Metaphors of Dissemination and Interaction of morphoCAs

Functional Analysis of the Graphematics of morphoCAs

Dr. phil Rudolf Kaehr

copyright © ThinkArt Lab Glasgow

ISSN 2041-4358

(work in progress, v. 0.5.5, Dec., Oct. 2015)

Diagrams and Dissemination

Initialization

Initialization

morphoCA requisites

Keywords

ECA, morphoCA, diagrams, reduction, minimization, flow charts, interaction, transaction, mediation, heterogeneous structures, poly-, dis-, intra-, trans-contexturality, contextuality, morphogrammatics, retro-grade recursion, memristivity

Motivation

“Physical computing media are asymmetric. Their symmetry is broken by irregularities, physical boundaries, external connections and so on. Such peculiarities, however, are highly variable and extraneous to the fundamental nature of the media. Thus, they are pruned from theoretical models, such as cellular automata, and reliance on them is frowned upon in programming practice. However, computation, like many other highly organized activities, is incompatible with perfect symmetry. Some standard mechanisms must assure breaking the symmetry inherent in idealized computing models.”

Leonid A. Levin, The Computer Journal Vol. 48, No. 6, 2005
Minimization and flow charts

It is not easy to explain how to understand the results of morphoCAs. It seems that there is a strong conflict between the millions of visualizations, sonifications and structurations managed by the approach of claviatures and the paradigmatic statement developed in the paper “Asymmetric Palindromes” for morphoCAs that “What you see is not what it is”.

Instead of studying the multitude of the products of morphoCAs, another approach that is more focused on the mechanism of the production process of morphoCAs might help to uncover the deep-structural significance of morphogrammatic based cellular automata.

This paper offers some insights into the mechanism of production by the application of reductions (minimizations) of the functional interpretations of the morphoCA rules and by designing the network of the actions of the morphic automata by some flow charts.

There is not yet an algorithmic approach to reduce morphic CA functions accessible. But the distinction between reducible and non-reducible morphoCAs is well defined.

Hence, instead of considering the multi-millions of morphoCA productions, some specific flow charts of the mechanism of production is presented to continue the studies of morphoCAs. With that a kind of reflection a kind of a meta-theory of morphoCAs is introduced.

From this meta-theoretical point of view, morphoCAs might be involved into an introspection between Kaluzhnine-Graph-Schemata of recursivity and poly-contextural memristivity.

A further approach to study the deep-structure of the meaning of morphoCAs will be sketched in a further paper by an analysis of their underlying poly-contextural logics.

Contexturality vs. contextuality

The term “polycontexturality” occurs frequently in sociological studies. Often as a synonyme or replacement of ‘polycentricity’ and linguistically, modal-logically or semiologically identified with ‘contextuality’.

Polycontexturality refers to a trans-classical paradigm of thinkind and writing that is not compatible with established concepts of science, while ‘polycentricity’ and ‘contextuality’ are parts of classical logic (say, modal logic), ontology and semiotics.

“Polycentricity is similar to the concept of polycontexturality in logic. Polycontexturality represents a many-system logic, in which the classical logic systems (called contexts) interplay with each other, resulting in a complexity that is structurally different from the sum of its components (Kaehr and Mahler 1996).”

(Rajendra Singh, Towards Information Polycentricity Theory: Investigation of a Hospital Revenue Cycle, 2011)

A similar approach, chosen out of the ‘polycentricity or contextualist movement’, is proposed e.g. by Lars Qvortrup:

“The implicit idea behind the first three theses is that we are on our way into a society, which is radically different from the so-called modern society. It has been described as “functionally differentiated” (Luhmann 1997), as “polycontextural” (Günther 1979) or as “hypercomplex” (Qvortrup 1998), emphasising that it does not offer one single point of observation, but a number of mutually competing observation points with each their own social context.”

Lars Qvortrup, THE AESTHETICS OF INTERFERENCE: From anthropocentrism to polycentrism and the reflections of digital art

http://www.hotelproforma.dk/Userfiles/File/artikler/lq.pdf

It might provoke some progress if the distinctions proposed in this paper would be applied to systems theory of intra-, inter- and trans-contextually mediated complex dis-contextural constellations and dynamics.

http://memristors.memristics.com/Morphospheres/Asymmetric%20Palindromes.html

http://scholarworks.gsu.edu/cgi/viewcontent.cgi?article=1003&context=ceprin_diss


http://works.bepress.com/cgi/viewcontent.cgi?article=1007&context=thinkartlab
Three kinds of morphoCA diagrams

Three kinds of morphoCA diagrams have to be distinguished:

1. Mono-contextural diagrams (intra)

![Diagram of mono-contextural diagrams]

This ‘technical’ diagram has a *meta-mathematical* representation in the graph schemata calculus for recursion. With this connection, all the meta-theoretical results about computability are ready to be applied.

**Graph scheme for mathematical recursion**

\[
\begin{align*}
\text{(In)} & \\
\downarrow & \\
\text{(D)} & \leftarrow \\
\leftarrow & \rightarrow \\
\text{(Q')} & \uparrow \\
\text{(Q)} & \\
\end{align*}
\]

\[
E(n, a) = (o, n, a), \\
\Gamma(i, n, \omega) = n = i \\
A(i, n, \omega') \\
\delta(i, n, \omega) = (i', n, \omega') \\
m' = m + 1, \quad m \in \mathbb{N}
\]


The first kind is covered by the classical diagrams. These diagrams hold for classical ECAs as much as for mono-contextural morphoCAs of different topological complexity. Morphogrammatically, they are supported by the ‘classical’ morphograms of complexity 2.

2. Poly-contextural diagrams as interaction (inter, trans)

![Diagram of poly-contextural diagrams as interaction]

The second kind is based on a distribution of the diagram of at least 3 loci. This distribution is basic for the interac-
tions between otherwise autonomous automata. The internal structure of the memory/logic unit of the single automata is intrinsically changed toward a chiastic, i.e. memristive behavior of internal and external events.

The interactional activity of the second kind of diagrams is supported by the morphograms of complexity 3. In this field of interactional activity of complexity 3, two different modi might be distinguished:

• **inter-actional** with morphograms mg[5], mg[10] and mg[14], and head(1,2,3) -> i, i=1,2,3

• **trans-contextural** mg[11], mg[12] and mg[13] with head ([1,2]) -> 3.

3. Poly-contextual diagrams as mediation

![Diagram](image)

The third kind is based on the second kind but is involving the whole structural complexion of the distributed morpho-CAs. Only with this configuration the full graphematic character of morphoCAs enters the trans-classic game of computation, interaction, reflection and mediation.

**Mediative** actions are supported by morphograms of the minimal complexity 4, represented by the morphogram mg[15] with head([1,2,3]) -> 4.

### Poly-contextual basic component

![Diagram](image)

**Examples**

**Functions**

```
intra : [0, 0, 0] \rightarrow [0] : sys1, 1, 1 \rightarrow sys1, 1, 1
trans : [0, 0, 1] \rightarrow [2] : sys1, 1, 1 \rightarrow sys3 | sys1 | sys3
inter : [1, 2, 1] \rightarrow [0] : sys2, 1, 2 \rightarrow sys1 | sys3 | sys1
med : [1, 0, 2] \rightarrow [3] : sys1, 2, 3 \rightarrow sys5 | sys4 | sys6
```

