Diamond Semiotics

An interplay of semiotic and graphematic diamonds

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Abstract

Some preliminary remarks about an interplay of semiotic and graphematic diamonds are sketched. A reconstruction of Alfred Toth's semiotic constructions of diamonds with the help of different notations is introduced. A distinction between the diamond properties of basic semiotic configurations and the composition of semiotic configurations as micro- and macro-analysis is proposed. The as-abstraction for semiotic connections is introduced and a mechanism to complement semiotic figures is proposed.

1. Mathematical semiotics and diamonds

1.1. Semiotics, again?

Thanks to the recent work of the semiotician Alfred Toth about mathematical semiotics and its application to polycontextural and kenogrammatic concepts, like chiasms and diamonds, a chapter of *semiotization* of diamonds and a diamondization of semiotics has to be added to the project of *Short Studies.*

This is a very first response to the profound work of Alfred Toth. It takes me back to the 70s/80s when I got involved in this headaching adventure of confronting Bense's semiotics with Gunther's polycontextural logic and kenogrammatics, both, at this time, quite in *status nascendi*, especially Gunther's project.

Semiotics is defined by Peirce and is elaborated in extenso by Bense and Toth as a triadic-trichotomic system of semiosis, i.e. as a scheme of generating signs. Obviously, it has not to be confused with other sign theoretical projects, like semiology (de Saussure, Barthes) or the pre-war Semiotik for formal systems by Manfred Schröter and Hans Hermes .

Diamonds are not triadic-trichotomic but genuinely tetradic, chiastic, antidromic and 4-fold. Hence, diamonds are not semiotical

Are semiotic diamonds semiotical?

First diamondization: internal or micro
The semiotic sign relation is a product of semiosis which can be modeled as a categorical composition of elementary sign relations. Hence, a diamondization of semiotics is a diamondization of the semiotic composition operation of elementary sign relations. This kind of diamondization shall be called *internal* (micro) diamondization in contrast to the *external* (macro) diamondization of the composition of full sign systems.

Basic work to the study of diamonds of elementary semiotic compositions had been published by the semiotician Alfred Toth.
Toth gives a solution for the diamondization of sign systems with the help of the *inversion* operation (INV) he introduced.

Second diamondization: external or macro
A second kind of diamondization is introduced with the diamondization of the composition of signs as it occurs, i.e. in the constructions of iterative and accretive compositions of sign schemes, e.g. *superposition* and *superisation* of signs.

Transpositions, dualizations, inversions and compositions are semiotic operations, diamondization consists of difference, saltisitions, bridges and complementarity.

1.2. Toth's semiotic diamonds

The semiotic composition operation of the elementary semiotic mappings, like (I->M), (M->O), (I->O), between the objects I, M, O, is *commutative* and *associative*. And obviously the *identity* mapping *id* is realized for the objects I, M, O.
Hence, semiotic composition can be studied as a mathematical category in the sense of category theory with objects I, M, O and its mappings (arrows) between the objects.

In concreto, it still has to be analyzed how the semiotic *matching conditions* for compositions are realized.
In the example above, the question is, how is "2.1" in (3.1 -> 2.1) as a *codomain* and in (2.1 -> 1.3) as a *domain* defined?

The new question which arises for abstract diamond theory is: How are the difference relations and hence the hetero-morphisms defined *in concreto*?

The papers of Toth are filling this gap with his *semiotic* modeling of diamonds.

Toth is suggesting an answer to the question, how to interpret the *difference* relations, with the introduction of the operation of *inversion* INV of a concrete sign scheme. Hence, Toth's interpretation of diamonds is joining together semiotic and diamond thematizations and notational systems.

Inversion
Where is this operation INV from?

The sign relation ZR is defined as a relation of *monadic*, *dyadic* and *triadic* relations:

ZR = (a, (a==>b), (a==>b==>c).

Sign values for ZR are:
a = {1.1, 1.2, 1.3}
b = {2.1, 2.2, 2.3}
c = {3.1, 3.2, 3.3}

ZR = <3.x, 2.y, 1.z> with $x,y,z \in \{1,2,3\}$ and $x<=y<=z$.

It is clear, that the semiotic inversion operation INV is a semiotic operation based on the elementary operations of transpositions and is not leading out of the semiotic domain.
INV is defined by:

INV(a.b c.d e.f) = (e.f c.d a.b).

In contrast, dualization is defined as:

DUAL(a.b c.d e.f) = (f.e d.c b.a).

The *abstract sign scheme* gets an interpretation by the introduction of the instances:
I = interpretant, M=medium and O=object.

Hence, the semiotic triad occurs as morphisms between the instances I, O, M and their combinations, called graph theoretic sign models.

1. (I-->O -->M)	4. (O-->M-->I)
2. (M-->O-->I)	5. (I-->M-->O), (M-->I-->O)
3. (I-->M--> O)	6. (O-->I-->M).

It is proposed by the Stuttgarter School of semiotics (Bense, Walter) that those triadic sign schemes can be composed by their dyadic relations (mappings). Most of the semiotic work is in German, thus it is easily possible that I will miss the correct terminology.

Example:
2. (M ==> O) (O ==> I) = (M ==> O • O==> I)
Hence, triadic-trichotomic sign relations are compositions of *dyadic*-dichotomic relations. This is a strong thesis, and I don't see the necessity of such a reduction.
Even more problematic, Elisabeth Walter (1979, S. 79), speaks of a *lattice theoretical* union of "(M ==> O) (O ==> I) = (M ==> O.O==> I)". (Toth, (2008b), p.11)
Bense (1976) is mentioning a category theoretical composition of the triadic sign conceived as a transition from the set theoretic and relational definition to a more adequate presentation (Darstellung) of semioticity .

