, u_2\, q \Leftrightarrow q\, u_2\, p$	inverse Stein–Identität
$p\, u_2\, (p\, u_2\, q) \Leftrightarrow q\, u_2\, p$	Stein–Identität
$(p\, u_2\, q)\, u_2\, q \Leftrightarrow p\, u_2 (p u_2 q)$	Distributivität
$q\, u_2\, (p\, u_2\, q) \Leftrightarrow p$	2. Absorptionsgesetz
$(p\, u_2\, q)\, u_2 (q\, u_2\, p) \Leftrightarrow p$	3. Absorptionsgesetz
$N_{10}\, (N_{10} p\, u_2\, N_{10} q) \Leftrightarrow N_{1\cdot 4}\, (p\, u_2\, q)$	De Morgansches Gesetz
$p\, u_2\, q \Leftrightarrow N_{3\cdot 4}\, (q\, u_3\, p)$	Antisymmetrie
$p\, u_1\, q \Leftrightarrow N_4\, (p\, u_2\, q)$	Definition
$p\, u_3 q \Leftrightarrow N_{1\cdot 6}\, (q\, u_1\, p)$	Definition

Die differenzierte totale Transjunktion u aus G^4 hat Halbgruppeneigenschaften und ist nach A. Sade idempotent, halb–symmetrisch, anti–abelsch, distributiv und entropisch. Ihre vorzügliche Eigenschaft ist die Stein–Identität.

At the climax of this academic enterprise the idempotency of A. Sade is celebrated for place-valued logics m≥4.

After more than 30 years I will let the cat out of the bag. It’s all about Extended Lo Shu Magic Squares.

http://rudys-chinese-challenge.blogspot.com/2006/09/on-chinese-mathematics_04.html

5 Gunther's "Cybernetic Ontology"

Cybernetic Ontology is one of the most important of Gunther's results at the BCL.

5.1 Gunther's "awkward formula"

Gunther's (short) definition of total transjunction by junctions and negations only.

Transjunctions defined by junctions and negations

$$X <><><> Y \; eq \; (X \vee\vee\wedge Y) \; .\vee\vee\wedge. \; \neg_{2.1}(X \; \wedge\wedge\vee Y)$$

$$X <><><> Y \; eq \; (X \wedge\wedge\vee Y) \; .\wedge\wedge\vee. \; \neg_{1.2}(X \; \vee\vee\wedge Y)$$

Some like it different

$$X \oplus\oplus\oplus Y := (X \vee\vee\wedge Y) \; \vee\vee\wedge \; N_1 N_2 (X \; \wedge\wedge\vee Y)$$

$$X \oplus\oplus\oplus Y := (X \wedge\wedge\vee Y) \; \wedge\wedge\vee \; N_2 N_1 (X \; \vee\vee\wedge Y)$$

It easily can be shown that in this way transjunctions (13) can be defined by disjunctions (1) or by conjunctions (4) and negations (N) only.

p[4,4,1]q =Def N1 (N1 p[1,1,1] N1 q) [4,4,4] N2 (N2 p[1,1,1] N2 q) (14)
p[1,1,4]q =Def N1 (N1 p[4,4,4] N1 q) [1,1,1] N2 (N2 p[4,4,4] N2 q) (15)

p[13,13,13]q =Def <N1(N1 p[4,4,4] N1 q) [1,1,1] N2 (N2 p[4,4,4] N2 q)>
N1(N1 p[4,4,4] N1 q) [1,1,1] N2 (N2 p[4,4,4] N2 q)
N2.1 < N1 (N1 p[1,1,1] N1 q) [4,4,4] N2 (N2 p[1,1,1] N2 q)> (16)

"By using the Formulas (14) and (15) we may, of course, reduce the awkward Formula (16) to the very simple formula:

[13,13,13] = ([1,1,4]) [1,1,4] (N2.1 [4,4,1]) (17)

and

[13,13,13] = ([4,4,1]) [4,4,1] (N1.2 [1,1,4]) (18)" Gunther

5.2 What's all about with morphograms?

"But negation in a many-valued system has, under certain conditions, an entirely different function from the corresponding operation in traditional logic. If we negate 1222 and obtain 2111 in classic logic we have negated the meaning of the original sequence. But if we apply negator N_2, thus changing 1222 to 1333, we insist that the second value-sequence carries exactly the same meaning as the first. What the operator did was only to *shift the meaning* from one given location in a system of reflection to some other place. A change of values in a many-valued order may under given circumstances produce a change of meaning. But it does not necessarily do so. In traditional logic a value has one and only one function. By negating one value it unavoidably accepts the other as the only possible expression of a choice. And by doing so it implicitly accepts the alternative that is offered by the given values. In this sense negation is a function of *acceptance* in the classic theory and the values "true" and "false" are acceptance values." Cybernetic Ontology, p. 381

Gunther's remark makes it clear that a shift by negation happens only if the negation is changing the values of a value-sequence. In polylogics, a shift can happen without value-change. Because the shift is a structural operation moving a value-sequence from one sub-system to another. Thus, say $N_1(\mathbf{F}) = \mathbf{F}$ for PVL, where a logical value is an entity while in PCL logical values are based on *differences*, thus $N_1(t_3, f_3)=(t_2, f_2)$.

5.3 Reconstruction and rehabilitation of the "awkward formula"

In the following steps I will reconstruct Gunther's "awkward" formula, and fill some

$$(p \wedge\wedge\vee q) \equiv N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q) \quad : ok$$

$$(p \vee\vee\wedge q) \equiv N_1(N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee N_2(N_2\ p \wedge\wedge\wedge N_2 q) \quad : ok$$

$$(p \oplus\oplus\oplus q) \equiv (p \wedge\wedge\vee q) \wedge\wedge\vee N_2 N_1\ (p \vee\vee\wedge p) \quad : ok$$

$$(p \oplus\oplus\oplus q) \equiv [N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q)] \wedge\wedge\vee [N_2 N_1\ (N_1(N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee N_2(N_2\ p \wedge\wedge\wedge N_2 q))] \quad : ok$$

The following formula is the dual to the Gunther formula (16) *of Cybernetic Ontology, p.* 366
But this is not a working formula, but an abbreviation, which then is producing Gunther's formulae (17) *and* (18).
Operationally, the formula is meaningless.

gaps. To find a DeMorgan like formula, the poly-form compound (&,&,v) has to be replaced by a mono-form formula with disjunction and conjunction, only.

$$(p \oplus\oplus\oplus q) \equiv$$

$$\underbrace{[N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q)]}_{(p)} \underbrace{\langle N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q)\rangle}_{(p \wedge\wedge\vee q)} \underbrace{[N_2 N_1\ (N_1(N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee N_2(N_2\ p \wedge\wedge\wedge N_2 q))]}_{(q)}$$

But this formula is an abbreviation only. It can not be considered as a well-formed formula. For strange reasons, this fact was never mentioned in the literature.

Thus, we have to transform this abbreviation into a well-formed formula. The coloring may be of help to visualize the steps of transformations. The result may still be awkward, but at least it will be a well-formed formula, accessible to further calculations.

$$(p \oplus\oplus\oplus q) \equiv\equiv\equiv$$

$$\left\langle \begin{array}{l} N_1\left(N_1[N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q)] \vee\vee\vee \langle N_2 N_1(N_1[\ N_1(N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee N_2(N_2\ p \wedge\wedge\wedge N_2 q)])\rangle\right) \\ \wedge\wedge\wedge \\ N_2\left(N_2\ [N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q)] \vee\vee\vee \langle N_2 N_1([N_2(N_1(N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee N_2(N_2\ p \wedge\wedge\wedge N_2 q))])\rangle\right) \end{array} \right\rangle$$

The same formula in a visually more accessible form.