**Action schemes**

```
The compound morphogram \text{ruleDM}[\{1, 3, 4, 11, 15\}] inscribes the deep-structure of the mediation of intra- and inter-contextural actions.

\begin{itemize}
  \item Poly-contextural logic

  Quite obviously, intra-contextural morphograms are representing the deep-structure of junctional mono- and poly-contextural operators.

  As a first remark, inter- and trans-contextural morphograms are representing the deep-structures of transjunctional poly-contextural operators.

  Morphogram mg[15] represents the full differentiations of the interplay of inter- and trans-contextural poly-contextural operators.

  The proof-theoretical metaphor of polycontextural interplays is not anymore just a ‘tree’ but a ‘forest of colored trees’.
\end{itemize}
Example: ternary 3-contextural transjunctions of ruleDCM([1, 2, 12, 13, 5])

\[
\begin{align*}
{0, 0, 0} &\rightarrow 0, {0, 0, 1} \rightarrow 0 : \text{disjunctions in syst1,} \\
{0, 0, 2} &\rightarrow 0 : \text{junction in syst3, "0\cdot(0\cdot2)\equiv 0"}, \\
{2, 2, 0} &\rightarrow 2 : \text{junction in syst3, "2\cdot(2\cdot0)\equiv 2"}, \\
{0, 1, 0} &\rightarrow 2, {1, 0, 0} \rightarrow 2 : \text{transjunctions from syst1 to syst2 || syst3}, \\
{1, 2, 1} &\rightarrow 0 \quad : \text{transjunctions from syst2 to syst1 || syst3} \\
{0, 2, 2} &\rightarrow 1, {2, 0, 0} \rightarrow 1 : \text{transjunctions from syst3 to syst1 || syst2,} \\
{0, 1, 2} &\rightarrow 0, {2, 1, 0} \rightarrow 2 : \text{transjunctions from syst1, 3, 2 to syst1 || syst3}.
\end{align*}
\]

1. Mono-contextural diagrams: ECA and morphoCA\((m,2,n)\)

Diagram of the ECA scheme

K. Salman’s paper “Elementary Cellular Automata (ECA) Research platform” gives an elaborated definition and explanation of the concept of ECAs.

For the purpose of an introduction of morphoCAs it suffice to connect to some of its terms and constructions.

“For ease of illustration we let the CA evolve according to one uniform neighborhood transition function and fixed radius which is a local function (rule) \(R_0: \mathbb{Q}^{2r+1} \rightarrow \mathbb{Q}\) where the CA evolves after a certain number of time steps \(T\).

In this case we have a total of \(2^{2r+1}\) distinct rules. It follows that a 1-D CA is a linear lattice or register of \(K \in \mathbb{N}\) memory cells. Each cell is represented by \(c_k\), where \(k = \{1: K\}, K \in \mathbb{N}\) and \(t = \{1: T\}, T \in \mathbb{Z}\) that describes the content of memory location at evolution time step \(t\).” (K. Salman)

Figure 5, Detailed Structure of a typical Cellular Automaton Cell for rule 30.


Diagram of the CA rule in respect of input and output cells in time \(t\) to \(t+1\)
Description of the mechanism of the CA calculation

The object \( D_k \) of \( C_k^{i+1} \) of Fig. 5 is a result of the calculation of the logical unit \( U \), i.e. Transition Rule Logic, in relation to its inputs \( C_{k+1}^i \) and \( C_{k+1}^j \), but it is also at the same time the initial value, \( Q_k \), in the Memory Cell, of a new calculation of a next step of the CA.

This new calculation might happen intra-contextually as a mapping from \( Q_k \) as \( Q' \) to the logic unit \( U \) or trans-contexturally as a mapping from \( Q_k \) as \( Q' \) to the new object \( D_k \) of \( D_k^{i+1} \) in CA\(^2\) where it becomes the new value of \( Q_k^{i+1} \) for a calculation in CA\(^3\).

The presumption of the classical model of ECAs is certainly that all components are from the same contexture, and having the same clock.

Mono-contextual CAs are homogeneous structures.

Classical Cellular Automata. Homogeneous Structures

By V. Z. Aladjev

Diagram of the sub-rule definition of ECAs

A sub-rule implementation of the ECA rules might augment its computational efficiency and reduce numeric complexity for programmable hybrid ECA compositions.
As it is well known, CAs are understood as parallel computing concepts and devices.

There is no doubt that the sketched sub-rule approach can be concretized and implemented as a ‘hybrid’ ECA on a hardware board like Spartan-6 FPGA Connectivity Kit or similar. (http://www.xilinx.com/products/boards-and-kits.html)

A further step in augmenting the granularity of CAs might be achieved with the sub-rule approach for ECA rules. Each ECA rule is defined in a sub-rule oriented approach as a composition of sub-rules. Thus all compatible sub-rules can be applied in parallel to realize a single ECA rule.

Also the sub-rule approach is defining the ECA rules is not yet showing the flow chart of the interactions of the sub-rules to build the ECA rule.

ECA-rule = [eca1, eca2, ..., eca8]

**Example**: ECA rule 210

```
C_k^{-1}, C_k^0, C_k^1
: intra
```

```
ruleECA[{6, 7, 3, 9, 10, 12, 13, 15}]
```

```
{{0, 0, 0} → 1,  
 {0, 0, 1} → 1,  
 {0, 1, 0} → 0,  
 {0, 1, 1} → 1,  
 {1, 0, 0} → 0,  
 {1, 0, 1} → 0,  
 {1, 1, 0} → 1,  
 {1, 1, 1} → 0}
```

```
FromDigits[{{1, 1, 0, 1, 0, 0, 1, 0}, 2] 210
```

Hence the ECA rule 210 is represented by the tuple (6,7,3,9,10,12,13,15) of ECA sub-rules.

Flow chart of the parallel realization of the 8 sub - rules of an ECA.

"If in a CA the same rule applies to all cells, then the CA is called a uniform CA; otherwise the CA is called a hybrid CA (Fig. 1)."

*Theory and Applications of Cellular Automata in Cryptography*

S.Nandi, B.K.Kar and P.Pal Chaudhuri
ECA sub-rule manipulators

The method of sub-rules for ECAs is an abstraction and parametrization of the components of the rule schemes that allows a micro-analysis of the ECAs. The ECA sub-rule manipulator manages all ECA rules of a 1D ECA. The sub-rule manipulators enables a micro-analysis of the behavior of all 2^8 ECA rules.

Each 1-D ECA rule number has a sub-rule number representation. There are just 8 disjunct pairs of sub-rules to define a 1D ECA rule.

The results are visualized below. The combination of the 8 sub-rules covers all the 256 well known ECA rules.
Mono-contextural ruleDCKF\((5,2,3)\)

Reduction (trivial)

\[
\text{ruleDCKF}\{\{1111, 1122, 1211, 2112, 1221, 2121, 2211, 2222\}\}
\]

Random
Scheme: \( (r_1, r_2, r_3, r_4) \rightarrow r_5, r_i = \{0, 1\} \)

2. Poly-contextural diagrams with interactions

**General approach**

**Internal structure of the memory unit of the second kind**

Following for example K. Salman’s classical modelling approach in “Elementary Cellular Automata (ECA) Research platform” a more explicit modelling of the mechanism of morphoCAs might be achieved.

A first crucial difference to the classical concept is the fact that the memory unit is not just passively receiving (D) and sending data (Q) but is also actively deciding to which system of its computational environment they belong and if the data remain in its domain or not. If not, the activity of the memory unit is deciding where that data belong and sends it to the evoked computational unit of the complexion.

In terms of actors, the memory unit is receiving, sending and deciding about the contexts of data. Classical memory actions are strictly intra-contextural. This holds in the same sense for multi-processor systems too. They are acting strictly intra-contexturally, keeping their distributed data together.

Hence, the logic devices in the modified diagram, Fig. 5b, have two function towards its memory units:

1. a decision operation over the logical operations, i.e. to decide if an operation stays inside the contexture or if it leaves trans-contexturally the contexture for another contexture on another layer of the complex poly-layered morphoCA system.

2. the intra-contextural function of producing the junctional values for the corresponding intra-contextural memory in the sense of ordinary logical functions, like NAND or NOR.

The object \( D_o \) of the CA diagram receives a value from the logic unit and it delivers it to \( Q_1 \) for the new calculation with \( c_i \) of the logical unit in time \( t + 1 \).

Secondly, \( Q \) receives the value from \( D \) as a value, not for \( Q^{1,1} \) in \( CA^1 \) but for \( Q^{2,1} \) of the neighbor layer \( CA^2 \). This new value is memorized in the neighbor \( CA^2 \) as the new positive value for calculation in \( CA^2 \), hence it is placed in \( CA^{2,1} \) and not as a genuine value of \( CA_k \) as \( CA^{2,2} \).

The result of the application of the rule in all 3 sub-systems is delivered with the multi-layered system as a whole, i.e. with morphoCA\(^{(3,3)}\) and its rules \( R^{(3,3)} \).

Obviously, the whole automaton with its different layers has to be designed in the epistemological mode of the ‘as-\( abstraction\); i.e. as ‘\( A \) as \( B \) is \( C \)’ and not in the mode of identity with ‘\( A \) is \( A \)’.

The modified diagram is introducing an environment to the original mono-contextural CA diagram that implies the possibility of interactions. The environment of a CA system is the primary condition for a possible self-reflection of the complex system of different and interacting CAs.

The logic behind this construction was first introduced by Gotthard Gunther’s ‘Cognition and Volition’ (1970) which gives a profound explanation of the new concept of the ‘proemial relation’.

**Modified diagram Fig. 5b**
Memristive properties of the memory/logic unit

Why and how is the behavior of the memory units of morphoCAs of second-order and memristive and not just defined as first-order actions of storage and transformations? The main strategy of the whole maneuver is to avoid 'information processing'. Interaction is prior to information exchange.

It could be said: morphoCAs without memristivity are reducible without loss to classical CAs.

\[
\begin{align*}
\text{internal:} & \quad D_k^{1.1} \implies \begin{cases} Q_k^{1.1} \implies D_k^{2.2} \\ Q_k^{1.2} \implies Q_k^{2.1} \end{cases} \\
\text{external:} & \quad U_k^{1.1} \implies D_k^{1.1} \implies \begin{cases} Q_k^{1.1} \implies D_k^{2.2} \implies Q_k^{2.1} \implies U_k^{2.2} \\ Q_k^{1.2} \implies Q_k^{2.1} \end{cases}
\end{align*}
\]

The diagram below, Fig. 1, shows again the chiasm interaction between operators \((M)\) and operands \(\sigma\) distributed over different loci of the kenomic matrix.

"M as \(\sigma\)" is obviously not the so called self-reference of "M is \(\sigma\)."

\[X(M, \sigma) = \begin{cases} M_{1.1} \implies \sigma_{2.1} & M_{2.2} \implies \sigma_{1.2} \\ \sigma_{2.2} \implies M_{1.2} & \sigma_{1.1} \implies M_{2.1} \end{cases}\]

http://memristors.memristics.com/semi-Thue/Notes%20on%20semi-Thue%20systems.html

Fig. 1 Chiasm \((M, \sigma)\)

Explanation of Fig. 1

"The wording here is not only "types becomes terms and terms becomes types" but "a type as a term becomes a term" and, at the same time, "a type as type remains a type"."

Thus, "a type as a term becomes a term and as a type it remains a type". And the same round for terms.

Full wording for a chiasm between terms and types over two loci

Explicitly, first the green round,

"A type \(\sigma_{1.1}\) as a term \(M_{2.1}\) becomes a term \(M_{2.1}\)
and as a type $\sigma_{1,1}$ it remains a type $\sigma_{1,1}$ for a term $M_{1,1}$.

And, “A type $\sigma_{2,2}$ as a term $M_{1,2}$ becomes a term $M_{1,2}$ and as a type $\sigma_{2,2}$ it remains a type $\sigma_{2,2}$ for a term $M_{2,2}$.

And simultaneously, mediated, the second round in red, the same for terms:

“A term $M_{1,1}$ as a type $\sigma_{2,1}$ becomes a type $\sigma_{2,1}$ and as a term $M_{1,1}$ it remains a term $M_{1,1}$ for a type $\sigma_{1,1}$”.

And, “A term $M_{2,2}$ as a type $\sigma_{1,2}$ becomes a type $\sigma_{1,2}$ and as a term $M_{2,2}$ it remains a term $M_{2,2}$ for a type $\sigma_{2,2}$”.

And finally, between terms $M_{1,1}$ and $M_{2,2}$ and types $\sigma_{1,1}$ and $\sigma_{2,2}$, a categorial coincidence is realized.

While between terms and types a morphism (order relation) exists.

Fig. 2 Complete interactional scheme

Hence, this kind of memory is a complexion of ‘memory’ and ‘logic’ as it is supposed for memristive behavior.

There are four basic components plus the clock in the interaction paradigm of morphoCAs.

Calculation:
send/receive,
accept/reject
in generalized time

In contrast to the classical CA with its send/receive properties, there are four basic components plus the clock in the paradigm of morphoCAs. The sens/receive or read/write mechanism is augmented in morphoCAs by a decision-making (trans-logical) component of accept/reject in regard of the sub-system property.

The contrast to Konrad Zuse’s conception of calculation is obvious:

“Rechnen heisst: Aus gegebenen Angaben nach einer Vorschrift neue Angaben bilden.” (Konrad Zuse)
The discontextually of morphoCAs is certainly also not in harmony with Karl Hewitt's monolithic actor approach to computation.

A systematic deconstruction has obviously to deconstruct all 4+1 components of the diagram.

The very first deconstruction happens by parametrizing the inputs. Each input/output, i.e. send/receive action might belong to a different contexture. Hence, the very first task of the automaton is to handle such profound diversity. This job is obsolete for classical CAs because all data are from/in the same contexture.

This contextual embodiment of the fourth term, $c_k^{-1}$, explains why the term is not just an extensional result of a mapping but is structurally depending on the conceptual 'history' of the 3 previous actions.

This understanding of the morphoCA rules relates back to the concept of the $\alpha/\nu$-structure of morphic objects and actions within the concept of the proposed memristive automata.

http://www.thinkartlab.com/pkl/media/SKIZZE-0.9.5-medium.pdf

http://works.bepress.com/thinkartlab/20/

http://transhumanism.memristics.com/Diagrammatik.ppt.htm

**From memristive flip-flop to memristive interactions**

*Finite state machines and morphoCAs*

"A Cellular Automaton (CA) is an infinite, regular lattice of simple finite state machines that change their states synchronously, according to a local update rule that specifies the new state of each cell based on the old states of its neighbors." (Kari)

http://users.utu.fi/jkari/ca/CAintro.pdf

"Furthermore, since the ECA is actually a finite state machine then the present state of the neighborhood $c_{k+1}, c_{k+1}, c_{k+1}^3$ of cell $c_k$ at time step $t$ and the next state $c_k^{t+1}$ at time step $t+1$, can be analyzed by the state transition table and the state diagram depicted in figure 4." (K. Salman)

![State machine analysis of Rule 30](http://www.slideshare.net/ijcsit/5413/ijcsit03)

**ECA Rule 30**