In the following, I will first follow this strategy, then I will focus on the composition of triadic-trichotomic sign structures as such.
The diamondization of the internal relations of signs might be called *micro*-analysis, the focus on the latter *macro*-analysis of semiotic diamonds.

These 6 graph theoretic sign models of I, O, M, get an interpretation by their corresponding numeric value occupancies.
Example
3.1 (I-->O-->M)
(3.1 2.1 1.1) (3.1 2.3 1.3)
(3.1 2.1 1.2) (3.2 2.2 1.2)
(3.1 2.1 1.3) (3.2 2.2 1.3)
(3.1 2.2 1.2) (3.2 2.3 1.3)
(3.1 2.2 1.3) (3.3 2.3 1.3)
(Thot, p. 2, 2008a)

The new question which arises now is: How are the *difference* relations of the semiotic diamond and hence the hetero-morphisms defined *in concreto*? More precisely, how are the difference relations between the domains/codomains of morphisms and hetero-morphisms of semiotic composition defined? And is the inversion INV operation strong enough to define the differentness of the new hetero-morphisms?

A semiotic diamond by Toth

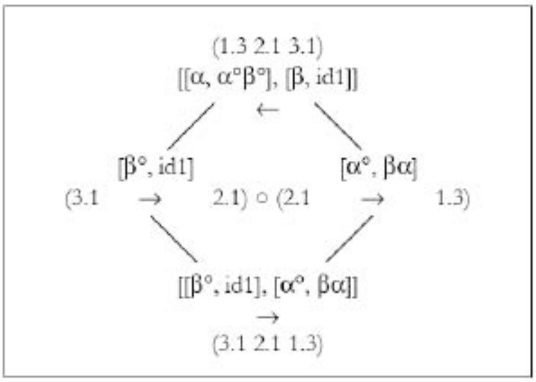
Semiotic diamond for (3.1 2.1 1.3) with
INV (3.1 2.1 1.3) = (1.3 2.1 3.1)
diff(2.1_{ω}) = (1.3)
diff (2.1_{α}) = (3.1)

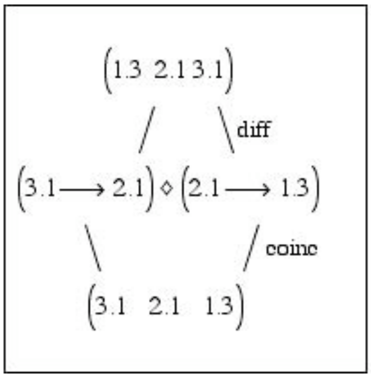
$(3.1_{\alpha} \rightarrow 2.1_{\omega}) \diamond (2.1_{\alpha} \rightarrow 1.3_{\omega})$: composition

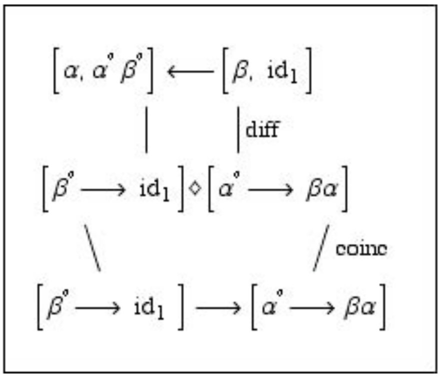
$\underline{(3.1_{\alpha} \rightarrow 1.3_{\omega})}$ | $\underline{(1.3 \longleftarrow 3.1)}$: acception | rejection
(3.1 2.1 1.3) | $(1.3_{2.1}\ 3.1)$: diamond result

In general, the sign class (3.a 2.b 1.c) and its inversion INV(3.a 2.b 1.c) = (1.c 2.b 3.a) are the components for the composition of semiotic diamonds (Thot, p.1, Saltatorien, 2008b).

Toth's presentation of the micro-structure of a semiotic diamond and additional notational explications.


(1.3 2.1 3.1)
[[α, α°β°], [β, id1]]
←
[β°, id1]
[α°, βα]
(3.1 → 2.1) ○ (2.1 → 1.3)
[[β°, id1], [α°, βα]]
→
(3.1 2.1 1.3)


(1.3 2.1 3.1)
diff
(3.1 ⟶ 2.1) ◇ (2.1 ⟶ 1.3)
coinc
(3.1 2.1 1.3)


[α, α° β°] ⟵ [β, id1]
diff
[β° ⟶ id1] ◇ [α° ⟶ βα]
coinc
[β° ⟶ id1] ⟶ [α° ⟶ βα]

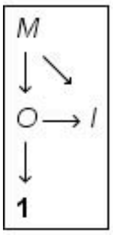
1.3. General micro-structure of semiotic diamonds

Example $M \longrightarrow O \longrightarrow I$

Semiotic composition:

$(M \longrightarrow O) \circ (O \longrightarrow I) \Longrightarrow (M \longrightarrow I).$

Conceptual graph for signs



Semiotics (Peirce, Bense, Toth) is fundamentally mono – contextural and it is blind

for its monocontexturality, *i.e.* the *uniqueness* property, **1**, is not part of the definition of semiot

Diamond composition:

$(M_\alpha \longrightarrow O_\omega) \diamond (O_\alpha \longrightarrow I_\omega) \Longrightarrow (M_\alpha \longrightarrow I_\omega) \,\Big|\, (O_\omega \longleftarrow O_\alpha)$