$$(p \oplus\oplus\oplus q) \equiv\equiv\equiv \left\langle \begin{array}{l} N_1 \left(\begin{array}{l} N_1[N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q)] \\ \vee\vee\vee \\ \langle N_2 N_1(N_1[\ N_1(N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee N_2(N_2\ p \wedge\wedge\wedge N_2 q)])\rangle \end{array} \right) \\ \wedge\wedge\wedge \\ N_2 \left(\begin{array}{l} N_2\ [N_1(N_1 p \vee\vee\vee N_1 q) \wedge\wedge\wedge N_2(N_2\ p \vee\vee\vee N_2 q)] \\ \vee\vee\vee \\ \langle N_2 N_1(N_2[N_1(N_1 p \wedge\wedge\wedge N_1 q) \vee\vee\vee N_2(N_2\ p \wedge\wedge\wedge N_2 q)])\rangle \end{array} \right) \end{array} \right\rangle$$

$$
\begin{aligned}
&(p \oplus\oplus\oplus\, q) \equiv\equiv\equiv \\
&\left\langle
\begin{aligned}
&N_1 \left(
\begin{aligned}
&N_1\Big[N_5\big[N_5[N_1(N_1p \vee\vee\vee N_1q)] \vee\vee\vee N_5[N_2(N_2\ p \vee\vee\vee N_2q)]\big]\Big] \\
&\vee\vee\vee \\
&\Big\langle N_2N_1\Big(N_1\Big[N_5\Big[N_1\big([N_5(N_1p)] \vee\vee\vee [N_5(N_1q)]\big)\Big]\ \vee\vee\vee\ N_5\Big[N_2\big([N_5(N_2p)] \vee\vee\vee [N_5(N_2q)]\big)\Big]\Big]\Big)\Big\rangle
\end{aligned}
\right) \\
&\wedge\wedge\wedge \\
&N_2 \left(
\begin{aligned}
&N_2\ \Big[N_5\big[N_5[N_1(N_1p \vee\vee\vee N_1q)] \vee\vee\vee N_5[N_2(N_2\ p \vee\vee\vee N_2q)]\big]\Big] \\
&\vee\vee\vee \\
&\Big\langle N_2N_1\Big(N_1\Big[N_5\Big[N_1\big([N_5(N_1p)] \vee\vee\vee [N_5(N_1q)]\big)\Big]\ \vee\vee\vee\ N_5\Big[N_2\big([N_5(N_2p)] \vee\vee\vee [N_5(N_2q)]\big)\Big]\Big]\Big)\Big\rangle
\end{aligned}
\right)
\end{aligned}
\right\rangle
\end{aligned}
$$

Transjunction in monoform disjunctions only.

$$
\begin{aligned}
&(p \oplus\oplus\oplus\, q) \equiv\equiv\equiv \\
&N_5 \left\langle
\begin{aligned}
&N_5 \left\langle N_1 \left(
\begin{aligned}
&N_1\Big[N_5\big[N_5[N_1(N_1p \vee\vee\vee N_1q)] \vee\vee\vee N_5[N_2(N_2\ p \vee\vee\vee N_2q)]\big]\Big] \\
&\vee\vee\vee \\
&\Big\langle N_2N_1\Big(N_1\Big[N_5\Big[N_1\big([N_5(N_1p)] \vee\vee\vee [N_5(N_1q)]\big)\Big]\ \vee\vee\vee\ N_5\Big[N_2\big([N_5(N_2p)] \vee\vee\vee [N_5(N_2q)]\big)\Big]\Big]\Big)\Big\rangle
\end{aligned}
\right) \right\rangle \\
&\vee\vee\vee \\
&N_5 \left\langle N_2 \left(
\begin{aligned}
&N_2\ \Big[N_5\big[N_5[N_1(N_1p \vee\vee\vee N_1q)] \vee\vee\vee N_5[N_2(N_2\ p \vee\vee\vee N_2q)]\big]\Big] \\
&\vee\vee\vee \\
&\Big\langle N_2N_1\Big(N_1\Big[N_5\Big[N_1\big([N_5(N_1p)] \vee\vee\vee [N_5(N_1q)]\big)\Big]\ \vee\vee\vee\ N_5\Big[N_2\big([N_5(N_2p)] \vee\vee\vee [N_5(N_2q)]\big)\Big]\Big]\Big)\Big\rangle
\end{aligned}
\right) \right\rangle
\end{aligned}
\right\rangle
\end{aligned}
$$

The aim of Gunther was to establish a DeMorgan relation between disjunction, conjunction and his new function *transjunction*. Cybernetic, p.369

He was not happy with the "awkward formula" (16), and used his *discomfort* to motivate a decision towards a morphogrammatic formulation of DeMorgan based on the new operator *reflector* in a mixed system of logic and morphogrammatics.

$$
(p \oplus\oplus\oplus\ q) \equiv\equiv\equiv N_2\ \Big\langle\Big(N_R R^2 R[\vee\vee\vee]\Big)\ [\vee\vee\vee]\ \Big(N_{1.2} R^1\ [\vee\vee\vee]\Big)\Big\rangle
$$

$$
(p \oplus\oplus\oplus\ q) \equiv\equiv\equiv N_1\ \Big\langle\Big(N_R R^1 R[\wedge\wedge\wedge]\Big)\ [\wedge\wedge\wedge]\ \Big(N_{2.1} R^2\ [\wedge\wedge\wedge]\Big)\Big\rangle
$$

Gunther was aware about the *what* and *how* of a DeMorgan formula for transjunction. That is, it makes a difference *how* we are defining a DeMorgan formula for transjunction: in a place-value logic or in a morphogrammatic system. But the rules of such a mixed system are not mentioned in the historical paper "Cybernetic Ontology".

Finally, a black&white presentation of the formula for non-color printers.

Print versions b/w.

Transjunction in monoform disjunction only:

$$
(p \oplus\oplus\oplus q) \equiv\equiv\equiv N_5 \left\langle \begin{array}{l}
N_5 \left\langle N_1 \left(\begin{array}{l}
N_1 \left[N_5 \left[N_5 \left[N_1 \left(N_1 p \vee\vee\vee N_1 q \right) \right] \vee\vee\vee N_5 \left[N_2 \left(N_2 \ p \vee\vee\vee N_2 q \right) \right] \right] \right] \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_1 \left[N_5 \left[N_1 \left(\left[N_5 \left(N_1 p \right) \right] \vee\vee\vee \left[N_5 \left(N_1 q \right) \right] \right) \right] \right] \ \vee\vee\vee N_5 \left[N_2 \left(\left[N_5 \left(N_2 p \right) \right] \vee\vee\vee \left[N_5 \left(N_2 q \right) \right] \right) \right] \right] \right] \right) \right\rangle
\end{array} \right) \right\rangle \\
\vee\vee\vee \\
N_5 \left\langle N_2 \left(\begin{array}{l}
N_2 \ \left[N_5 \left[N_5 \left[N_1 \left(N_1 p \vee\vee\vee N_1 q \right) \right] \vee\vee\vee N_5 \left[N_2 \left(N_2 \ p \vee\vee\vee N_2 q \right) \right] \right] \right] \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_1 \left[N_5 \left[N_1 \left(\left[N_5 \left(N_1 p \right) \right] \vee\vee\vee \left[N_5 \left(N_1 q \right) \right] \right) \right] \right] \ \vee\vee\vee N_5 \left[N_2 \left(\left[N_5 \left(N_2 p \right) \right] \vee\vee\vee \left[N_5 \left(N_2 q \right) \right] \right) \right] \right] \right] \right) \right\rangle
\end{array} \right) \right\rangle
\end{array} \right\rangle
$$

Internal formula with monoform disjunction only:

$$
(p \oplus\oplus\oplus q) \equiv\equiv\equiv \left\langle \begin{array}{l}
N_1 \left(\begin{array}{l}
N_1 \left[N_5 \left[N_5 \left[N_1 \left(N_1 p \vee\vee\vee N_1 q \right) \right] \vee\vee\vee N_5 \left[N_2 \left(N_2 \ p \vee\vee\vee N_2 q \right) \right] \right] \right] \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_1 \left[N_5 \left[N_1 \left(\left[N_5 \left(N_1 p \right) \right] \vee\vee\vee \left[N_5 \left(N_1 q \right) \right] \right) \right] \right] \ \vee\vee\vee N_5 \left[N_2 \left(\left[N_5 \left(N_2 p \right) \right] \vee\vee\vee \left[N_5 \left(N_2 q \right) \right] \right) \right] \right] \right] \right) \right\rangle
\end{array} \right) \\
\wedge\wedge\wedge \\
N_2 \left(\begin{array}{l}
N_2 \ \left[N_5 \left[N_5 \left[N_1 \left(N_1 p \vee\vee\vee N_1 q \right) \right] \vee\vee\vee N_5 \left[N_2 \left(N_2 \ p \vee\vee\vee N_2 q \right) \right] \right] \right] \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_1 \left[N_5 \left[N_1 \left(\left[N_5 \left(N_1 p \right) \right] \vee\vee\vee \left[N_5 \left(N_1 q \right) \right] \right) \right] \right] \ \vee\vee\vee N_5 \left[N_2 \left(\left[N_5 \left(N_2 p \right) \right] \vee\vee\vee \left[N_5 \left(N_2 q \right) \right] \right) \right] \right] \right] \right) \right\rangle
\end{array} \right)
\end{array} \right\rangle
$$

Transjunction in mixed, monoform conjunctions and disjunctions:

$$
(p \oplus\oplus\oplus q) \equiv\equiv\equiv \left\langle \begin{array}{l}
N_1 \left(\begin{array}{l}
N_1 \left[N_1 \left(N_1 p \vee\vee\vee N_1 q \right) \wedge\wedge\wedge N_2 \left(N_2 \ p \vee\vee\vee N_2 q \right) \right] \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_1 \left[\ N_1 \left(N_1 p \wedge\wedge\wedge N_1 q \right) \vee\vee\vee N_2 \left(N_2 \ p \wedge\wedge\wedge N_2 q \right) \right] \right) \right\rangle
\end{array} \right) \\
\wedge\wedge\wedge \\
N_2 \left(\begin{array}{l}
N_2 \ \left[N_1 \left(N_1 p \vee\vee\vee N_1 q \right) \wedge\wedge\wedge N_2 \left(N_2 \ p \vee\vee\vee N_2 q \right) \right] \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_2 \left[N_1 \left(N_1 p \wedge\wedge\wedge N_1 q \right) \vee\vee\vee N_2 \left(N_2 \ p \wedge\wedge\wedge N_2 q \right) \right] \right) \right\rangle
\end{array} \right)
\end{array} \right\rangle
$$