```math
FromDigits[{0, 0, 0, 1, 1, 1, 1, 0}, 2]
30

FromDigits[kAryFromRuleTable[
    ruleECA[[1, 2, 3, 9, 5, 11, 13, 15]], 2]
30
```
ruleECA[\{1, 2, 3, 9, 5, 11, 13, 15\}] = rule30

<table>
<thead>
<tr>
<th>rule30</th>
<th>111</th>
<th>110</th>
<th>101</th>
<th>100</th>
<th>011</th>
<th>010</th>
<th>001</th>
<th>000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(5)</td>
<td>: 1</td>
<td>(9)</td>
<td>: 1</td>
<td>(3)</td>
</tr>
<tr>
<td>0</td>
<td>(15) : 0</td>
<td>(13) : 0</td>
<td>(11) : 0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(1) : 0</td>
</tr>
</tbody>
</table>

http://memristors.memristics.com/MorphoFSM/Finite
%20 State %20 Machines %20 and %20 Morphogrammatics.html

System of elementary kenomic cellular automata rules in trito-difference form

**Interpretation**

**Difference scheme**

The difference scheme is a *scheme of differences*, and not just a relational mapping from \( C^1 \) to \( C \).

Also an evolution from \([ C_{k-1}^1, C_k, C_{k+1}^1] \) to \( C_{k+1}^1 \) is defined by all previous elements of time \( t \) of the specified CA rule there is no concrete differentiation between the new state of \( C_{k+1}^1 \) and the previous states defined.

Hence, the new state \( C_{k+1}^1 \) of a classical CA might incorporate any arbitrary value from a pre-given set of values and is not retro-recursive characterized by the differences of the previous constellation it depends.
Monomorphic prolongation

First aspect: iteration
Given a morphogram MG, which is always a localized pattern in a kenomic matrix, a prolongation (successor, evolution) of the morphogram is achieved with the successor operator $s$. To each prolongation a further prolongation is defined by the iterated application of the operator $s$.

The morphogrammatic succession $(MG \xrightarrow{s} MG)$ is founded by its model $(gm \xrightarrow{h} gm)$ and the morphism $f$, guaranteeing the commutativity of the construction.

FSA Example

$C_{k-1}, C_k, C_{k+1} \Rightarrow C_{k-1}, C_k, C_{k+1} + 1$

With $d = \{ \epsilon, \nu \}$, $\epsilon = \text{equal}$, $\nu = \text{non-equal}$

$d_1 = \text{diff} (C_{k-1}, C_k)$,
$d_2 = \text{diff} (C_{k-1}, C_{k+1})$,
$d_3 = \text{diff} (C_k, C_{k+1})$,
$d_4 = \text{diff} (C_{k-1}, C_{k+1} + 1)$,
$d_5 = \text{diff} (C_{k+1}, C_{k+1} + 1)$,
$d_6 = \text{diff} (C_k, C_{k+1} + 1)$.

http://memristors.memristics.com/CA-
Overview/Short-%20Overview%20of%20Cellular%20Automata.pdf
As a third rule, the iterability of the successor operation is arbitrary, which is characterised by the commutativity of the diagram. Hence, the conditions for a (retrograde) recursive formalisation are given.

**Second aspect: anti-dromicity**

Each prolongation is realized simultaneously by an iterative progression and an antidromic retrogression. That is, the operation of prolongation of a morphogram is defined retro-grade by the possibilities given by the encountered morphogram. A concrete prolongation is selecting out of those possibilities its specific successions. All successions are to be considered as being realized at once.

**Third aspect: simultaneity and interchangeability**

This simultaneity of different successions defines the range of the prolongation. This definition of morphogrammatic prolongation is not requiring an alphabet and a selection of a sign out of the alphabet. Hence, the concept of morphogrammatic prolongation is defined by the two aspects of iteration and antidromic retro-gradeseness of the successor operation. The simultaneity of the prolongations is modeled by the interchangeability of its actions.

**Fourth aspect: diamond characterization of antidromicity**

Both aspects together, repeatability and antidromicity with its simultaneous and interchangeable realizations, are covered by the diamond-theoretic concept of combination of operations and morphisms, i.e. composition and saltisition, between morphogrammatic prolongations.

The philosophical status of morphoCA's has yet to be determined. “What’s after digitalism?” might give a hint.

https://www.academia.edu/1873531/Digital_Philosophy_Formal_Philosophy_and_Knowledge_Representation_in_Cellular_Automata

**Internal structure of the morphogrammatic transition rule**

**Recall definitions: classical transition rule**

“Rigid computations have another node parameter: location or cell. Combined with time, it designates the event uniquely. Locations have structure or proximity edges between them. They (or their short chains) indicate all neighbors of a node to which pointers may be directed.

“CA are a parallel rigid model. Its sequential restriction is the Turing Machine (TM). The configuration of CA is a (possibly multi-dimensional) grid with a fixed (independent of the grid size) number of states to label the events. The states include, among other values, pointers to the grid neighbors. At each step of the computation, the state of each cell can change as prescribed by a transition function of the previous states of the cell and its pointed-to neighbors. The initial state of the cells is the input for the CA. All subsequent states are determined by the transition function (also called program).” Leonid A. Levin. Fundamentals of Computing.

http://www.cs.bu.edu/fac/lnd/toc/z/z.html

**Morphogrammatic transition rule**

http://memristors.memristics.com/Memristive%20Cellular%20Automata/Memristive%20Cellular%20Automata.html

<table>
<thead>
<tr>
<th>General scheme</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>rule set, start string</td>
<td>rule set = {1, 7, 8, 4}, start string</td>
</tr>
<tr>
<td>↓ string pos = (Nr., 1)</td>
<td>↓ [bob]: string at pos (Nr., 1)</td>
</tr>
<tr>
<td>↓ ReLabel</td>
<td>↓ ReLabel</td>
</tr>
<tr>
<td>ReLabel (string)</td>
<td>[aba]</td>
</tr>
<tr>
<td>↘ ↘ NextGen</td>
<td>↗ ↗ NextGen</td>
</tr>
<tr>
<td>NextGen (ReLabel (string))</td>
<td>[abaa] [abab] [abac]</td>
</tr>
<tr>
<td>↘ ↘ ε rule - set?</td>
<td>↗ ↗ [abaa] ε rule - set?</td>
</tr>
<tr>
<td>⟨yes; no⟩</td>
<td>⟨yes; no⟩</td>
</tr>
<tr>
<td>↓ apply rule</td>
<td>↓ apply: [abaa] rule7</td>
</tr>
<tr>
<td>result</td>
<td>result = [a]</td>
</tr>
</tbody>
</table>

NextGen is in this morphoCA context a retrograde recursive action and not to be confused by a classical recursion.

What makes the difference?
1. retro-grade recursivity
2. irreducible heterogeneity
3. interactivity and reflectionality

**Morphogrammatic example**

\[
\forall i \in (mg (MG)), 1 \leq i \leq mg (MG) + 1
\]

\[
\begin{align*}
\text{second} & \quad \text{retrogression} \quad \text{first} \\
\downarrow \text{selection} & \quad \uparrow \text{choice} \\
\left[ \text{mg}_1 \ldots \text{mg}_i \ldots \text{mg}_n \right] & \rightarrow \left[ \text{mg}_1 \ldots \text{mg}_i \ldots \text{mg}_n \right] \\
\downarrow \text{choice} & \quad \uparrow \text{selection} \\
\text{first} & \rightarrow \text{progression} \quad \text{second}
\end{align*}
\]

http://memristors.memristics.com/MorphoReflection/Morphogrammatics
%20 of %20 Reflection.html

**Flow charts for morphoCAs**

**Full mediation of input**

Basic scheme: Explanation for morphoCA\(^{(3,3)}\)

\[
\text{Clock}^{(3,3)} = \text{synch} \left( \text{Clock}^{(1,1)}, \text{Clock}^{(2,2)}, \text{Clock}^{(3,3)} \right)
\]

\[
\text{Calculation}^{(3,3)} = \text{mediation} \left( \text{TRL1.1}, \text{TRL2.2}, \text{TRL3.3} \right)
\]

\[
\left\{ \text{TRL}^1 \mid \text{TRL}^2 \mid \text{TRL}^3 \right\} = \begin{bmatrix}
\text{TRL}^1 & - & - \\
- & \text{TRL}^2 & - \\
- & - & \text{TRL}^3
\end{bmatrix}
\]

\[
\text{intra}^{(3,3)} = \left\{ \begin{array}{c}
\text{TRL1.1} (\text{Ct1}k, \text{Ct1}k + 1, \text{Ct1}k - 1) \\
\text{TRL2.2} (\text{Ct2}k, \text{Ct2}k + 1, \text{Ct2}k - 1)
\end{array} \right\}
\]
\[ TRL3.3 ((\text{Ct3k}, \text{Ct3k} + 1, \text{Ct3k} - 1)) \]

\[ \text{ENV}^{(3,3)} = \left[ \text{ENV}^1 \parallel \text{ENV}^2 \right] \parallel \text{ENV}^3 = \]

\[
\begin{array}{c}
\text{Ct1k} + 1 \parallel \text{Ct3k} + 1 \\
\text{Ct2k} - 1 \parallel \text{Ct2k} + 1 \\
\text{Ct3k} - 1 \parallel \text{Ct3k} - 1
\end{array}
\]

\[
\begin{pmatrix}
\text{ENV}^1 - \\
\text{ENV}^2 - \\
\text{ENV}^3 -
\end{pmatrix}
\]

**Arrows**

- Directed arrows: input/output arrows,
- Open headed arrows: inter- and trans-action, mediation arrows

**Explanation for morphoCA\((4,4)\)**

\[
\text{morphoCA}^{(4,4)} =
\]

Full interaction and mediation table for morphoCA\(^{(3,3)}\)

<table>
<thead>
<tr>
<th></th>
<th>S11</th>
<th>S21, 31</th>
<th>S22</th>
<th>S12, 32</th>
<th>S33</th>
<th>S13, 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11</td>
<td></td>
<td></td>
<td>S22</td>
<td></td>
<td>S33</td>
<td></td>
</tr>
<tr>
<td>S22</td>
<td></td>
<td></td>
<td>S12</td>
<td></td>
<td>S13</td>
<td></td>
</tr>
<tr>
<td>S33</td>
<td></td>
<td></td>
<td>S13</td>
<td></td>
<td>S11</td>
<td></td>
</tr>
</tbody>
</table>
Discontexturality of distributed CAs

Poly-layered grid structure

An interpretation of the discontexturality diagram shows that the grid structure of distributed CAs of the morphoCAs are in fact not 1-D CAs but disseminated 1-D CAs. It also shows that disseminated CAs are not necessarily 2- or 3-dimensional or higher. What we see as a linear 1-D grid by the visualization of morphoCA actions is in fact a composition of different parallel 1-D grids projected onto an 1-D grid of an uninterpreted output.