Diamond relations as rules

Semiotic composition rule

$$\frac{(M \longrightarrow O) \circ (O \longrightarrow I)}{M \longrightarrow I}$$

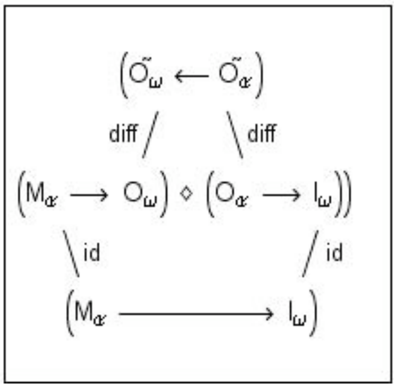
Null

Diamond composition rule

$$\frac{(M_\alpha \longrightarrow O_\omega) \diamond (O_\alpha \longrightarrow I_\omega)}{(M_\alpha \longrightarrow I_\omega) \,\Big|\, (\tilde{O}_\omega \longleftarrow \tilde{O}_\alpha)}$$

$\tilde{O}_\alpha \equiv \text{diff}(O_\alpha)$

$\tilde{O}_\omega \equiv \text{diff}(O_\omega).$

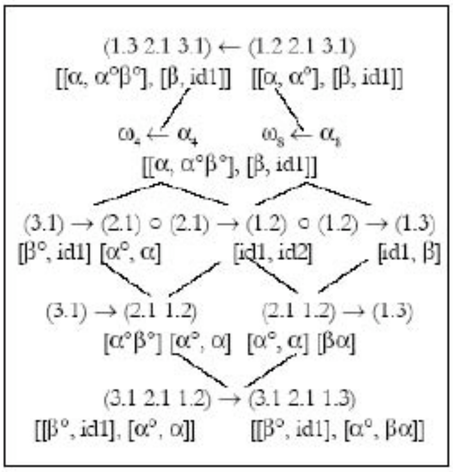


1.4. Compositions of semiotic diamonds

Diamond composition rule $\left(A_\alpha \longrightarrow B_\omega\right) \diamond \left(B_\alpha \longrightarrow C_\omega\right) \diamond \left(C_\alpha \longrightarrow D_\omega\right)$

$$\left(A_\alpha \longrightarrow D_\omega\right) \mid \left(\tilde{B_\omega} \longleftarrow \tilde{B_\alpha}\right) \| \left(\tilde{C_\omega} \longleftarrow \tilde{C_\alpha}\right)$$

Toth’s Example (SemDiamanten, p. 14, 2008b)



A full definition of diamonds in diamond theory requires at least 3 basic morphisms with 2 corresponding basic compositions.
Categories are defined by 2 morphisms and 1 composition. Between 3 categorical morphisms the property of associativity holds naturally. All other properties are inherited by the basic definition of a category.
It could be said that the gaps between hetero-morphisms occurs automatically with the extension of single diamonds to compound diamonds. But the jump operation, between different hetero-morphisms is not automatically given by the extension.
Categories
Structure: *composition* and *identities*, for the
Properties: *unit* and *commutativity* axioms. All based on
Data: arrows, with source and target. Fulfilling the matching conditions for arrows.
Saltatories
Data:inverted arrows
Properties: *diversity* and *jump-law*
Structure: *saltisition* and *differences*.
Diamonds
complementarity
interplay
bi-objects.

Thus, not only hetero-morphisms, with *antidromic* directionality and differences, are new and not covered by a semiotic modeling but saltisitions (jump-operations) too. Further unknown operation to semiotics are *bridge* and *bridging.*

Toth’s application of the diamond strategies (diamonds, chiasms) to semiotics has discovered some important new features, structures and dynamics in the field of semiotics per se.

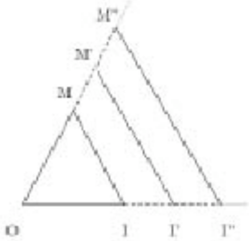
The following paragraphs will present a kind of a reconstruction of Toth’s approach to *semiotic diamonds* and some further ideas to a diamondization of semiotics.

1.5. Semiotic operations

It seems that a more genuine semiotic approach to diamondize the semiotic sign relation might be introduced not as an *internal* reflection inside the sign definition but as an operation between signs as such. That is, the binary (bivariate) operation of *adjunction*, *superisation* and *iteration*. (Obviously, it has to be 3 types of combinations.)

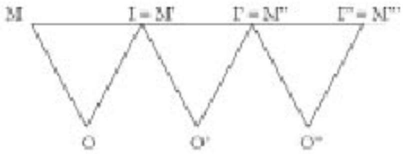
Adjunction

"**Adjunktion** ist eine Zeichenoperation mit reihendem, verkettendem Charakter" (Bense und Walther 1973, S. 11).



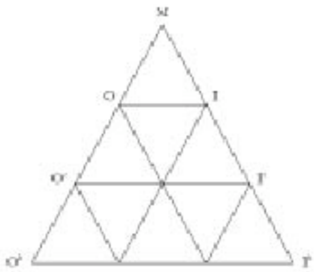
Superisation

"**Superisation** ist ein Zeichenprozess im Sinne der zusammenfassenden Ganzheitsbildung einer Menge von einzelnen Zeichen zu einer 'Gestalt', einer 'Struktur' oder einer 'Konfiguration'" (ibid., S. 106).



Iteration

"**Iteration** ist eine Operation, die alle Teilmengen des Zeichenrepertoires gewinnt, als Potenzmengenbildung darstellbar ist" (ibid., S. 46).