Some people don't like the awkwardness of linearity.

$$
\left(p \oplus\oplus\oplus q\right) \equiv\equiv\equiv \left\langle \begin{array}{l}
N_1 \left(\begin{array}{l}
\left\langle N_1 \left[\begin{array}{l} N_1\left(N_1 p \vee\vee\vee N_1 q\right) \\ \quad \wedge\wedge\wedge \\ N_2\left(N_2\ p \vee\vee\vee N_2 q\right) \end{array} \right] \right\rangle \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_1 \left[\begin{array}{l} N_1\left(N_1 p \wedge\wedge\wedge N_1 q\right) \\ \quad \vee\vee\vee \\ N_2\left(N_2\ p \wedge\wedge\wedge N_2 q\right) \end{array} \right] \right) \right\rangle
\end{array} \right) \\
\wedge\wedge\wedge \\
N_2 \left(\begin{array}{l}
\left\langle N_2 \left[\begin{array}{l} N_1\left(N_1 p \vee\vee\vee N_1 q\right) \\ \quad \wedge\wedge\wedge \\ N_2\left(N_2\ p \vee\vee\vee N_2 q\right) \end{array} \right] \right\rangle \\
\vee\vee\vee \\
\left\langle N_2 N_1 \left(N_2 \left[\begin{array}{l} N_1\left(N_1 p \wedge\wedge\wedge N_1 q\right) \\ \quad \vee\vee\vee \\ N_2\left(N_2\ p \wedge\wedge\wedge N_2 q\right) \end{array} \right] \right) \right\rangle
\end{array} \right)
\end{array} \right\rangle
$$

From Ruby to Rudy, we learn: General Limits of Multi-Processor-Systems for PCL

The main reason why it is not possible to *realize* polycontextural computing systems with multi-processor systems is embedded in the definition of the ALU of those systems. ALUs are containing in their logic junctions and negations, say as NAND or NOR operations. They are, obviously, not equipped with transjunctional operators. Transjunctions are for PCL systems the main logical operators of interactivity. On the other hand, it is not possible in polycontextural logics to define transjunctions with junctions and negations only. Otherwise there would be a chance to build a polycontextural computing systems out of a combination of distributed processors, organized as a special kind of a multi-processor system with distributed conjunctions and negations, poly-NANDs and poly-NORs.

This statement is in strict contrast to the genuine approach of Gunther in his main paper *"Cybernetic Ontology"* (1962) where transjunctions are defined by conjunctions and negations only. A reduction which seems to contradict his own aim to deliver a cybernetic theory of subjectivity. This result says, that subjectivity is definable in objective terms only. The new distinction Gunther introduced to define subjectivity, the possibility of *rejection* by transjunctions in contrast to acceptance by junctions, is reduced by this transformation to acceptance only. Thus, subjectivity is reducible to objectivity. This may be true for dead subjective systems but not for living systems.

These two facts, restriction of ALU and non-definability of transjunctions by junctions and negations only, are forcing the attempt to realize polycontextural computation on multi-processor systems from the attempt of *realization* to the attempt to *emulate/simulate* such processes. The other chance would be to design new processor types, not necessarily based on electronics, which would be able to realize directly transjunctional operations. It seems, that there are no "meta"-physical obstacles for that.

6 Unification for place-valued logics

$$\frac{T\ X \supset\supset\supset Y}{T\ X \;\Big|\; F\ Y \;\Big|\; \begin{array}{c} F\ X \\ F\ Y \end{array}} \qquad \frac{F\ X \supset\supset\supset Y}{\begin{array}{c} T\ X \\ F\ Y \end{array}} \qquad \frac{F\ X \supset\supset\supset Y}{\begin{array}{c} F\ X \\ F\ Y \end{array} \Big| \begin{array}{c} T\ X \\ F\ Y \end{array}}$$

$$\frac{T\ X \supset\supset\supset Y}{T\ \beta_1 \;\Big|\; F\ \beta_2 \;\Big|\; \begin{array}{c} F\ \alpha_1 \\ F\ \alpha_2 \end{array}} \qquad \frac{F\ X \supset\supset\supset Y}{\begin{array}{c} T\ \alpha_1 \\ F\ \alpha_2 \end{array}} \qquad \frac{F\ X \supset\supset\supset Y}{\begin{array}{c} F\ \alpha_1 \\ F\ \alpha_2 \end{array} \Big| \begin{array}{c} T\ \alpha_1 \\ F\ \alpha_2 \end{array}}$$

$$\frac{T\ X \wedge\wedge\wedge Y}{\begin{array}{c} T\ X \\ T\ Y \end{array}} \qquad \frac{F\ X \wedge\wedge\wedge Y}{\begin{array}{c} F\ X \\ T\ Y \end{array} \Big| \begin{array}{c} T\ X \\ F\ Y \end{array} \Big| \begin{array}{c} F\ X \\ F\ Y \end{array}} \qquad \frac{F\ X \wedge\wedge\wedge Y}{F\ X \mid F\ Y}$$

$$\frac{T\ X \wedge\wedge\wedge Y}{\begin{array}{c} T\ \alpha_1 \\ T\ \alpha_2 \end{array}} \qquad \frac{F\ X \wedge\wedge\wedge Y}{\begin{array}{c} F\ \alpha_1 \\ T\ \alpha_2 \end{array} \Big| \begin{array}{c} T\ \alpha_1 \\ F\ \alpha_2 \end{array} \Big| \begin{array}{c} F\ \alpha_1 \\ F\ \alpha_2 \end{array}} \qquad \frac{F\ X \wedge\wedge\wedge Y}{F\ \beta_1 \mid F\ \beta_2}$$

Because the tableaux are reflecting the global mapping of the truth-values, not much information about the local situations can be obtained. Laws of *conjugation* between alpha and beta terms is only partly to establish. Analogies to classic many-valued logics may give some hints.

As in classical multiple-valued logics structural studies about logical value matrices as such can be done and interesting features about the transformations of matrices can be studied. But this has in fact noting to do with logic but with matrix analysis. Because the obtained results are not naturally to be translated into logical functions such analysis is not helpful for the study of the complexity of poly-contextural logics in general.

The distribution of alpha and beta terms for place-valued tableaux can be used, additionally to a limited value, for logical analysis, e.g., classification reasons, too.

Polylogical definitions are delivering exact information about their local dualism and self-dualisms.

6.1 Gunther's classificaction of the binary functions of $G^{(3, 2)}$

148 Information, Communication and Many-valued Logic 157

f) Subsystems 1 ←→ 2 and 1 ←→ 3

dto		2646	constants

g) All subsystems

Frame:

Irref. 1		124	constants
ref. 1	2 x 124 =	248	constants
Irref. 2-5	4 x 124 =	496	constants
ref. 2–5	4 x 620	2480	constants
		3348	constants
All constants of B II		17442	constants

This classification Table is capable of generalization if either more variables or more values or more of both are introduced. It is asserted that it implies all basic logical concepts which refer to the process of communication. Moreover, it indicates that among the

1728	constants of A I and II
513	constants of B I
17442	constants of B II
19683	constants

G. Gunther, Information, Communication and Many-Valued Logic
Memorias del XIII. Congreso Internacional de Filosofia, México 1963, Communicaciones Libres Vol. V, p. 143-157
Better copy: http://www.vordenker.de/ggphilosophy/gg_inf-comm-many-val-logic.pdf

This classification gives a helpful overview over the 19683 logical functions of the system (3, 2). The method of classification can be applied to systems with more than 3 values. But it is still restricted to binary functions, excluding ternary functions and higher.

Nevertheless, the classification is not giving much information about the *logical* structure of the system under consideration. It is a *combinatorial* classification of the global functions augmented by an interpretation of some situations.

With the following characteristics:
value: acceptance, rejection, symmetric and asymmetric,
frame: irreflexive, reflexive,
place: 1–2, 2–3, 1–3.

More combinatorics: http://www.thinkartlab.com/pkl/tm/MG-Buch.pdf

7 What to study in the complexity of polylogical systems?

7.1 Imitating classical aspects

One special interest surely is to simulate classical logical features, laws, rules, characteristics, definitions etc. in PCL-systems.

DeMorgan
Definitions
Normal forms
Duality
Deduction
Lattices

7.2 Tangled mirroring

Because of the special kind of mediated duality between L_1 and L_2 in $L^{(3)}$ studies have to take into consideration an equal tangled way of comparison of features between the systems not available in classical systems.