Hence, in functional terms, there is no mapping from $\{0, 1, 2, 3\}^3 \rightarrow \{0,1,2,3\}$ but a composition of partial sub-maps.

Nevertheless, poly-layered grids are not multi-layered, because the layers of a multi-layered system are unified under the umbrella of First-Order Logic with Modal logic and General Ontology (Upper Ontology). While discontexturality implies an interplay of a multitude of irreducibly different logics, each containing their inter- and trans-logical operators, additionally to the full set of intra-logical operators too.

Multi-layered systems are logically defined by the basic intra-logical operations only. Poly-layered systems are involved in an interplay of dis-contextual operations of inter- and trans-contextual actions.

This discontextual approach obviously is in strict conflict with Proposition 1 of category theory and its unique universe $U$:

$$\text{If } x \in U \text{ and } y \subseteq x, \text{ then } y \in U.$$

As usual in such fundamental situation, the proposition is circular. It presumes the uniqueness of its logical universe to work for a definition of its unique category-theoretical universe which is taken as the base for the definition of First-Order Logic and its unique universe of terms.

“Polycontexturality alone is not enough to realize the interwoven dynamics a new world-view is desperate for. Gotthard Gunther introduced his proemial relationship to dynamize his contexts, albeit still restricted to a uni-directional movement. The concept of metamorphosis as part of the diamond strategies, based on polycontexturality and disseminated over the kenomic matrix, is a further step to realize a radical paradigm change in our way of thinking and designing futures.”
The projection marks the difference of the deep-structure and the surface structure of the productions of morphoCAs.

It makes it clear, again, that “What you see is not what it is”. Hence, any ontologizing will fail.

\[
\text{projection} \rightarrow C_{1t+1k} | C_{2t+1k} | C_{3tk-1} | C_{4t+1k}
\]

The difference between multi-layered and poly-layered systems got a conceptual sketch with the paper:

Memristics: Dynamics of Crossbar Systems
Strategies for simplified polycontextural crossbar constructions for memristive computation

“Interchangeability is part of a new axiomatics of poly-categorical diamond systems still to be developed. Interchangeability is defined intra-contextural for composition and juxtaposition, and trans-contextural for interactions, like mediation, replication, iteration and transposition.”

http://www.thinkartlab.com/Memristics/Poly-Crossbars/Poly-Crossbars.pdf
Claviatures for morphoCAs

Claviature: ruleDM

![Claviature: ruleDM Diagram]

Claviature: Random ruleDM

![Claviature: Random ruleDM Diagram]
Claviature: ruleDCKV

Analysis of ruleDM[{1,11,3,9,x}]

\[
\text{ruleDM}\left[\{1, 11, 3, 9, 5\}\right]
\]

\[
\text{reduced ruleDM}\left[\{1, 11, 3, 9, x\}\right]
\]

<table>
<thead>
<tr>
<th>Subrules</th>
<th>Subsystems</th>
</tr>
</thead>
<tbody>
<tr>
<td>ruleDM[1] : {0, 0, 0} \rightarrow 0, S1, 3 : yellow</td>
<td></td>
</tr>
<tr>
<td>ruleDM[3] : {0, 1, 0} \rightarrow 0, S1</td>
<td></td>
</tr>
<tr>
<td>ruleDM[9] : {1, 0, 0} \rightarrow 0, S1</td>
<td></td>
</tr>
<tr>
<td>ruleDM[3] : {0, 2, 0} \rightarrow 0, S3</td>
<td></td>
</tr>
<tr>
<td>ruleDM[9] : {2, 0, 0} \rightarrow 0, S3</td>
<td></td>
</tr>
<tr>
<td>ruleDM[11] : {0, 0, 1} \rightarrow 2, S1 \rightarrow S2, 3 : blue</td>
<td></td>
</tr>
<tr>
<td>ruleDM[11] : {0, 0, 2} \rightarrow 1, S3 \rightarrow S1, 2 : red</td>
<td></td>
</tr>
</tbody>
</table>
Distribution density: [5,1,1]

Flow chart for ruleDM[{1,11,3,9,x}]

Explicit transition system table for ruleDM[{1,11,3,9,x}]

TRL1 .1 || TRL3 .3
(0, 0, 0) -> 0
: D1 .1 -> Q1 .1 -> TRL1 .1 || D3 .3 -> Q3 .3 -> TRL3 .3

TRL1 .1
(0, 1, 0) -> 0,
{1, 0, 0} -> 0
: D1.1 -> Q1.1 -> TRL1.1

TRL3.3
{0, 2, 0} -> 0,
{2, 0, 0} -> 0
: D3.3 -> Q3.3 -> TRL3.3

TRL2.2 || TLR3.3
{0, 0, 1} -> 2
: Q1.2 -> Q3.2 -> TLR3.3 || Q1.2 -> Q2.1 -> TLR22

TRL1.1 || TLR2.2
{0, 0, 2} -> 1
: Q3.1 -> Q1.3 -> TLR1.1 || Q3.2 -> Q2.1 -> Q2.2 -> TLR22

Example: ruleDM[{1, 11, 3, 9, 15}] step-wise realization

start (init) : yellow, red
step 37: \( \{0, 0, 0\} \rightarrow 0, \{0, 1, 0\} \rightarrow 0, \{1, 0, 0\} \rightarrow 0 \):

\( TRL1 \) || \( TRL3 \)

\( \{0, 0, 0\} \rightarrow 0 \)

\( D1 \) || \( D3 \)

\( \{0, 0, 0\} \rightarrow 0 \)

Start of the morphoCA with init \( \{(1), 0\} \) producing the entry “red” with an environment “yellow” with the properties defined at step 37 by the rules:

\( \{(0,0,0)\} \rightarrow 0 \) of sys1\|sys3 and \( \{(0,1,0)\} \rightarrow 0 \) of sys1.

construction: blue, yellow, red, yellow

step 44: \( \{0, 0, 1\} \rightarrow 2 \):

\( TLR2 \) || \( TLR3 \)

\( \{0, 0, 1\} \rightarrow 2 \)

\( Q1 \) || \( Q2 \)

\( \{0, 0, 1\} \rightarrow 0 \)

At step 44, the memory decides that the received value “2” doesn’t belong to its range, i.e. the system 1, defined by the values \( \{0\} \).

The value “2” of system 3 defines a new start at the system 3 with the properties of \( \{0,0,2\} \rightarrow 1 \), \( \{0,2,0\} \rightarrow 0 \), \( \{2,0,0\} \rightarrow 0 \).

step 66: \( \{0, 0, 2\} \rightarrow 1 \):

\( TLR2 \) || \( TLR3 \)

\( \{0, 0, 1\} \rightarrow 2 \)

\( Q1 \) || \( Q2 \)

\( \{0, 0, 1\} \rightarrow 0 \)

Again, at the step 66, the decider of the memory unit of system 3 decides that the value “1” doesn’t belong to its range, i.e. the system 3, defined by the values \( \{0,2\} \).

The value “1” of memory 3 defines a continuation in the system 1 with the background properties of \( \{0,2,0\} \rightarrow 0 \), \( \{2,0,0\} \rightarrow 0 \).

The background is symbolized numerically by 0, i.e. yellow. But “0” belongs to 2 different sub-systems defined by \( \{0\} \) and \( \{0,1\} \).

What counts is not just the value in a system but its contextual relation or difference to other values. Hence the presupposed rule: \( \{0,0,0\} \rightarrow 0 \), holds in general but its significance depend on its context.

iteration of construction
Concerning The wide For A

step 88: \( \{0, 0, 1\} \rightarrow 2 \):

\[
\begin{align*}
\text{TLR2.2} & \mid | \text{TLR3.3} \\
\{0, 0, 1\} & \rightarrow 2 \\
\{0.2 \rightarrow Q3.2 \rightarrow TLR3.3 \mid | Q1.2 \rightarrow Q2.1 \rightarrow TLR22
\end{align*}
\]

At step 88, the memory decides that the received value “2” doesn’t belong to its range, i.e. the system 1, defined by the values \( \{0, 1\} \).
The value “2” of system 3 defines a new start at the system 3 with the properties of \( \{0,0,2\} \rightarrow 1, \{0,2,0\} \rightarrow 0, \{2,0,0\} \rightarrow 0 \).

step 110: \( \{0, 0, 2\} \rightarrow 1, \{0, 2, 0\} \rightarrow 0, \{2, 0, 0\} \rightarrow 0 \),

\[
\begin{align*}
\text{TLR1.1} & \mid | \text{TLR2.2} \\
\{0, 0, 2\} & \rightarrow 1 \\
\{0.3 \rightarrow Q1.3 \rightarrow TLR1.1 \mid | Q3.2 \rightarrow Q2.1 \rightarrow Q2.2 \rightarrow TLR22
\end{align*}
\]

\[
\begin{align*}
\text{TRL3.3} & \rightarrow 0, \rightarrow D3.3 \rightarrow Q3.3 \rightarrow TRL3.3 \\
\{2, 0, 0\} & \rightarrow 0
\end{align*}
\]

Again, at the step 110, the decoder of the memory unit of system 3 decides that the value “1” doesn’t belong to its range, i.e. the system 3, defined by the values \( \{0,2\} \). The value “1” of memory 3 defines a continuation in the system 1 and in system 2 with the properties of \( \{0,1,0\} \rightarrow 0, \{1,0,0\} \rightarrow 0 \).

And so on.

Unfortunately it is necessary to go through these tedious phenomenological interpretations of the mechanism of morphoCAs because without this kind of modelling it isn’t possible to understand the nature of their outcome. Just to enjoy interesting pictures and listening to unheard sounds is not yet enough to understand the novelty of the morphogrammatic approach towards cellular automata and automata in general.

The switch from one automaton to the net of automata is not just ruled by the clock but also by the logic of the unit. If there is a transjunctural result of the logical unit, the calculations have to switch to another automaton. Different types of polycontextural transjunctions are ruling such interactions. Otherwise, without a switch, it stays inside the domain of the automaton for further intra-contextural calculations.


**PCA, programmable CAs**

“As the matter of fact, PCA are essentially a modified CA structure. It employs some control signals on a CA structure. By specifying certain values of control signals at run time, a PCA can implement various functions dynamically in terms of different rules.”


For morphoCAs, the range of reconfiguring processors is not limited to the range of classical CAs but spans over a wide range of trans-classical paradigms of morphoCAs also including classical CAs.

The specification of morphoCAs shows clearly the paradigmatical difference between morphoCAs, ECAs and PCAs.

Concerning the sub-rule approach, morphoCAs might be seen as ‘hybrid’ CAs with transjunctural functions and mediation to be considered.

**3. PCL diagrams for morphoCA\(^{(3,3)}\) with interaction and**
mediation

Analysis of minimized ruleDCM[\{1, 2, 12, 13, 5\}]