The global sign relation or sign scheme seems to be ruled by an interplay of iteration, superisation and adjunction. Those operators are defining the *complexity* and *complication* of a composed sign structure.
Unfortunately, the semiotic literature is not giving much information about their definition and how their internal mechanism is working. (Cf. Toth, pp.14-15, 2008a)

2. General macro-structure of semiotic diamonds

2.1. Dyads and triads (n-är vs. n-adisch)

The above considerations about formalization strategies for semiotics are supposing the possibility of composing the triadic-trichotomic sign relation out of dyadic sign relations. This is a common approach and might go back to Elisabeth Walter.
Are triads, composed by dyads, still those highly privileged objects Peirce tried to introduce into mathematics with his trichotomic mathematics and trichotromic semiotics? Schroeder has written his famous book about relations on the base of dyadic relations. And from this point of view it is easy to proof that all n-adic relations can be reduced to binary relations. But Peirce wasn't in love with dyads but with triads.

Metacritics

Metatheoretical comments to Toth's mathematical semiotics as well as to the complex Bense/Walter is simply this: it is mathematical. Mathematics is not triadic-trichotomic, hence all the applications of set theory, logic, category theory, etc. are artificial. As long as this situation would be critically reflected in the semiotic studies it would be adequate as a kind of modeling, simulation and formalization. But this is not the case! All such simulative applications comes with Bense’s scholastic authority. Hence, this strategy is sabotaging its own intention of developing a complex system of semiotics.

Nothing is changed with the involvement of the so called “qualitative mathematics” (Kronthaler, Toth). Not only because the dichotomy of quantitative/qualitative is Aristotelian but as a consequence, most operators introduced are “quantitative” per se. Even worse, there is no such thing as a ‘quantitative mathematics’. There is nothing “quantitative” with group and set theory and much less with category theory.

2.2. Semiotic operations

It seems that a more genuine semiotic approach to diamondize the semiotic sign relation might be introduced not as an *internal* reflection inside the sign definition but as an operation between signs as such. That is, the unary or binary (bivariate) operation of *adjunction*, *superisation* and *iteration* as a starting point leads to interesting diamond constructions. (Obviously, it has to be 3 types of combinations.)

Adjunction, superisation and iteration are operations on the general sign model (O, I, M). Such unary operations, like successor operations, can be set into a binary formulation, i.e. as adding an element to an existing element or complexion. This addition, concatenation or composition can be specified as adjunctive, superitative and iterative. All 3 types of compositions have to fulfil their matching conditions to enable the specific composition. Such a specification is considering the internal structure of the sign, depending on its I, O, M constellations (cf. Toth’s “Makrosemiotische Zeichenzusammenhänge").
But operations as compositions or combinations can be understood as diamonds, i.e. as having the acceptional composat and its rejectional sign as results.

General diamond combination scheme

$$\mathrm{Op}_{\mathrm{diam}}\left(\mathrm{sign}_1, \mathrm{sign}_2\right) = \mathrm{sign}_3 \,\Big|\, \overline{\mathrm{sign}_4}$$

$$\mathrm{sign}_1 = \left(O, I, M\right)_1$$

$$\mathrm{sign}_2 = \left(O, I, M\right)_2$$

$$\left(\mathrm{sign}_1 \diamond \mathrm{sign}_2\right) = \left[\mathrm{sign}_3 \,\Big|\, \overline{\mathrm{sign}_4}\right]$$

$$\frac{\mathrm{sign}_1 \diamond \mathrm{sign}_2}{\mathrm{sign}_3 \,\Big|\, \overline{\mathrm{sign}_4}}$$

2.3. Semiotic monadic sign compositions

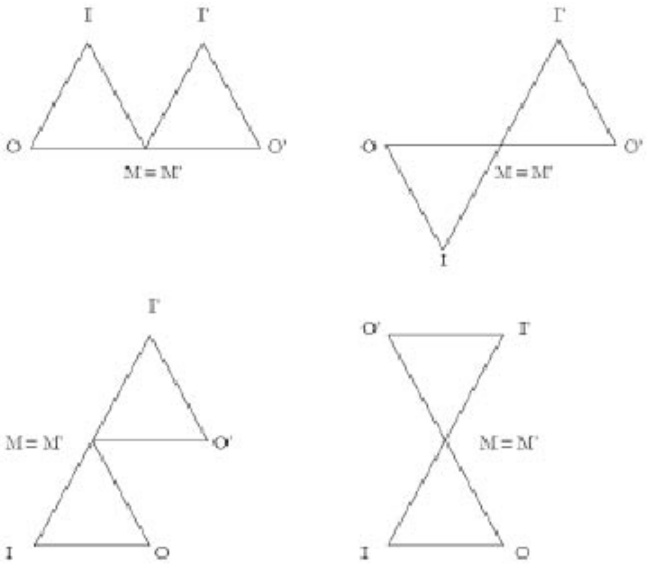
Matching conditions for monadic sign compositions:

$X \equiv X'$, with $X, X' \in \left\{I, O, M, I', O', M'\right\}$

Semiotic examples are taken from: (Toth, 2008 *a*)

Example: 3.1.2.1. M ≡ M' (Toth, p. 20, 2008a)

3.1.2.1. M ≡ M'



$$\frac{(O, I, M) \circ (M', I', O'), \text{ with } MC = M \equiv M'}{(O, I, MM', I', O')}$$

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M \equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix}, \begin{bmatrix} \square & \square & I' \\ O & MM' & O' \\ \square & I & \square \end{bmatrix}, \begin{bmatrix} I' & \square \\ MM' & O' \\ I & O \end{bmatrix}, \begin{bmatrix} O & \square & I \\ \square & MM' & \square \\ I & \square & O \end{bmatrix}}$$