The mediated duality between L_1 and L_2 in $L^{(3)}$ has to be distinguished from the immanent duality of classical systems. Mediated duality happens between distributed contextures. Therefore, there is some kind of a queer or tangled duality to study and to compare with the straight duality of classical systems. In other words, classical duality is between two systems, both representing the logic under consideration. Mediated duality happens inside a complexion of mediated logical systems, i.e., between intra-logical systems.

7.3 Independencies of distributed systems

PCL systems with more than 3 contextures offer space for a more independent behavior of local systems.

Because of the commutativity of the negations N_1 and N_4 classical features can be repeated in separation without any involvement into permutative system-changes while simulating classical features.

$$\neg_1 \left(\neg_4 \ X^{(4)}\right) = \neg_4 \left(\neg_1 \ X^{(4)}\right)$$

Classical many-valued logics are focussed on one and only one truth-value situation to determine *tautologies*. This narrowness happens in place-valued systems for m=3, too. But with m>3, separated designational values are introduced, enabling separated systems of tautology. Say, tautology in system S_1 and a different tautology in system S_4. Thus, a kind of a place-valued parallelism is placed.

$$\neg_1 \left(\neg_4 \ X^{(4)}\right) \ \vee^{(4)} \ Y^{(4)} = X^{(4)} \rightarrow \oplus\oplus \rightarrow \oplus\oplus \ Y^{(4)}$$

Logical functions too, can be defined in separation, realizing some independency.

In the formula, the classic correspondence between implication and disjunction plus negation is preserved at the locations L_1 and L_4 in $PCL^{(4)}$.

In this case, and the following, parallelisms of, say, Boolean systems, can be studied and easily applied to mathematics and programming languages.

7.4 Some 3-lattices in PCL$^{(3)}$

Some interesting excerices could be started here onn the ground of given introductions.

3-Lattice$^{(3)}$ = < lattice^1, lattice^2, latice^3 >

lattice = < Iden, Comm, Assoc, Distrib, Modus Ponens > for junctions.

A strictly parallel definition of a distrbutive 3-lattice is given by the homogeneous mediation of its components.

```
3-lattice_distr =
< Iden(3), Com(3), Assoc(3), Distrib(3), MP(3) >
```

This kind of lattices should be, for obvious reasons, called *"lettuce-lattice",* or short: *n-lettuce.* There are *domesticated* and *wild* lettuce-lattices. Domesticated lettuces are strictly harmonic and supportive to their conditions of mediation. Wild lettuces are accepting the risk of disturbing their mediation rules and braking the harmony between the sub-systems making the whole system instable.

7.4.1 Logical strength vs. logical order

Following Rudolf Carnap, Heinz von Foerster introduced the term *"logical strength"* to study his Bio-Logic.

Gunther, then, followed this terminology to classify different strength of place-valued implications. As mentioned before, on an interpretational level, distinctions like conjunctive implication vs. disjunctive implications tried to clarify the situation.

A more structural analysis suggests to understand such situations as different mappings from the full system to its sub-systems, e.g.,: $S^{(3)}$ —> ($S^1S^1S^3$) or ($S^1S^3S^3$).

Thus, in the example, the junctor *"implication"* is distributed by reduction over the sub-systems and here is no need to introduce different types of logical implications.

As a consequence, different kinds of implications, i.e., *implicational orders* and different *deduction rules* based on them can be introduced.

Modus ponens $PM^{1,1,3}$ *for* $G^{(3,2)}_{neg,\ impl}$

$$\frac{\left(X^{(3)} \supset^1\supset^1\supset^3 Y^{(3)}\right) \in ag^{1,1,3},\ X^{(3)} \in ag^{1,1,3}}{Y^{(3)} \in ag^{1,1,3}}$$

Modus ponens $PM^{1,3,3}$ *for* $G^{(3,2)}_{neg,\ impl}$

$$\frac{\left(X^{(3)} \supset^1\supset^3\supset^3 Y^{(3)}\right) \in ag^{1,3,3},\ X^{(3)} \in ag^{1,3,3}}{Y^{(3)} \in ag^{1,3,3}}$$

$$\left\langle \left(\left(X^{(3)} \supset\supset\supset Y^{(3)} \right) \wedge\wedge\wedge\ X^{(3)} \right) \supset\supset\supset Y^{(3)} \right\rangle \in ag^{(3)}$$

The two examples may be read as:

If the implicational order imp^{113} of X and Y in $G^{(3)}$ is tautological for ag^{113} and X is tautological in ag^{113}, *then* Y is tautological in ag^{113}. The same with: imp^{133}.

Obviously, the meta-logical *then* has to be read as *then$^{(3)}$* and *ag* as "allgemeingültig" (tautological) as part of a polylogical meta-logic.

Given an axiom system Axa$^{(3)}$ for different implicational orders the corresponding deductions systems Ded$^{(3)}$ = (Axa$^{(3)}$, MP$^{(3)}$) with MP$^{(3)}$ as the different Modi ponens are defined.

http://www.vordenker.de/ggphilosophy/gg_formal-logic-totality.pdf

7.5 Programming languages based on place-valued logics.

7.5.1 Implementing the reflectional behavior of logical objects

The patterns, below, are not necessarily conform with the common introductions of truth-values in programming languages. I follow, partly, the approach of Georg Loczewski as developed for A++. The difference is, obviously, that I'm dealing with polycontextural constellations. The other difference is, that I'm introducing truth-values as *reflectional objects*. Therefore, both values (truth and false) appear in one definition, and are distributed over reflectional positions (contexture$_{i.i}$ and contexture$_{i.j}$). This presentation may be new, compared to ConTeXtures.

Classical languages are not offering an explicit place for the truth-value "false". It seems not to be necessary because the aim is to produced truth-related sentences and the false are simply the opposite. (IF ... THEN, ELSE). To give a reflectional place to the values is a result of the difference-motivated approach to values which is in contrast to a entity-semantic approach.

The modeling follows the introductions given in ConTeXtures et al.

$$samba^{(3)}(\,id,\ id,\ id\,)$$

$$\begin{bmatrix} thematize\ truth\text{-}values(\,T,\ F,\ \mathbf{F}\,) \\ \begin{bmatrix} \begin{bmatrix} identify\ contexture_{1.1} \\ \begin{bmatrix} define\ T \\ \begin{bmatrix} lambda\ (\ a^1\ b^1\ ) \\ \quad (\ if\ a^1\ a^1\ b^1\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \varnothing & \begin{bmatrix} identify\ contexture_{3.1} \\ \begin{bmatrix} define\ T \\ \begin{bmatrix} lambda\ (\ a^3\ b^3\ ) \\ \quad (\ if\ a^3\ a^3\ b^3\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} identify\ contexture_{1.2} \\ \begin{bmatrix} define\ F \\ \begin{bmatrix} lambda\ (\ a^1\ b^1\ ) \\ \quad (\ if\ b^1\ a^1\ b^1\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} identify\ contexture_{2..2} \\ \begin{bmatrix} define\ F \\ \begin{bmatrix} lambda\ (\ a^2\ b^2\ ) \\ \quad (\ if\ a^2\ a^2\ b^2\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \varnothing \\ \varnothing & \begin{bmatrix} identify\ contexture_{2.,3} \\ \begin{bmatrix} define\ \mathbf{F} \\ \begin{bmatrix} lambda\ (\ a^2\ b^2\ ) \\ \quad (\ if\ b^2\ a^2\ b^2\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} identify\ contexture_{3.3} \\ \begin{bmatrix} define\ \mathbf{F} \\ \begin{bmatrix} lambda\ (\ a^3\ b^3\ ) \\ \quad (\ if\ b^3\ a^3\ b^3\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

$$\begin{bmatrix} identify\ contextures^{(m)} \\ \begin{bmatrix} define\ elector \\ \begin{bmatrix} lambda\ (\ contextures^{(m)}\ ) \\ \begin{bmatrix} elect\ contexture \\ \begin{pmatrix} define\ selector \\ \begin{pmatrix} lambda\ (\ a\ b\ ) \\ \quad (\ sel\ a\ b\ ) \end{pmatrix} \end{pmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Because place-valued logics are based on a mix of many-valued and polylogical strategies, the definitions of the selector/elector mechanism which belongs to the polycontextural situations can be used. The operator *if* then is based on the selector *sel.*

Two-step reduction of mediated truth-values to a chain of place values

$$\begin{array}{ccc} poly-logic\ values & & place-valued,\ with/without\ index \\ \begin{bmatrix} t_1 & \longrightarrow & f_1 & & \\ | & & \Updownarrow & & \\ & & t_2 & \longrightarrow & f_2 \\ | & & & & | \\ t_3 & & \longrightarrow & & f_3 \end{bmatrix} & \xrightarrow[reduction]{} & \left\{ \begin{array}{c} \left(T_{1,3} \longrightarrow F_{1,2} \longrightarrow \mathbf{F}_{2,3} \right) \\ \Downarrow \\ \left(T \longrightarrow F \longrightarrow \mathbf{F} \right) \end{array} \right\} \end{array}$$

Gunther's values are not truth-values in a classical sense (cf. Parsons), but are indicating levels of reflection in a reflectional system. Thus, from T to F and to **F,** a hierarchy of reflectional positions (levels) is marked. But the chiastic structure of reflection is omitted. This again, is a hint, that mediation is supposed in place-valued logics but not realized as a working formalism.