```
ArrayPlot[CellularAutomaton[
   {0, 0, 0} \rightarrow 0,
   {0, 0, 1} \rightarrow 0, {0, 0, 2} \rightarrow 0, {2, 2, 0} \rightarrow 2,
   {1, 2, 1} \rightarrow 0, {0, 1, 0} \rightarrow 2,
   {0, 2, 2} \rightarrow 1, {1, 0, 0} \rightarrow 2, {2, 0, 0} \rightarrow 1,
   {0, 1, 2} \rightarrow 0, {2, 1, 0} \rightarrow 2
},
   \{(1), 0\}, 11\],
ColorRules \rightarrow \{1 \rightarrow \text{Red}, 0 \rightarrow \text{Yellow}, 2 \rightarrow \text{Blue}, 3 \rightarrow \text{Green}\},
Mesh \rightarrow \text{True}, \text{ImageSize} \rightarrow 100]
```

Analysis

```
ruleDM[\{1, 11, 3, 9, x\}]
```

Distribution density: [7,2,2]

The *distribution density* of a morphoCA constellation gives a simple measure for classification and comparison of morphoCAs. It holds for reduced and non-reduced morphoCA constellations.

```
Analysis of the interaction patterns
```

```
\{(0, 0, 0) \rightarrow 0, : \text{sys1}
\{(0, 0, 1) \rightarrow 0,
\{(0, 0, 2) \rightarrow 0, : \text{sys3}
\{(2, 2, 0) \rightarrow 2,
\{(0, 0, 0) \rightarrow 0
```

Metaphors of Dissemination.nb
Diagram scheme for ruleDCM[{1, 2, 12, 13, 5}]

Simplified diagram of interactions and mediation for morphoCA

Analysis of ruleDM[{1, 2, 12, 13, 5}]
ArrayPlot[CellularAutomaton[
{
{0, 0, 0} → 0,
{0, 0, 1} → 0, {0, 0, 2} → 0, {0, 0, 3} → 0,
{2, 2, 0} → 2,
{3, 2, 1} → 0, {0, 1, 0} → 2,
{2, 2} → 3, {1, 0, 0} → 2, {2, 0, 0} → 1, {3, 2, 2} → 0,
{0, 2, 1} → 0, {0, 3, 2} → 0, {2, 1, 0} → 2, {3, 2, 1} → 3
},
{1, 0}, 11],
ColorRules -> {1 -> Red, 0 -> Yellow, 2 -> Blue, 3 -> Green},
Mesh → True, ImageSize → 100]

Analysis

intra + inter + trans

ruleDM[(1, 2, 12, 13, 5)]

Distribution density: [7,1,3]

Analysis of the interaction patterns

{0, 0, 0} → 0, : sys1
{0, 0, 1} → 0
{0, 0, 2} → 0, : sys3
{2, 2, 0} → 2,
{0, 0, 0} → 0
{0, 0, 3} → 0, : sys6
{0, 0, 0} → 0
4. PCL diagrams with interactions and mediations: morphoCA\(^{(4,3,3)}\)

Analysis of ruleDM[\{1,11,3,4,15\}]

\[
\text{ruleDM}[\{1, 11, 3, 4, 15\}]
\]

Reduced
Random (restricted by reduction)

Analysis

Analysis of ruleDM([1,11,3,4,15])

<table>
<thead>
<tr>
<th>ruleDM([1,11,3,4,15])</th>
</tr>
</thead>
<tbody>
<tr>
<td>x/yz 00 01 10 02 03 20 30</td>
</tr>
<tr>
<td>0  0; sys1, 3 2; sys2, 3 0; sys1 1; sys1 3; sys2, 3 2; sys 0; sys2 0; sys3</td>
</tr>
<tr>
<td>1  1; sys1 1; sys1 - 3; sys2, 3 2; sys - -</td>
</tr>
<tr>
<td>2  - 3; sys - 2; sys 1; sys2, 4 - -</td>
</tr>
<tr>
<td>3  - 2; sys - 2; sys 3; sys3 - -</td>
</tr>
</tbody>
</table>

Distribution density: [4,5,4,3]

<table>
<thead>
<tr>
<th>ruleDM([1,11,3,4,15])</th>
</tr>
</thead>
<tbody>
<tr>
<td>x/yz 00 01 10 02 03 20 30</td>
</tr>
<tr>
<td>0  0; sys1, 3 2; sys2, 3 0; sys1 1; sys1 3; sys2, 3 2; sys 0; sys2 0; sys3</td>
</tr>
<tr>
<td>1  1; sys1 1; sys1 - 3; sys2, 3 2; sys - -</td>
</tr>
<tr>
<td>2  - 3; sys - 2; sys 1; sys2, 4 - -</td>
</tr>
<tr>
<td>3  - 2; sys - 2; sys 3; sys3 - -</td>
</tr>
</tbody>
</table>

Analysis of the interaction patterns

Calculation: intra-contextural action

\[
\begin{align*}
0, 0, 0 \rightarrow 0, & : \text{sys1} \\
0, 1, 0 \rightarrow 0, & \\
1, 0, 1 \rightarrow 1, & \\
1, 0, 0 \rightarrow 1, & \\
0, 2, 0 \rightarrow 0, & : \text{sys3} \\
2, 0, 2 \rightarrow 2, & \\
0, 0, 0 \rightarrow 0 & \\
\end{align*}
\]

Alteration: trans-contextural action from sys1 to sys2||sys3 and from sys3 to sys2||sys1
Mediation: poly-layered action

\[
\begin{align*}
\{0, 0, 1\} & \rightarrow 2: \text{sys1} \rightarrow \text{sys3} \mid \text{sys1} \mid \text{sys3} \\
\{0, 0, 2\} & \rightarrow 1: \text{sys3} \rightarrow \text{sys1} \mid \text{sys2} \mid \text{sys1}
\end{align*}
\]

Interpretation of mediation

\[
\begin{align*}
\{1, 0, 2\} & \rightarrow 3: \text{sys1}, 2, 3 \rightarrow \text{sys5} \mid \text{sys6} \mid \text{sys4} \\
\{1, 0, 3\} & \rightarrow 2: \text{sys1}, 6, 5 \rightarrow \text{sys2} \mid \text{sys3} \mid \text{sys4} \\
\{2, 0, 3\} & \rightarrow 1: \text{sys3}, 6, 4 \rightarrow \text{sys2} \mid \text{sys1} \mid \text{sys5} \\
\{2, 0, 1\} & \rightarrow 3: \text{sys3}, 6, 4 \rightarrow \text{sys2} \mid \text{sys1} \mid \text{sys5} \\
\{3, 0, 1\} & \rightarrow 2: \text{sys6}, 5, 1 \rightarrow \text{sys4} \mid \text{sys1} \mid \text{sys3} \\
\{3, 0, 2\} & \rightarrow 1: \text{sys6}, 4, 3 \rightarrow \text{sys5} \mid \text{sys2} \mid \text{sys1}
\end{align*}
\]

Transition system table for ruleDM[\{1, 11, 3, 4, 15\}]

\[
\begin{array}{c|c|c|c|c|c|c}
S1 & S1 & S2 & S3 & S5 & S6 & 3 \\
S2 & S2 & - & S3 & - & - & - \\
S3 & S3 & S1 & - & - & - & - \\
\end{array}
\]

\[
\begin{align*}
\{0, 1\} = \text{sys1}, \{1, 2\} = \text{sys2}, \{2, 3\} = \text{sys4} \\
\{0, 2\} = \text{sys3}, \{1, 3\} = \text{sys5}, \{0, 3\} = \text{sys6}
\end{align*}
\]

Diagram scheme for ruleDM[\{1, 11, 3, 4, 15\}]

The rules placed in the first half are the rules of intra-contextural actions. They don’t refer to other contextures. The rules in the upper part represent the trans-contextural actions between different contextures depicted as directed arrows.
The compound morphogram of \textbf{ruleDM\{1, 3, 4, 11, 15\}} reflects the mediation of intra- and inter-contextural actions of the flow chart. It is the morphogram compound of the flow chart of the actions of the morphoCA \textbf{ruleDM\{1, 3, 4, 11, 15\}}.

\begin{center}
\includegraphics[width=0.4\textwidth]{morphogram.png}
\end{center}

\section*{Non-reducible examples}

Non-reducible automata definitions might be used as complete irreducible building-blocs for complex morphoCAs. For complete irreducible building-blocs, all entries of the transition table are occupied. In other terminology, all intra-, inter- and trans-contextural sections of the flow-chart scheme are occupied.

Irreducible rules are playing the same role for morphoCAs as the irreducible binary functions like NAND, XOR for binary reductions. With NAND or NOR, all other two-valued binary function are defined. Because they are not reducible they are used as elementary devies in electronic circuit constuctions.

Unfortunately, there is not yet an algorithmic procedure to minimize (reduce) the functional representation of morphoCA rules.

The question for morphic patterns arises: How many irreducible patterns exist for morphoCA\{3,9\}? In analogy:

\textit{"No logic simplification is possible for the above diagram. This sometimes happens. Neither the methods of Kar-naugh maps nor Boolean algebra can simplify this logic further. […] Since it is not possible to simplify the Exclusive-OR logic and it is widely used, it is provided by manufacturers as a basic integrated circuit (7486)."}

http://www.allaboutcircuits.com


\section*{Example : ruleDM\{1, 2, 12, 4, 15\}}

\subsection*{reducible to steps 22}

\textbf{ruleDM\{1, 2, 12, 4, 15\}}

\begin{center}
\includegraphics[width=0.6\textwidth]{example.png}
\end{center}

\textbf{Reducts}

\begin{align*}
{1, 1, 3} & \rightarrow 1, \quad \{3, 3, 0\} \rightarrow 3, \quad \{0, 2, 0\} \rightarrow 1, \\
{1, 0, 1} & \rightarrow 2, \quad \{3, 1, 3\} \rightarrow 2, \\
{2, 1, 1} & \rightarrow 2 (3, 0, 0) \rightarrow 3, \quad \{3, 2, 2\} \rightarrow 3
\end{align*}
Not reduced

\[ \text{ruleDM}[[1, 2, 12, 4, 15]] \]

Random

Analysis

Analysis of the interaction patterns
Computation: intra-contextural actions

\[
\begin{array}{c}
0, 0, 0 \rightarrow 0, \text{ : sys1} \\
0, 0, 1 \rightarrow 0, \\
0, 1, 1 \rightarrow 0, \\
1, 0, 0 \rightarrow 0, \\
1, 0, 1 \rightarrow 1, \\
1, 1, 0 \rightarrow 1, \\
1, 1, 1 \rightarrow 1,
\end{array}
\]

\[
\begin{array}{c}
1, 1, 1 \rightarrow 1, \text{ : Sys2} \\
1, 1, 2 \rightarrow 1, \\
1, 2, 1 \rightarrow 1, \\
2, 1, 1 \rightarrow 2, \\
2, 2, 1 \rightarrow 2, \\
2, 2, 2 \rightarrow 2.
\end{array}
\]

\[
\begin{array}{c}
0, 0, 0 \rightarrow 0, \text{ : Sys3} \\
0, 0, 2 \rightarrow 0, \\
0, 2, 2 \rightarrow 0, \\
2, 0, 0 \rightarrow 2, \\
2, 2, 2 \rightarrow 2.
\end{array}
\]

\[
\begin{array}{c}
2, 2, 2 \rightarrow 2, \text{ : Sys4} \\
2, 2, 3 \rightarrow 2, \\
2, 3, 2 \rightarrow 2, \\
3, 2, 2 \rightarrow 3, \\
3, 3, 2 \rightarrow 3, \\
3, 3, 3 \rightarrow 3.
\end{array}
\]

\[
\begin{array}{c}
1, 1, 1 \rightarrow 1, \text{ : Sys5} \\
1, 1, 3 \rightarrow 1, \\
1, 3, 1 \rightarrow 1, \\
3, 1, 1 \rightarrow 3, \\
3, 3, 1 \rightarrow 3, \\
3, 3, 3 \rightarrow 3.
\end{array}
\]

\[
\begin{array}{c}
0, 0, 0 \rightarrow 0, \text{ : Sys6} \\
0, 0, 3 \rightarrow 0, \\
0, 3, 3 \rightarrow 0, \\
3, 0, 0 \rightarrow 3, \\
3, 3, 0 \rightarrow 3, \\
3, 3, 3 \rightarrow 3.
\end{array}
\]

Alternation: inter-contextural actions
| Example: ruleDM[{1, 11, 12, 9, 15}] |

ruleDM[{1, 11, 12, 9, 15}] : reducible with |3,3,3| \rightarrow |3| for steps <33

ruleDM[{1, 11, 12, 9, 15}]
Non-reducible for steps > 22

Example: ruleDM[{1, 11, 12, 4, 15}]
Analysis

<table>
<thead>
<tr>
<th>x / yz</th>
<th>00</th>
<th>01</th>
<th>02</th>
<th>03</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>11</th>
<th>12</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>31</th>
<th>32</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>01</td>
<td>11</td>
<td>12</td>
<td>21</td>
<td>02</td>
<td>03</td>
<td>12</td>
<td>22</td>
<td>32</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>23</td>
<td>33</td>
<td>00</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>22</td>
<td>23</td>
<td>33</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>32</td>
<td>03</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>24</td>
<td>34</td>
<td>01</td>
</tr>
<tr>
<td>2</td>
<td>23</td>
<td>33</td>
<td>02</td>
<td>12</td>
<td>22</td>
<td>23</td>
<td>33</td>
<td>03</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>24</td>
<td>34</td>
<td>00</td>
</tr>
<tr>
<td>3</td>
<td>33</td>
<td>03</td>
<td>02</td>
<td>01</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>24</td>
<td>34</td>
<td>00</td>
<td>01</td>
<td>02</td>
<td>03</td>
<td>04</td>
</tr>
</tbody>
</table>

Distribution density: [14,18,15,10]

<table>
<thead>
<tr>
<th>0 / 14</th>
<th>1 / 18</th>
<th>2 / 15</th>
<th>3 / 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>100</td>
<td>200</td>
<td>300</td>
</tr>
<tr>
<td>011</td>
<td>002</td>
<td>001</td>
<td>201</td>
</tr>
<tr>
<td>212</td>
<td>202</td>
<td>101</td>
<td>102</td>
</tr>
<tr>
<td>312</td>
<td>302</td>
<td>003</td>
<td>210</td>
</tr>
<tr>
<td>213</td>
<td>203</td>
<td>103</td>
<td>120</td>
</tr>
<tr>
<td>121</td>
<td>303</td>
<td>010</td>
<td>311</td>
</tr>
<tr>
<td>221</td>
<td>020</td>
<td>130</td>
<td>012</td>
</tr>
<tr>
<td>321</td>
<td>220</td>
<td>211</td>
<td>021</td>
</tr>
<tr>
<td>022</td>
<td>320</td>
<td>013</td>
<td>322</td>
</tr>
<tr>
<td>123</td>
<td>230</td>
<td>113</td>
<td>333</td>
</tr>
<tr>
<td>223</td>
<td>111</td>
<td>222</td>
<td></td>
</tr>
<tr>
<td>231</td>
<td>112</td>
<td>031</td>
<td></td>
</tr>
<tr>
<td>132</td>
<td>313</td>
<td>131</td>
<td></td>
</tr>
<tr>
<td>033</td>
<td>122</td>
<td>331</td>
<td></td>
</tr>
<tr>
<td></td>
<td>023</td>
<td>233</td>
<td></td>
</tr>
<tr>
<td></td>
<td>323</td>
<td>032</td>
<td></td>
</tr>
<tr>
<td></td>
<td>032</td>
<td>133</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{DistrDense}(\text{ruleDM}([1, 11, 12, 4, 15])) = (14, 18, 15, 10) \]

Analysis of the interaction patterns
Computation: intra-contextural actions

\[
\begin{align*}
(0, 0, 0) &\rightarrow 0, \text{ sys1} \\
(0, 1, 1) &\rightarrow 0 \\
(1, 0, 0) &\rightarrow 1, \\
(1, 0, 0) &\rightarrow 1, \\
(1, 1, 1) &\rightarrow 1,
\end{align*}
\]

\[
\begin{align*}
(1, 1, 1) &\rightarrow 1, \text{ Sys2} \\
(1, 2, 1) &\rightarrow 2, \\
(1, 2, 2) &\rightarrow 1, \\
(2, 1, 1) &\rightarrow 2, \\
(2, 2, 2) &\rightarrow 2,
\end{align*}
\]

\[
\begin{align*}
(0, 0, 0) &\rightarrow 0, \text{ Sys3} \\
(0, 2, 2) &\rightarrow 0, \\
(2, 0, 0) &\rightarrow 2, \\
(2, 2, 2) &\rightarrow 2,
\end{align*}
\]

\[
\begin{align*}
(2, 2, 2) &\rightarrow 2, \text{ Sys4} \\
(2, 3, 3) &\rightarrow 2, \\
(3, 2, 2) &\rightarrow 3, \\
(3, 3, 3) &\rightarrow 3,
\end{align*}
\]

\[
\begin{align*}
(1, 1, 1) &\rightarrow 1, \text{ Sys5} \\
(1, 3, 3) &\rightarrow 1, \\
(3, 1, 1) &\rightarrow 3, \\
(3, 3, 3) &\rightarrow 3,
\end{align*}
\]

\[
\begin{align*}
(0, 0, 0) &\rightarrow 0, \text{ Sys6} \\
(0, 3, 3) &\rightarrow 0, \\
(3, 0, 0) &\rightarrow 3, \\
(3, 3, 3) &\rightarrow 3,
\end{align*}
\]

Alternation: inter-contextural actions

\[
\begin{align*}
(0, 0, 1) &\rightarrow 2, \text{ sys1} \rightarrow \text{ sys2} \mid \text{ sys3} \\