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M \equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} O & \longrightarrow & O' \end{bmatrix}, \begin{bmatrix} O & \longrightarrow & O' \\ I & \longrightarrow & I' \end{bmatrix}, \begin{bmatrix} I \longrightarrow & I' \end{bmatrix}, \begin{bmatrix} I & \longrightarrow & I'' \\ O & \longrightarrow & O' \end{bmatrix}}$$

Example : 3.1 .2 .2. *M'* ≡ *M*

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M' \equiv M}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} I' & \square & I \\ O' & M'M & O \end{bmatrix}, \begin{bmatrix} \square & \square & I \\ O' & M'M & O \\ \square & I' & \square \end{bmatrix}, \begin{bmatrix} O & \square \\ M'M & I \\ O' & I' \end{bmatrix}, \begin{bmatrix} I & \square & O \\ \square & MM' & \square \\ O' & \square & I' \end{bmatrix}}$$

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M'\equiv M}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} O' & \longrightarrow & O \end{bmatrix}, \begin{bmatrix} O' & \longrightarrow & O \\ I' & \longrightarrow & I \end{bmatrix}, \begin{bmatrix} I' \longrightarrow & I \end{bmatrix}, \begin{bmatrix} I' & \longrightarrow & I \\ O' & \longrightarrow & O \end{bmatrix}}$$

Example : 3.1 .2 .3 $M \equiv O'$

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M\equiv O'}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MO' & M' \end{bmatrix}, \begin{bmatrix} \square & \square & I' \\ O & MO' & M' \\ \square & I & \square \end{bmatrix}, \begin{bmatrix} I' & \square \\ MO' & M' \\ I & O \end{bmatrix}, \begin{bmatrix} M' & \square & I' \\ \square & MO' & \square \\ I & \square & O \end{bmatrix}}$$

ETC: to 3.1.2.18. I'≡I (Toth, 2008a, p.32)

2.3.1. Semiotic complementations

Given monadic compositions complementations/supplementations/derivations of the compositions are naturally produced and interpreted as a kind of semiotic deductions.
A reasonable complementation operation has to rely on the *as-abstraction* to build its matching conditions. The existing concept of matching conditions is object-based, i.e. the coincidence of domain and codomain is not yet ruled by the as-abstraction, which is a mechanism to redefine and re-frame the functionality of the encountered objects.

This approach of complementations leads to the question how the different matching types are inter-related.
How, e.g. can Type $A \equiv C$ be generated from Type1, $M \equiv M'$?

Toth (2008a) gives a complex classification of different semiotic constellations. Complementations of one type is generating other types, hence a complementation-based inter-relation between different semiotic constellations becomes accessible for further studies.

Transformation of semiotic structural types by the as-abstraction
As an example, *Type1 M≡M'*, if *M≡M'*, the matching condition for *M≡M'* is naturally realized. If *I* becomes *I"* and *I'* functions as *M"* and *M≡M'* becomes *O"* then the concept of matching conditions is involved into a different kind of interpretation than the *is-abstraction.* Thus, the object-related or set-theoretical interpretation of objects as domains and codomains is not flexible enough to deal such a situation. In other words, the simple categorical identity relation for objects *id* has to be replaced by a more complex operation.

Hence, a reframing of M≡M' of Type1 is generating new matching possibilities and thus a transformation of Type1 M≡M' to other existing or new semiotic constellation types.
The additional matchings for Type1M≡M' are:
$\{I \equiv I',\ M \equiv O" \equiv M',\ M" \equiv I'\}$, which are generating $\begin{bmatrix} II" & \square & M"I' \\ O & MO"M' & O' \end{bmatrix}$.

Example

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 1}\\ M\equiv M'}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1 \end{bmatrix}}$$

First supplement of $\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1 \end{bmatrix}$

$\text{MC}: \; M \equiv M^1 \implies \{ I \equiv I^2, \; M \equiv M^1 \equiv O^2, \; M^2 \equiv I^1 \}$:

$$\frac{\text{supplem}_1\left(\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1 \end{bmatrix}\right)}{\begin{bmatrix} (I)\, I^2 & \square & (I^1)\, M^2 \\ \square & (MM^1)\, O^2 & \square \\ (O) & \square & (O^1) \end{bmatrix}}$$

Second supplement of $\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1 \end{bmatrix}$

$\text{MC}: \; M \equiv M^1 \implies \{ O \equiv I^2, \; M \equiv M^1 \equiv M^2, \; O^1 \equiv O^2 \}$:

$$\frac{\text{supplem}_2\left(\begin{bmatrix} I & \square & I^1 \\ O & MM^1 & O^1 \end{bmatrix}\right)}{\begin{bmatrix} (I) & \square & (I^1) \\ \square & (MM^1)\, M^2 & \square \\ (O)\, I^2 & \square & (O^1)\, O^2 \end{bmatrix}}$$

Compositions of supplements

a. $\text{supplem}_3 = \text{supplem}_1 \oplus \text{supplem}_2$:

$$\frac{\text{supplem}_3\left(\begin{bmatrix} (I)\, I^2 & \square & (I^1)\, M^2 \\ \square & (MM^1)\, O^2 & \square \\ (O) & \square & (O^1) \end{bmatrix} \oplus \begin{bmatrix} (I) & \square & (I^1) \\ \square & (MM^1)\, M^2 & \square \\ (O)\, I^2 & \square & (O^1)\, O^2 \end{bmatrix}\right)}{\begin{bmatrix} (I I^2) & \square & (M^2\, I^1\, I^2) \\ \square & (MM^1\, O^2)\, M^3 & \square \\ (O)\, I^3 & \square & (O^1\, O^2)\, O^3 \end{bmatrix}}$$

The supplementation *supplem*$_4$ seems to saturate the possibilities of complementing figure Type1 $M \equiv M'$. A saturation is achieved if all possible semiotic knots are connected. A saturated semiotic figure or constellation can then be reduced or it can be augmented in complexity by iterative and accretive operations. The labelling of the semiotic knots with I, M, O might be at first just arbitrary and only depending on the as-abstraction, e.g. $(MM^1\ O^2\ M^3)$ reads as " $(MM^1\ O^2)$ as M^3". Thus, different labelling decisions are possible.