Polylogical definition of truth markers

$$samba^{(3)}(\ id,\ id,\ id\ )$$

$$\begin{bmatrix} thematize\ truth\text{-}values(\ t_{1,3},\ f_{1,}\ t_2,\ f_{2,3}) \\ \begin{bmatrix} \begin{bmatrix} identify\ contexture_{1.1} \\ \begin{bmatrix} define\ t_1 \\ \begin{bmatrix} lambda\ (\ a^1\ b^1\ ) \\ (\ if\ a^1\ a^1\ b^1\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \varnothing & \begin{bmatrix} identify\ contexture_{3.1} \\ \begin{bmatrix} define\ t_3 \\ \begin{bmatrix} lambda\ (\ a^3\ b^3\ ) \\ (\ if\ a^3\ a^3\ b^3\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} identify\ contexture_{1.2} \\ \begin{bmatrix} define\ f_1 \\ \begin{bmatrix} lambda\ (\ a^1\ b^1\ ) \\ (\ if\ b^1\ a^1\ b^1\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} identify\ contexture_{2..2} \\ \begin{bmatrix} define\ t_2 \\ \begin{bmatrix} lambda\ (\ a^2\ b^2\ ) \\ (\ if\ a^2\ a^2\ b^2\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \varnothing \\ \varnothing & \begin{bmatrix} identify\ contexture_{2.3} \\ \begin{bmatrix} define\ f_2 \\ \begin{bmatrix} lambda\ (\ a^2\ b^2\ ) \\ (\ if\ b^2\ a^2\ b^2\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \begin{bmatrix} identify\ contexture_{3.3} \\ \begin{bmatrix} define\ f_3 \\ \begin{bmatrix} lambda\ (\ a^3\ b^3\ ) \\ (\ if\ b^3\ a^3\ b^3\ ) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

7.6 Comparison between place-valued and polylogical implementations

7.6.1 The IF-THEN-ELSE distribution

Short for: Polylogical IF-THEN-ELSE

```
[identify contextures(3) boole(3)
 [[(define if(3) - control
   (lambda (expr(3) stmt(3))
    (if ⟨boole1,3 - expression⟩
     then ⟨true1,3 - statement⟩
     else ⟨false1 - statement⟩
     or
     (if ⟨boole2,3 - statement⟩
      then ⟨true2 - statement⟩
      else ⟨false2,3 - statement⟩)))))]]
```

Place-valued IF-THEN-ELSE

```
[identify contextures(3) boole(3)
 [[(define if(3) - control
   (lambda (expr(3) stmt(3))
    (if ⟨boole(3) - expression⟩
     then ⟨T - statement⟩
     else ⟨F - statement⟩
     else ⟨𝐅 - statement⟩))))]]
```

Take a metaphor. Place-valued decisions are like a change in a railway junction. Polycontextural changes are not so convenient. They have to make a jump into another kind of railway logic. And there is even a possibility to choose both at once.

Parallelism in place-valued systems

```
[identify contextures(4) boole(4)
 [[(define if(4)par - control
   (lambda (expr(4) stmt(4))
    (if ⟨boole(4) - expression⟩
     [then ⟨T - statement⟩    ] / / / [then ⟨𝐅 - statement⟩  ]
     [else ⟨F - statement⟩    ]       [else ⟨𝐅'– statement⟩  ]))))]]
```

Because of the nice commutativity of the negation N_1 in sub-system S_1 and negation N_4 in sub-system S_4, with:

$$N_1(N_4X^{(4)})=N_4(N_1X^{(4)})$$

a kind of a control-parallelism can be introduced on the base of place-valued logics.

The above metaphor is in this case not simple, the train comes in parallel with another train, then they change destination. One goes along the information of T-statement, the other take the information from the **F**-statement.

7.6.2 Junctions and transjunctions in comparison

Non-reflectional implementation of mono-form conjunction

$$\begin{bmatrix} thematize\ (\wedge\wedge\wedge) \\ \begin{bmatrix} \begin{bmatrix} identify\ contexture_{1.1} \\ \begin{bmatrix} define\ \wedge_1 \\ \begin{bmatrix} lambda\ (a^1\ b^1) \\ (if\ a^1\ a^1\ b^1) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \varnothing & \varnothing \\ \varnothing & \begin{bmatrix} identify\ contexture_{2..2} \\ \begin{bmatrix} define\ \wedge_2 \\ \begin{bmatrix} lambda\ (a^2\ b^2) \\ (if\ a^2\ a^2\ b^2) \end{bmatrix} \end{bmatrix} \end{bmatrix} & \varnothing \\ \varnothing & \varnothing & \begin{bmatrix} identify\ contexture_{3.3} \\ \begin{bmatrix} define\ \wedge_3 \\ \begin{bmatrix} lambda\ (a^3\ b^3) \\ (if\ a^3\ a^3\ b^3) \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

The distribution matrix for place-valued logics is in fact 1-dimensional and reduced to the main-diagonal of the polycontextural matrix. This lack of space to place logical sub-systems is the source of many, not specially intuitive, interpretations of the basic 15 morphograms (Reflexionsmuster) in the setting of the place-valued logic.

Polylogical scenario **Place-valued scenario**

$$\frac{t_1\ X \vee\!\rightarrow\!\vee\ Y}{f_1\ X \mid f_1\ Y \parallel f_3\ X \mid f_3\ Y} \qquad \frac{f_1\ X \vee\!\rightarrow\!\vee\ Y}{\begin{matrix} f_1\ X \parallel f_3\ X \\ f_1\ Y \parallel f_3\ Y \end{matrix}}$$

$$\frac{T\ X \vee\!\rightarrow\!\vee\ Y}{T\ X \mid T\ Y} \qquad \frac{F\ X \vee\!\rightarrow\!\vee\ Y}{\begin{matrix} F\ X \mid \mathbf{F}\ X \mid \mathbf{F}\ X \\ F\ Y \mid F\ Y \mid \mathbf{F}\ Y \end{matrix}} \qquad \frac{\mathbf{F}\ X \vee\!\rightarrow\!\vee\ Y}{\begin{matrix} F\ X \\ \mathbf{F}\ Y \end{matrix}}$$

$$\frac{t_2\ X \vee\!\rightarrow\!\vee\ Y}{f_2\ X \mid t_2\ Y} \qquad \frac{f_2\ X \vee\!\rightarrow\!\vee\ Y}{\begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}}$$

PM	O1	O2	O3
M1	log1	∅	∅
M2	∅	log2	∅
M3	∅	∅	log3

PM	O1	O2	O3
M1	or	∅	∅
M2	∅	impl	∅
M3	∅	∅	or

PM	O1	O2	O3
M1	log1	∅	∅
M2	∅	log2	∅
M3	log3	∅	∅

PM	O1	O2	O3
M1	or	∅	∅
M2	∅	impl	∅
M3	or	∅	∅

Also there is only one implication, but different implicational orders, it may be convenient, to use, sometimes, different symbols for implications (arrow or folding), depending on their distribution in the logical system.

It wasn't unknown to Gunther that there is a little problem of distribution/mediation which needs a special explanation. Gunther’s solution insisted correctly that the value-sequence of sub-system S_3 is still a disjunction because it is based in the morphogram [1] for disjunction. But it was slightly shifted to a value-sequences corresponding to a value-sequence of sub-system S_1. To solve this point, an interpretation was introduced: it was called a *disjunctive disjunction.* Such *interpretative* solutions had been widely used to justify logical functions in place-valued logics. But they are in no way operational. In polylogical systems such problems are solved naturally by distribution over the polycontextural matrix.