(1, 1, 0) &\rightarrow 2, \text{ sys1} \rightarrow \text{ sys3} \mid \text{ sys2} \\
(0, 0, 1) &\rightarrow 2, \\
(1, 0, 1) &\rightarrow 2, \\
(0, 0, 2) &\rightarrow 1, \text{ sys3} \rightarrow \text{ sys1} \mid \text{ sys2} \\
(2, 2, 0) &\rightarrow 1, \\
(2, 0, 2) &\rightarrow 1, \\
(0, 2, 0) &\rightarrow 1, \\
(0, 0, 3) &\rightarrow 2, \text{ sys6} \rightarrow \text{ sys3} \mid \text{ sys4} \\
(3, 3, 0) &\rightarrow 2, \\
(0, 3, 0) &\rightarrow 2, \\
(3, 0, 3) &\rightarrow 1, \\
(1, 1, 2) &\rightarrow 0, \text{ sys2} \rightarrow \text{ sys1} \mid \text{ sys3} \\
(2, 2, 1) &\rightarrow 0, \\
(1, 2, 1) &\rightarrow 0, \\
(2, 1, 2) &\rightarrow 0, \\
(2, 2, 3) &\rightarrow 0, \text{ sys4} \rightarrow \text{ sys3} \mid \text{ sys6} \\
(3, 3, 2) &\rightarrow 1, \text{ sys4} \rightarrow \text{ sys5} \mid \text{ sys2} \\
(2, 3, 2) &\rightarrow 1, \\
(3, 2, 3) &\rightarrow 1, \\
(1, 1, 3) &\rightarrow 2, \text{ sys5} \rightarrow \text{ sys2} \mid \text{ sys4} \\
(3, 3, 1) &\rightarrow 0, \text{ sys5} \rightarrow \text{ sys6} \mid \text{ sys1} \\
(3, 1, 3) &\rightarrow 2, \\
(1, 3, 1) &\rightarrow 2,
\end{align*}
\]
Mediation: poly-layered trans-contextural action

\[
\begin{align*}
\{0, 2, 1\} & \rightarrow 3, \\
\{0, 1, 2\} & \rightarrow 3, \\
\{1, 0, 2\} & \rightarrow 3, \\
\{1, 2, 0\} & \rightarrow 3, \\
\{2, 0, 1\} & \rightarrow 3, \\
\{2, 1, 0\} & \rightarrow 3 : \text{sys1, 2, 3 } \rightarrow \text{sys5 } | | \text{sys6 } | | \text{sys4} \\
\{0, 3, 1\} & \rightarrow 2, \\
\{0, 1, 3\} & \rightarrow 2, \\
\{1, 0, 3\} & \rightarrow 2, \\
\{1, 3, 0\} & \rightarrow 2, \\
\{3, 1, 0\} & \rightarrow 2, \\
\{3, 0, 1\} & \rightarrow 2 : \text{sys1, 6, 5 } \rightarrow \text{sys2 } | | \text{sys3 } | | \text{sys4} \\
\{0, 2, 3\} & \rightarrow 1, \\
\{0, 3, 2\} & \rightarrow 1, \\
\{2, 3, 0\} & \rightarrow 1, \\
\{2, 0, 3\} & \rightarrow 1, \\
\{3, 0, 2\} & \rightarrow 1, \\
\{3, 2, 0\} & \rightarrow 1 : \text{sys3, 6, 4 } \rightarrow \text{sys2 } | | \text{sys1 } | | \text{sys5} \\
\{1, 2, 3\} & \rightarrow 0, \\
\{1, 3, 2\} & \rightarrow 0, \\
\{2, 1, 3\} & \rightarrow 0, \\
\{2, 3, 1\} & \rightarrow 0, \\
\{3, 1, 2\} & \rightarrow 0, \\
\{3, 2, 1\} & \rightarrow 0 : \text{sys4, 5, 2 } \rightarrow \text{sys6 } | | \text{sys3 } | | \text{sys1}
\end{align*}
\]

Example: ruleDM[{1, 11, 8, 4, 15}]

\[
\text{ruleDM[\{1, 11, 8, 4, 15\}]
\]

Random

Analysis
Analysis of the interaction patterns

**Computation:** intra-contextural action

- \{0, 0, 0\} \rightarrow 0, : sys1
- \{0, 1, 0\} \rightarrow 0
- \{0, 1, 1\} \rightarrow 0
- \{1, 0, 0\} \rightarrow 1
- \{1, 0, 1\} \rightarrow 1
- \{1, 1, 1\} \rightarrow 1

- \{1, 1, 1\} \rightarrow 1, : Sys2
- \{1, 2, 1\} \rightarrow 2
- \{1, 2, 2\} \rightarrow 1
- \{2, 1, 1\} \rightarrow 2
- \{2, 2, 2\} \rightarrow 2

- \{0, 0, 0\} \rightarrow 0, : Sys3
- \{0, 2, 0\} \rightarrow 2
- \{2, 0, 0\} \rightarrow 2
- \{2, 0, 2\} \rightarrow 0
- \{2, 2, 2\} \rightarrow 2

- \{2, 2, 2\} \rightarrow 2, : Sys4
- \{2, 3, 2\} \rightarrow 3
- \{2, 3, 3\} \rightarrow 2
- \{3, 2, 2\} \rightarrow 3
- \{3, 3, 3\} \rightarrow 3

- \{1, 1, 1\} \rightarrow 1, : Sys5
- \{1, 3, 1\} \rightarrow 3
- \{1, 3, 3\} \rightarrow 1
- \{3, 1, 1\} \rightarrow 3
- \{3, 1, 3\} \rightarrow 1
- \{3, 3, 3\} \rightarrow 3

- \{0, 0, 0\} \rightarrow 0, : Sys6
- \{0, 3, 0\} \rightarrow 3
- \{0, 3, 3\} \rightarrow 0
- \{0, 3, 0\} \rightarrow 3
- \{3, 0, 3\} \rightarrow 0
- \{3, 3, 3\} \rightarrow 3

**Alternation:** inter-contextural actions
Mediation: poly-layered trans-contextural action
ruleDCKV reduction examples

ruleDCKV5 \(\{1111, 1121, 1213, 1223, 1231, 2111, 2121, 2131, 2211, 2223, 2234, 2311, 2321, 2331, 2344\}\)

Analysis

distribution density: \([11,2,4,2]\)

Analysis of the interaction patterns

\[
\begin{array}{c|c|c|c}
0, 0, 0, 0 & 0, 0, 2, 0 & 0, 0, 0, 0 & 0, 2, 0, 2 \\
0, 0, 0, 1 & 1, 0, 2, 0 & 0, 0, 1, 0 & 0, 0, 2, 0 \\
0, 1, 0, 0 & 1, 0, 2, 0 & 0, 0, 1, 0 & 0, 2, 0, 0 \\
0, 0, 0, 2 & 2, 0, 2, 0 & 1, 0, 0, 0 & 0, 0, 0, 0 \\
0, 0, 0, 3 & 2, 0, 0, 0 & 0, 0, 0, 2 & 0, 0, 0, 0 \\
0, 1, 0, 1 & 0, 2, 0, 1 & 0, 2, 0, 2 & 0, 2, 0, 2 \\
0, 2, 0, 1 & 0, 2, 0, 1 & 0, 2, 0, 1 & 0, 2, 0, 1 \\
0, 2, 0, 3 & 0, 2, 0, 3 & 0, 2, 0, 3 & 0, 2, 0, 3 \\
\end{array}
\]

\(\text{DitrDense}(\text{ruleDCKV5}[\{1111, 1121, 1213, 1223, 1231, 2111, 2121, 2131, 2211, 2223, 2234, 2311, 2321, 2331, 2344\}]) = (11, 2, 4, 2)\)
Further example: non-reducible

\begin{verbatim}
ruleDCKV5[{{1111, 1121, 1213, 1223, 1232, 2111, 2121, 2133, 2211, 2223, 2230, 2311, 2323, 2332, 2300}}]
\end{verbatim}

\begin{verbatim}
(1, 1, 1, 1) \rightarrow 1, \{2, 2, 2, 2\} \rightarrow 2, \{3, 3, 3, 3\} \rightarrow 3,
\end{verbatim}

Not reduced

Random
Layers of morphoCA sub-systems of ruleDM[[1,1,3,4,15]]
Step-wise developments of ruleDM[{1,11,3,4,15}]

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>1</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l</td>
<td>2</td>
<td>7</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>3</td>
<td>8</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>4</td>
<td>9</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>o</td>
<td>5</td>
<td>10</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

cell:
initial:

22 : red
66 : blue 68 : red
110 : red 112 : green 114 : red
154 : blue 156 : blue 158 : blue 160 : red
198 : red 200 : blue 202 : blue 204 : green 206 : red

Transition graphs of reduced ruleDM[{1,11,3,4,15}]

Additionally to the difference of reduced and non-reduced morphoCA rule in respect to their seed structure, there is also an interesting difference between ArrayPlot visualizations and transition graph representations by the GraphPlot of reduced morphoCA rules to observe. All reductions are conserving the full visualization of the original, while the transition structure is significantly reduced.

ruleDM[{1, 11, 3, 4, 15}]

Reduction steps
Full pattern

Without \{1, 1, 1\} -> 1, \{2, 2, 2\} -> 2, \{3, 3, 3\} -> 3,

Without:
\{0, 0, 3\} -> 2,
\{1, 1, 0\} -> 2, \{1, 1, 2\} -> 0, \{1, 1, 3\} -> 2, \{2, 2, 3\} -> 0,
\{2, 2, 0\} -> 1, \{2, 2, 1\} -> 0, \{3, 3, 2\} -> 1, \{3, 3, 0\} -> 2, \{3, 3, 1\} -> 0,

\{3, 2, 0\} -> 1, \{3, 1, 2\} -> 0, \{1, 2, 3\} -> 0

Fully reduced