Redundant supplementation

A saturated figure, like supplem$_3$, might iteratively augmented by supplem$_4$ as a *redundant* iteration of an existing sub-figure, say $(I^3,\ M^3,\ O^3)$.

Redundant supplementation is based on a semiotic operation, which isn't common in semiotic literature. It is well based on the polycontextural *as-abstraction.* Hence, it enables to thematize semiotic objects *"X as Y is Z"* , e.g. I as I^1 is (II^1), I as O^1 is (IO^1), I as M^1 is (IM^1) and O as O^1 is (OO^1), etc.

Hence, again, the category theoretic identity operation *id* of classical semiotics is transformed to a difference operation. Identity, i.e. the is-abstraction, derives naturally from the as-abstraction with "X as X is X".

$$\frac{\mathbf{op}_{\mathbf{redun}}\begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} II^1 & \square \\ OO^1 & MM^1 \end{bmatrix}, \begin{bmatrix} IO^1 & \square \\ OM^1 & MI^1 \end{bmatrix}, \begin{bmatrix} IM^1 & \square \\ OI^1 & MO^1 \end{bmatrix}, \ldots, \begin{bmatrix} IM^1 & \square \\ OO^1 & MI^1 \end{bmatrix}}$$

2.4. Diamond monadic sign compositions

$$\frac{(O, I, M) \diamond (M', I', O'),\ \text{with } MC = M \equiv M'}{(O, I, M) \circ (M', I', O') \,\big|\, (M,\ M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'),\ M \equiv M'}{(O \longrightarrow I \longrightarrow M) \circ (M' \longrightarrow I' \longrightarrow O') \,\big|\, (M \longleftarrow M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'),\ M \equiv M'}{(O \longrightarrow M) \circ (M' \longrightarrow O') \,\big|\, (M \longleftarrow M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'),\ M \equiv M'}{(O \longrightarrow O') \,\big|\, (M \longleftarrow M')}$$

$$\frac{\Pi^{(2)}_{\substack{\mathbf{Type\,1} \\ M \equiv M'}}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix} \,\Big|\, (M \longleftarrow M')}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 1} \\ M \equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} \square & \square & I' \\ O & MM' & O' \\ I & \square & \square \end{bmatrix} \Bigg| \left(M \longleftarrow M' \right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 3} \\ M \equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} \square & I' \\ MM' & O' \\ I & O \end{bmatrix} \Bigg| \left(M \longleftarrow M' \right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type 4} \\ M \equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} O' & I' \\ MM' & \square \\ I & O \end{bmatrix} \Bigg| \left(M \longleftarrow M' \right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type } A \\ \textbf{MC } M \equiv M', \\ O \equiv O'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & \square \\ MM' & OO' \\ I' & \square \end{bmatrix} \Bigg| \left(M \longleftarrow M' \right) \Big\| \left(O \longleftarrow O' \right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type } B \\ \textbf{MC } M \equiv M', \\ O \equiv O'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & MM' \\ OO' & I' \end{bmatrix} \Big| \left(\overline{O} \longleftarrow \overline{O'} \right) \Big\| \left(\overline{M} \longleftarrow \overline{M'} \right)}$$

$$\frac{\Pi^{(2)}_{\substack{\textbf{Type } c\\ \textbf{MC } M\equiv M',\\ O\equiv O'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} MM' & I' \\ M & OO' \end{bmatrix} \Big| \left(\overline{M} \longleftarrow \overline{M}'\right) \Big\| \left(\overline{O} \longleftarrow \overline{O}'\right)}$$

2.5. Dyadic sign compositions

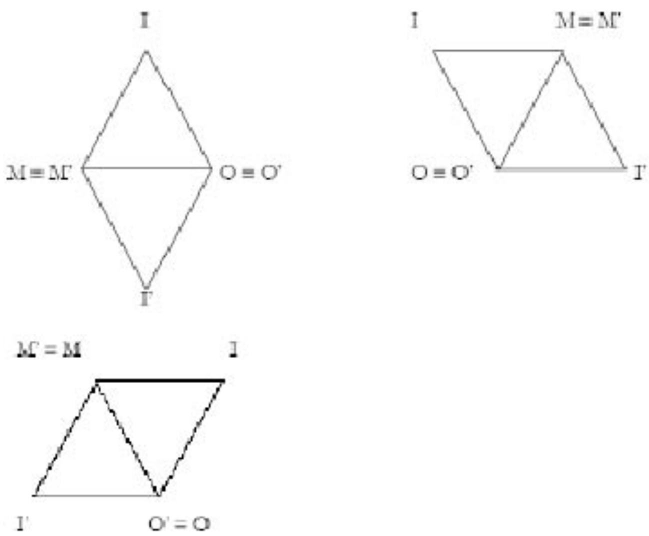
2.5.1. Dyadic semiotic sign compositions

Matching conditions for dyadic sign compositions:

MC: $X / Y \equiv X' / Y'$, with $X, X' Y, Y' \in \{I, O, M, I', O', M'\}$

Example: 3.2.2.1. M/O ≡ M'/O' (Toth, p. 33, 2008a)

3.2.2.1. M/O ≡ M'/O'



$$\frac{\Pi^{(2)}_{\substack{O\equiv O' \\ M\equiv M'}} \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I \longrightarrow & & M \end{pmatrix}}{\begin{bmatrix} \square & I \\ MM' & OO'' \\ \square & I'' \end{bmatrix}, \begin{bmatrix} I & MM'' \\ OO' & I' \end{bmatrix}, \begin{bmatrix} MM' & I \\ I' & O'O \end{bmatrix}}$$

2.5.2. Diamond dyadic sign compositions

$$\frac{(M \longrightarrow I \longrightarrow O) \diamond (O' \longrightarrow I' \longrightarrow M'),\ \text{with MC} = M / O \equiv M' / O'}{(M \longrightarrow I \longrightarrow O) \circ (O' \longrightarrow I' \longrightarrow M') \,\Big|\, (O \longleftarrow O') \,\Big\|\, (M \longleftarrow M')}$$

$$\frac{(O \longrightarrow I \longrightarrow M) \diamond (M' \longrightarrow I' \longrightarrow O'),\ \text{with MC} = M / O \equiv M' / O'}{(O \longrightarrow I \longrightarrow M) \circ (M' \longrightarrow I' \longrightarrow O') \,\Big|\, (M \longleftarrow M') \,\Big\|\, (O \longleftarrow O')}$$

$$\frac{(O, I, M) \diamond_{M,O} (M', I', O'),\ \text{with MC} = M / O \equiv M' / O'}{(O, I, M) \circ_{M,O} (M', I', O') \,\Big|\, (M \longleftarrow M') \,\Big\|\, (O \longleftarrow O')}$$

$$\textbf{a.}\ \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,M} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } M \big/ O \equiv M' \big/ O'$$

$$\textbf{b.}\ \begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ M & \longrightarrow & I \end{pmatrix} \circ_{O,M} \begin{pmatrix} O' & \longrightarrow & I' \\ \downarrow & \nearrow & \\ M' & & \end{pmatrix} \Bigg\| \left(M \longleftarrow M'\right) \,\|\, \left(O \longleftarrow O'\right)$$

$$\textbf{c.}\ \begin{bmatrix} \left(O \equiv O'\right) & \longrightarrow & I \\ \downarrow & x & \square \\ \left(M \equiv M'\right) & \longrightarrow & I' \end{bmatrix} \Bigg\| \left(M \longleftarrow M'\right) \,\|\, \left(O \longleftarrow O'\right)$$

$$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,M,I} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } M \big/ O \equiv O' \big/ I'$$

$$\begin{bmatrix} \left(M \equiv O'\right) & \longrightarrow & I \\ \downarrow & x & \square \\ \left(O \equiv I'\right) & \longrightarrow & M' \end{bmatrix} \Bigg\| \left(M \longleftarrow O'\right) \,\|\, \left(O \longleftarrow I'\right)$$

$$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,M,I} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } O \big/ I \equiv O' \big/ I'$$

$$\begin{bmatrix} \left(O \equiv O'\right) & \longrightarrow & M \\ \downarrow & x & \square \\ \left(I \equiv I'\right) & \longrightarrow & M' \end{bmatrix} \Bigg\| \left(O \longleftarrow O'\right) \,\|\, \left(I \longleftarrow I'\right)$$

$$\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M \end{pmatrix} \diamond_{O,I,M} \begin{pmatrix} O' & & \square \\ \downarrow & \searrow & \\ I' & \longrightarrow & M' \end{pmatrix}, \text{ with } O \big/ I \equiv M' \big/ I'$$

$$\begin{bmatrix} \left(O \equiv M'\right) & \longrightarrow & M \\ \downarrow & x & \square \\ \left(I \equiv I'\right) & \longrightarrow & O' \end{bmatrix} \Bigg\| \left(O \longleftarrow M'\right) \,\|\, \left(I \longleftarrow I'\right)$$

$$\frac{\textbf{Type 1 = 1: } \text{MC}: M \equiv M',\ O' \equiv M'',\ O'' \equiv M''' \ (\text{Toth, p. 63, 2008 a}) \left(\begin{array}{cc} O & \square \\ \downarrow & \searrow \\ I \longrightarrow & M \end{array}\right)}{\left[\begin{array}{ccccccccc} I & \square & \square & I' & \square & \square & I'' & \square & \square \\ \downarrow & \searrow & \square & \downarrow & \searrow & \square & \downarrow & \searrow & \square \\ O & \longrightarrow & M \equiv M' & \longrightarrow & O' \equiv M'' & \longrightarrow & & & O'' \equiv M''' \end{array}\right] \left| \begin{array}{c} (O' \longleftarrow M'') \parallel \\ (M \longleftarrow M') \\ \square \end{array}\right.}$$

$$\frac{\textbf{Type 1 = 1: } \text{MC}: M \equiv M',\ O' \equiv M'',\ O'' \equiv M''' \qquad \Pi^{(3)}_{\diamond_{O,I,M}} \left(\begin{array}{cc} O & \square \\ \downarrow & \searrow \\ I \longrightarrow & M \end{array}\right)}{\left[\begin{array}{ccc} \square & I & \square \\ O' & M'M & O \\ \square & I'M'' & \square \\ O'' & \square & I'' \end{array}\right] \left| (M \longleftarrow O'') \right.}$$

$$\frac{\textbf{Type 1 = 1: } \text{MC}: M \equiv M',\ O' \equiv M'',\ O'' \equiv M''' \qquad \Pi^{(3)}_{\diamond_{O,I,M}} \left(\begin{array}{cc} O & \square \\ \downarrow & \searrow \\ I \longrightarrow & M \end{array}\right)}{\dfrac{(O \longrightarrow M,\ M' \longrightarrow O',\ M'' \longrightarrow O'' \mid (M \longleftarrow O'')}{(O \longrightarrow O'' \mid (M \longleftarrow O'')}}$$