Polylogical and place-valued mono-form implication

$$\frac{t_1\ X \rightarrow\!\rightarrow\!\rightarrow Y}{f_1\ X \,|\, t_1\ Y \,\|\, f_3\ X \,|\, t_3\ Y} \qquad \frac{f_1\ X \rightarrow\!\rightarrow\!\rightarrow Y}{\begin{array}{c||c} t_1\ X & t_3\ X \\ f_1\ Y & f_3\ Y \end{array}}$$

PM	O1	O2	O3
M1	*impl*	*impl*	∅
M2	∅	∅	∅
M3	*impl*	∅	∅

$$\frac{t_3\ X \rightarrow\!\rightarrow\!\rightarrow Y}{f_2\ X \,|\, t_2\ Y} \qquad \frac{f_3\ X \rightarrow\!\rightarrow\!\rightarrow Y}{\begin{array}{c} t_2\ X \\ f_2\ Y \end{array}}$$

$$\frac{T\ X \rightarrow\!\rightarrow\!\rightarrow Y}{T\ X \,\Big|\, T\ Y \,\Big|\, \begin{array}{c} F\ X \\ F\ Y \end{array}} \qquad \frac{F\ X \rightarrow\!\rightarrow\!\rightarrow Y}{\begin{array}{c|c|c} T\ X & TX & \mathbf{F}\ X \\ F\ Y & \mathbf{F}\ Y & \mathbf{F}\ Y \end{array}}$$

PM	O1	O2	O3
M1	*impl*	∅	∅
M2	∅	*impl*	∅
M3	∅	∅	*impl*

$$\frac{\mathbf{F}\ X \rightarrow\!\rightarrow\!\rightarrow Y}{\begin{array}{c} F\ X \\ \mathbf{F}\ Y \end{array}}$$

Place-valued "distribution and mediation" of junctions and transjunction

$$
\begin{array}{l}
chose\ place-valued\ logic\ G^{(3)} \\
thematize\ (\ \oplus \vee \wedge\ ) \\
\quad identify\ contexture_{1.1} \\
\quad define\ \oplus_1 \\
\quad\quad lambda\ (\ a^{(3)}\ b^{(3)}) \\
\quad\quad\quad (\ if\ a^1\ a^1\ a^1) \\
\quad\quad\quad (\ if\ b^2\ a^1\ b^1) \\
\quad\quad\quad (\ if\ b^2\ b^1\ a^1) \\
\quad\quad\quad (\ if\ b^1\ b^1\ b^1\ )
\end{array}
\quad \varnothing \quad \varnothing
$$

PVL – distribution matrix

$(\oplus, \vee, \wedge)$	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	S_2	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	S_3

$$
\varnothing \quad
\begin{array}{l}
identify\ contexture_{2..2} \\
define\ \vee_2 \\
\quad lambda\ (\ a^2\ b^2\ ) \\
\quad\quad (\ if\ a^2\ a^2\ b^2\ )
\end{array}
\quad \varnothing
$$

$$
\varnothing \quad \varnothing \quad
\begin{array}{l}
identify\ contexture_{3.3} \\
define\ \wedge_3 \\
\quad lambda\ (\ a^3\ b^3\ ) \\
\quad\quad (\ if\ b^3\ a^3\ b^3\ )
\end{array}
$$

Despite the importance of transjunctions with their mechanism of acception and rejection of truth-values from neighbor systems there is nothing to observe of an interactional and reflectional pattern enabling parallelism and concurrency of logical functions.

The only shift from one system to another is a mutual permutation, say from a conjunction in sub-system S_i to a subsystem S_j.

That is, the transjunction is placed at subsystem position S_1 and has the values (T**FF**F), where the values (**FF**) are so called "Fremdwerte".

Polycontextural dissemination of transjunction and junctions

$$
\begin{array}{l}
samba^{(3)}\ (\ bif,\ id,\ id\ ) \\
thematize\ (\ trans,\ or,\ and\ ) \\
identify\ contextures^{(3)} \\
lambda\ (a^{(3)}\ b^{(3)}) \\
\quad define\ trans^1 \\
\quad\quad elect_{1.3} \\
\quad\quad\quad (\ if\ \ a\ a\ a) \\
\quad\quad elect_1 \\
\quad\quad\quad (\ if\ \ b\ b\ b) \\
\quad\quad elect_2 \\
\quad\quad\quad (\ if\ \ a\ b\ b) \\
\quad\quad elect_{2.3} \\
\quad\quad\quad (\ if\ \ b\ a\ b) \\
\quad\quad\quad (\ if\ \ b\ b\ a)
\end{array}
$$

$(trans, or, and)$	$O1$	$O2$	$O3$
$M1$	S_1	$\varnothing$	$\varnothing$
$M2$	S_2	S_2	$\varnothing$
$M3$	S_3	$\varnothing$	S_3

$$
\begin{array}{l}
identify\ contexture^2 \\
define\ or^2 \\
\quad lambda\ (\ a\ b\ ) \\
\quad\quad (\ if\ a\ a\ b)
\end{array}
\quad
\begin{array}{l}
identify\ contexture^3 \\
define\ and^3 \\
\quad lambda\ (\ a\ b\ ) \\
\quad\quad (\ if\ b\ a\ b)
\end{array}
$$

7.6.3 Distributed logic units in place-valued logics

As mentioned before, interesting features for place-valued logics are emerging with truth-value sets, m>3. And obviously, there are interesting features for logics with a complication of n>2.

A first feature was mentioned as a place-value parallelism of control-structures based on a certain independency of negations in systems with m>3. This feature can be exploited to very interesting implementations in both soft- and hardware.

Junctions, but also transjunctions, are distributed in place-valued logics over different places defined by the number of sub-systems. For m>3 we can separated sub-systems, say S1 and S4, in a way they behave independently.

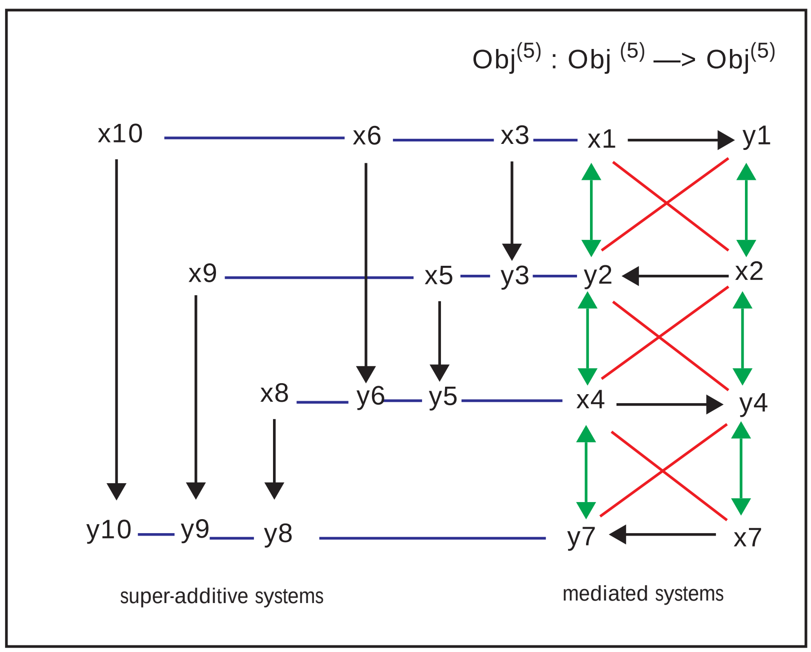
It is only a small step to define for such independent logical sub-systems equally independent operators like NOR or NAND. That is, the Boolean expressions, say in control-structures, are logically expressing the place-valued situations of sub-system-independency.

Thus, there shouldn't be systematic problems denying the introduction of distributed parallel logical circuits.

Nevertheless, such distributed and independent sub-systems are not isolated, they are embedded in the network of mediating sub-systems belonging to the system as a whole. They also can be shifted from one place to another.

In addition to Gunther's diagram of the relation between classical multi-valued and place-valued logics I add the following diagram from PolyLogics. The focus should be on the distributed sub-systems and not too much on the chiastic structure of mediation. The diagram show the mode of distribution and the numbering. Some formulas about the numbering are following.

Super-additivity of mediated systems



The numbering of the sub-systems is given for place-valued logics by the following arithmetical formulas.

Enumeration of sub-systems

3.4 Enumeration

In the PCL literature the following enumeration of the $n = \binom{m}{2}$ subsystems $L_1, \ldots, L_n$ of an m-valued PCL is used:

For the values $1, 2, \ldots, m$ each unordered pair $\{i, j\}$ determines exactly one subsystem L_k; its "number" is defined by $k = \binom{j}{2} - i + 1$. So, we obtain $n = \binom{m}{2}$ subsystems $L_1, \ldots, L_n$ in an m-valued PCL system.

The truth values i, j of L_k are given by: $i = j(j-1)/2 - k + 1$.
and $j = [3/2 + \sqrt{2k - 7/4}]$ (the integer part), cf. $[K2, p.\ 264]$.
Within L_k the two (classical) values are then determined by $T_k = \min(i, j)$, $F_k = \max(i, j)$.
Locally, this defines an order on these two values.
The global values $1, 2, \ldots, m$ then determine the corresponding identifications of local truth values: for example, if $T_k = i$ and $F_l = i$, then $T_k \equiv F_l$.

Obviously, there are independencies between different sub-systems, depending on the classification. But clearly it is the case for the following sub-systems.

S1(x1, y1)/S4(x4, y4) and for
S2(x2, y2)/S7(x7, y7).

If we include the super-additive sub-systems, the same holds for

S3(x3, y3)/S8(x8, y8) and for
S6(x6, y6)/S8(x9, y9).
S1(x1, y1)/S8(x8, y8)

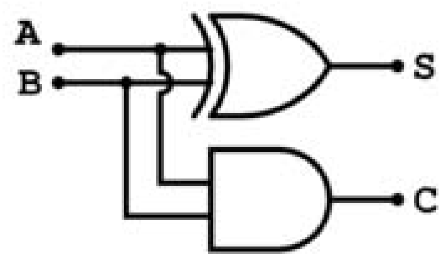
Thus, we might distribute arithmetical devices over different separated sub-systems.

Half adder

is a device which will perform the addition, S, of two numbers. In computing, the adder is part of the ALU, and some ALUs contain multiple adders.

A half adder has two inputs, generally labelled A and B, and two outputs, the sum S and carry output Co. S is the two-bit xor of A and B, and Co is the two-bit and of A and B. Essentially the output of a half adder is the two-bit arithmetic sum of two one-bit numbers, with Co being the most significant of these two outputs.

A half adder is a logical circuit that performs an addition operation on two binary digits. The half adder produces a sum and a carry value which are both binary digits.



S = A xor B
C = A and B

Following is the logic table for a half adder:

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

http://en.wikipedia.org/wiki/Adder_%28electronics%29

Distribution of Half Adders

"Computer scientists are familiar with options in which there are no middle choices between true and false. The lack of such choices is inconvenient – even critical – for example, when determining whether the status of a computer system is *go* or *no-go*. Multi[ple-valued logic is concerned with these intermediate choices." p. 87
G. Epstein, et al.
The development of multiple-valued logic as related to computer science.
in: David C. Rine (Ed.), Computer Science and Multiple-Valued Logic, 1984

In contrast, place-valued logics and polylogics are concerned with the management of *multiple-source, -channel and -layer* computing systems. For each sub-system, the techniques developed in the framework of classical multiple-valued logic can be applied.

Because of the friendly parallelism of sub-system$_1$ and sub-subsystem$_4$ in place-valued logics, adders can be distributed in $G^{(4)}$ without special concerns.

S1 = A1 xor B1 S4 = A4 xor B4
xor1 = N1(or) xor4 = N4(or)
C1 = A1 and B1 C4 = A4 and B4

Following are the logic tables for the S1 and S4 adders:

A	B	C	S	A	B	C	S
0	0	0	0	2	2	2	2
0	1	0	1	2	3	2	3
1	0	0	1	3	2	2	3
1	1	1	0	3	3	3	2

S1 has a binary arithmetic with {0, 1}
S4 has a binary arithmetic with {2, 3}
Arithmetically, arithmetic$_1$ is isomorph to arithmetic$_4$.

What would be the merits of separated but mediated complex control systems?

Today we need control! Especially in the UK where we are in permanent great danger of terrorist attacks. As I have shown several times, ambiguity, polysemy or relevancy of notions are not included in such control and surveillance systems. There is no tolerance for interpretation and negotiation in computerized control and surveillance systems.

So, I have to warn you! If you are one of the crazy young women with a computer, an internet access and a faible for fancy Djihad literature, don't load it down. Go to the *British Library*, ask the *Encyclopædia Britannica*. Otherwise our control system will catch you and classify you under the Terrorism Act 2006. And its all over Beethoven!

Another summary

But we shouldn't forget that Gunther's place-valued logic as it was developed, even with its *generalizations* and *repairs*, is not giving structural information enough about the behavior of the local sub-systems. Therefore, place-valued logics are not properly supporting the job of *formalizing*, *implementing* and *realizing* the complexity and complication of the systems in question. The presentation given goes back to the 70s.

Such questions are, I think, well placed and supported in a polylogical and polycontextural setting as we can know them now.

Some Contributions to PCL

ConTeXtures
Programming Dynamic Complexity.
http://www.thinkartlab.com/pkl/lola/ConTeXtures.pdf

From Ruby to Rudy
http://www.thinkartlab.com/pkl/lola/From Ruby to Rudy.pdf

FIBONACCI in ConTeXtures
http://www.thinkartlab.com/pkl/lola/FIBONACCI.pdf

poly-Lambda Calculus
Lambda Calculi in polycontextural Situations.
http://www.thinkartlab.com/pkl/lola/poly-Lambda_Calculus.pdf

PolyLogics
Towards a formalization of polycontextural Logics.
http://www.thinkartlab.com/pkl/lola/PolyLogics.pdf

8 Numeric representations, reduction strategies, normal forms

The hints and pieces about tedious numeric and normal form presentations given below may be the kind of stuff engineers at the BCL and the AFOSR had been missing after the Mansfield Amendment came into force. But now, it's too late. The hints given may be of actuality but the topic had never enjoyed to be my hobby-horse.

8.1 Pfalzgraf's decomposition strategies

"This is the point where we argue that our decomposition method can be usefully exploited in the following direction. Assuming that the decomposition of a given many valued space into a fibering can also be transformed canonically into the polynomial algebra case leads to the following aspects. A given decision problem then can be fully parallelized, i.e. manipulations can be done fiberwise in parallel, and even more, in each fiber (component) we have in many cases classical logical formulas to handle that means that the corresponding polynomials have degree not greater than two (!). If we have to deal with certain transjunctions (as discussed in examples above) we only have to consider a well known restricted class of operations and, again, the corresponding polynomials have bounded degree (maximally degree four). Having bounded small polynomial degrees might be a big advantage in Gr¨obner basis applications, because high polynomial degrees can cause heavy problems to the performance of computer algebra systems. Thus, the possibility to represent many-valued logics by logical fiberings provides a decomposition, parallelization approach for many-valued connectives yielding fiberwise simpler expressions. This, consequently, leads to an overall reduction of complexity, especially in the corresponding symbolic computation applications.

Polynomial representation

$$X \wedge Y \mapsto X.Y$$
$$X \vee Y \mapsto XY + X + Y$$
$$X \Rightarrow Y \mapsto XY + X + 1$$
$$\neg X \mapsto X + 1$$

This representation clearly demonstrates the decomposition (parallelization) of the original polynomial into smaller components with lower degree."

Jochen Pfalzgraf, On Logical Fiberings and Automated Deduction in Many-valued Logics Using Gröbner Bases
http://www.rac.es/ficheros/doc/00158.pdf

$$X^{(3)} \vee\vee\vee Y^{(3)} = \left(x_1 \vee y_1, x_2 \vee y_2, x_3 \vee y_3\right)^T$$
$$T_{or} = 2X_1^2X_2^2 + X_1^2X_2 + X_1X_2^2 + X_1X_2 + X_1 + X_2$$
$$binom(OR) = XY + X + Y$$
$$binom(T_{OR}) = \left(X_1Y_1 + X_1 + Y_1, X_2Y_2 + X_2 + Y_2, X_3Y_3 + X_3 + Y_3\right)^T$$
$$= \left[(XY + X + Y)^1, (XY + X + Y)^2, (XY + X + Y)^3\right]^T$$

Since we know how to compose and decompose place-valued logical functions, as well poly-contextural functions, it is a natural step to use *numeric techniques* instead of purely logical to represent logical functions, say for technical purposes like multiple-valued switching functions. Numerical techniques had been widely used in classical 2- and many-valued logics.

.As an example we may introduce the reduction rule for mono-form disjunctions as

$$Reduction-Rule: \quad \frac{2X_1^2X_2^2 + X_1^2X_2 + X_1X_2^2 + X_1X_2 + X_1 + X_2}{(XY + X + Y)^1 \,\big|\, (XY + X + Y)^2 \,\big|\, (XY + X + Y)^3}$$

proposed by Pfalzgraf. The super-operator involved is the identity operator *id*.

This rule gives a kind of a transformation from a global multiple-valued to a local place-valued treatement of the mono-form disjunction.

What is still needed is an algorithm to decompose the T-function into its parts. Until now, there are simply two different interpretations of a multiple-valued matrix.

The numeric definitions have to be connected with the super-operators of the poly-contextural logic. That is, negation is not only a numeric formula but also a permutation of sub-systems. This holds for all super-operators, reduction, bifurcation, etc.

Polynomial presentation for Negations

$$\neg_1 X^{(3)} \mapsto \left[(X+1)^1,\, X^3,\, X^2\right]$$
$$\neg_2 X^{(3)} \mapsto \left[X^3,\, (X+1)^2,\, X^1\right]$$
$$\neg_5 X^{(3)} \mapsto \left[(X+1)^2,\, (X+1)^1,\, (X+1)^3\right]$$

This example is decoomposing multiple-valued negations into place-valued negations by splitting it into its sub-system parts. This partition is mapping the permutions of the sub-systems by the super-operator *perm*, too.

As a consequence, the polycontextural matrix of the distribution of sub-systems has to be involved as the framework for the mappings of the *super-operators* and the placing of the numeric formulas in it. An isolated placing is denying mediation. Similar numerical representations exist for mono-form implication and negations. As was shown before, monoform transjunction can be defined by implication and negation. Hence, the numerical representation, based on this, can be derived directly. A nice consequence from the generalized definition of n-ary place-valued function occurs as a super-efficient method of numeric representation of such functions. But we have to take into consideration that classic multiple-valued logics are dealing with full global n-ary functions without decomposition techniques.

> **Abstract**. The concept of logical fiberings is briefly summarized. Based on experiences with concrete examples an algorithmic approach is developed which leads to a representation of a many-valued logic as a logical fibering. The Stone isomorphism for expressing classical logical operations by corresponding polynomials can be extended to m-valued logics. On the basis of this, a classical deduction problem can be treated symbolically as a corresponding ideal membership problem using computer algebra support with the method of Gr¨obner bases. A logical fibering representation in this context provides a parallelization of the original problem and leads to (fiberwise) simpler polynomials and thus to a reduction of complexity. (Pfalzgraf)

About Gröbner bases and the Buchberger algorithm.

> **Abstract**. We discuss the question of whether the central result of algorithmic Gröbner bases theory, namely the notion of S–polynomials together with the algorithm for constructing Gröbner bases using S–polynomials, can be obtained by “artificial intelligence”, i.e. a systematic (algorithmic) algorithm synthesis method.
> Bruno Buchberger, Towards the Automated Synthesis of a Gröbner Bases Algorithm
> http://www.rac.es/ficheros/doc/00149.pdf

> A Gröbner basis is a set of multivariate polynomials that has desirable algorithmic properties. Every set of polynomials can be transformed into a Gröbner basis. This process generalizes three familiar techniques: Gaussian elimination for solving linear systems of equations, the Euclidean algorithm for computing the greatest common divisor of two univariate polynomials, and the Simplex Algorithm for linear programming.
> Bernd Sturmfels, What is …. a Gröbner Basis?, Notices of the Ams, November 2005
> http://math.berkeley.edu/~bernd/what-is.pdf

8.2 Disjunctive Normal Form for $G^{(3)}$

Another kind of representation is given by the *normal forms* for conjunction and disjunction. This is working well for junctions but is not very practical for transjunctions. The direct analogy would have to be augmented by a more adequate representation mechanism for transjunctions.

Disjunctive normal form for mono-form disjunction and conjunction

$$X \vee\vee\vee Y \equiv \begin{bmatrix} \left((A_1 \wedge A_2) \vee (A_1 \wedge \neg A_2) \vee (\neg A \wedge A_2)\right)^1 \\ \vee \left((A_1 \wedge A_2) \vee (A_1 \wedge \neg A_2) \vee (\neg A \wedge A_2)\right)^2 \\ \vee \left((A_1 \wedge A_2) \vee (A_1 \wedge \neg A_2) \vee (\neg A \wedge A_2)\right)^3 \end{bmatrix}$$

$$X \wedge\wedge\wedge Y \equiv \left[(A_1 \wedge A_2)^1 \vee (A_1 \wedge A_2)^2 \vee (A_1 \wedge A_2)^3\right]$$

The sub-system indices for the variables and operators of the 3-valued binary functions are omitted for notational reasons but guaranteed by the index of the main brackets of the sub-systems.

Such formulas are awfully extensional, as they have to be, and tedious. But even in this little exercise some reductional economy is working if compared to the standard setting for global many-valued functions. Poly-form functions are easily constructed.

Disjunctive normal form for binary poly-form conjunction

$$X \wedge\wedge\vee Y \equiv \left[(A_1 \wedge A_2)^1 \vee (A_1 \wedge A_2)^2 \vee \left((A_1 \wedge A_2) \vee (A_1 \wedge \neg A_2) \vee (\neg A \wedge A_2)\right)^3\right]$$

The same scheme holds for ternary functions. But for reason of decomposability we have to consider the conditions of mediation for n-ary functions. But the normal form strategies applied are independent from the structural properties of n-ary functions.

Disjunctive normal form for ternary mono-form disjunction

$$(X \vee\vee\vee Y) \vee\vee\vee Z \equiv$$

$$\begin{bmatrix} \left((A_1 \wedge A_2 \wedge A_3) \vee (A_1 \wedge A_2 \wedge \neg A_3) \vee (\neg A_1 \wedge A_2 \wedge A_3)(\neg A_1 \wedge A_2 \wedge \neg A_3)(A_1 \wedge \neg A_2 \wedge A_3)(A_1 \wedge \neg A_2 \wedge \neg A_3)\right)^1 \\ \vee \left((A_1 \wedge A_2 \wedge A_3) \vee (A_1 \wedge A_2 \wedge \neg A_3) \vee (\neg A_1 \wedge A_2 \wedge A_3)(\neg A_1 \wedge A_2 \wedge \neg A_3)(A_1 \wedge \neg A_2 \wedge A_3)(A_1 \wedge \neg A_2 \wedge \neg A_3)\right)^2 \\ \vee \left((A_1 \wedge A_2 \wedge A_3) \vee (A_1 \wedge A_2 \wedge \neg A_3) \vee (\neg A_1 \wedge A_2 \wedge A_3)(\neg A_1 \wedge A_2 \wedge \neg A_3)(A_1 \wedge \neg A_2 \wedge A_3)(A_1 \wedge \neg A_2 \wedge \neg A_3)\right)^3 \end{bmatrix}$$

There is no surprise to go on with *transjunctions*. But the nice parallelism is intertwined and the presentation should be changed in the direction of a *tabular* representation form.

Disjunctive normal form for binary poly-form transjunction with conjunction

$$X \oplus\oplus\wedge Y \equiv$$

$$\left[\left((A_1 \wedge A_2) \vee \left((A_1 \wedge \neg A_2) \vee (\neg A_1 \wedge A_2)\right)^2\right)^1 \vee (A_1 \wedge A_2)^2 \vee \left((A_1 \wedge A_2) \vee \left((A_1 \wedge \neg A_2) \vee (\neg A_1 \wedge A_2)\right)^2\right)^3\right]$$

At least at this point the method of linearized normal forms for place-valued logics becomes tedious and in some kind obsolete, too.

Obviously, we have inside a sub-system bracket, compounds of other sub-systems, say $((...)^3 \dots (...)^2)^3$. This fact is naturally pushing for a tabular representation instead of the common linear presentation form.

8.3 Numeric representations

$$global - Matrix \longrightarrow numeric(sops(functions))$$

The global multiple-valued matrix is mapped onto the numeric representation of the distributed place-valued functions.

As an example I modify the approach of Stephen Y. H. Su and Peter T. Cheung.

These *hints* are not intending to give all the necessary definitions but show only the main differences to the presented work to read in Rhine's multiple-valued logic bible.

Complement (negation) in MVL

$$c_i = (a_i)^- \quad iff \quad c_{ij} = \begin{cases} 1 & for\ a_{ij} = 0 \\ 0 & otherwise \end{cases}$$

Complement (negation) definition for classic multiple-valued logic.

Negation schemes for m=3 in PVL

$$c_i = (a^1{}_i a^2{}_i a^3{}_i)^{-1} = ((a^1{}_i)^-, a^3{}_i, a^2{}_i)$$

$$c_i = (a^1{}_i a^2{}_i a^3{}_i)^{-2} = (a_i{}^3, (a^1{}_i)^-, a^1{}_i)$$

$$c_i = (a^1{}_i a^2{}_i a^3{}_i)^{-5} = ((a^2{}_i)^-, (a^1{}_i)^-, (a_i{}^3)^-)$$

Application of the classic definition for negation to polylogics. The complement function is distributed and the permutation of the sub-systems is ruled by the super-operator *perm*.
The values $(1,0)^i$ are corresponding to the truth-value (T_i, F_i), i=1,2,3.

Negations N_1, N_2 and N_5 representation

$$c_i = (a^1{}_i a^2{}_i a^3{}_i)^{-1} \quad iff \quad c_{ij} = \left(\begin{Bmatrix} 1 & for\ a^1{}_{ij} = 0 \\ 0 & otherwise \end{Bmatrix}^1, a^3{}_i, a^2{}_i \right)$$

$$c_i = (a^1{}_i a^2{}_i a^3{}_i)^{-2} \quad iff \quad c_{ij} = \left(a^3{}_i, \begin{Bmatrix} 1 & for\ a^2{}_{ij} = 0 \\ 0 & otherwise \end{Bmatrix}^2, a^1{}_i \right)$$

$$c_i = (a^1{}_i a^2{}_i a^3{}_i)^{-5} \quad iff \quad c_{ij} = \left(\begin{Bmatrix} 1 & for\ a^2{}_{ij} = 0 \\ 0 & otherwise \end{Bmatrix}^2, \begin{Bmatrix} 1 & for\ a^1{}_{ij} = 0 \\ 0 & otherwise \end{Bmatrix}^1, \begin{Bmatrix} 1 & for\ a^3{}_{ij} = 0 \\ 0 & otherwise \end{Bmatrix}^3 \right)$$

The emphasis of this little exercise lies in the *permutation* of the numeric representations of the sub-systems of the place-valued negation.

Compounds of junctions and reductions are trivially mirrored. More interesting are transjunctions because of their bifurcation properties.

Stephen Y. H. Su, Peter T. Cheung, Computer simplifications of multi-valued switching functions, in: Rhine, p. 202, 1984

David Rhine (Ed.), Computer Science and Multiple-Valued Logic.
Theory and Application. Elesevier1984