Type 4 = 4:

MC: $I' \equiv O''$, $M \equiv M'$, $M' \equiv M''$, $O \equiv I'$ (Toth, p. 65, 2008 a)

$$\frac{\Pi^{(4)}_{\textbf{Type 4=4}} \left(\begin{array}{cc} O & \square \\ \downarrow & \searrow \\ I \longrightarrow & M \end{array}\right)}{\left[\begin{array}{ccc} O' & I'O'' & I'' \\ MM' & \square & M'M'' \\ I & OI'' & O' \end{array}\right] \left| \begin{array}{c} (I' \longleftarrow O'') \parallel \\ (M \longleftarrow M') \parallel \\ (M' \longleftarrow M'') \parallel \\ (O \longleftarrow I') \end{array}\right.}$$

Type 1 = 2 :

MC : M ≡ M', O' ≡ O", M" ≡ M"' (Toth, *p.* 65, 2008 *a*)

$$\Pi^{(4)}_{\text{Type }1=2}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M\end{pmatrix} \quad \begin{bmatrix} I & \square & \square & I' & \square & \square & I'' & \square & \square & I''' & \square \\ \downarrow & \searrow & \square & \downarrow & \searrow & \square & \uparrow & \searrow & & \uparrow & \searrow \\ O & \longrightarrow & M & M' & \longrightarrow & O' & O'' & \longrightarrow & M'' & M''' \longrightarrow & O \end{bmatrix}$$

$$\begin{pmatrix} \square & I & I' & \square & \square \\ O & MM' & O'O'' & M''M''' & O''' \\ \square & I'M'' & \square & I''' & \square \end{pmatrix} \Bigg| \; (M'' \longleftarrow M''') \,\|\, (M \longleftarrow M') \,\|\, (O' \longleftarrow O'')$$

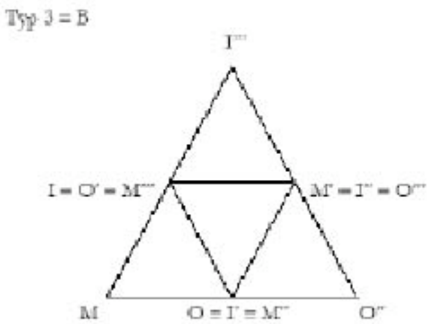
Type *A* ≡ *A* = *A* ≡ *B* : (Toth, 2008 *a*, *p.* 68)

MC$_I$: $I' \equiv I''$, $M \equiv M'$, $O \equiv O' \equiv O''$

$$\Pi^{(4)}_{\text{Type }(A\equiv A=A\equiv B)\,\text{MC}_I}\begin{pmatrix} O & & \square \\ \downarrow & \searrow & \\ I & \longrightarrow & M\end{pmatrix}$$

$$\begin{bmatrix} \square & II'' & M'' \\ MM' & OO'O'' & I'' \\ \square & I' & \square \end{bmatrix} \Bigg| \begin{matrix} (I \longleftarrow I'') \,\| \\ (M \longleftarrow M') \,\| \\ (O' \longleftarrow O'') \,\| \\ (O \longleftarrow O') \,\| \\ (O \longleftarrow O'') \end{matrix}$$

Type 3 ≡ *B* : (Toth, 2008 *a*, *p.* 71)



$$\mathbf{MC}_k : I \equiv O' \equiv M''', \; M' \equiv I''' \equiv O''', \; O \equiv I' \equiv M''$$

$$\frac{\Pi^{(4)}_{\substack{\text{Type } (3\equiv B)\\ \text{MC}_k}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M & \longrightarrow & O \end{pmatrix}}{\begin{bmatrix} \square & I''' & \square \\ IO'M'' & \square & M'I''O'' \\ M & OI'M'' & O'' \end{bmatrix} \left| \begin{array}{l} (O' \longleftarrow M'') \parallel \\ (I \longleftarrow O') \parallel \\ (I'' \longleftarrow O'') \parallel \\ (M' \longleftarrow I'') \parallel \\ (I' \longleftarrow M'') \parallel \\ (O \longleftarrow I'). \end{array} \right.}$$

2.6. Local-/global-heteromorphisms

Sign functions, understood as a composition of elementary sign relations are producing their *local* diamond structures according to the matching conditions of the compositions of elementary sign morphisms.

Sign systems, understood as combinations of sign functions are producing their *global* diamonds in accordance to the matching conditions of their combined signs.

As an interpretation, this semiotic difference of *local* and *global* or *micro/macro* diamond structures can be connected with the distinction of *inner* and *outer* environments of semiotic diamond systems.

$$\frac{\textbf{Micro – analysis of } (\boldsymbol{I}, \; \boldsymbol{O}, \; \boldsymbol{M}) \; (I_\alpha \rightarrow O_\omega) \circ (O_\alpha \rightarrow M_\omega)}{\begin{array}{l} (I_\alpha \rightarrow M_\omega) \mid (\tilde{O}_\omega \longleftarrow \tilde{O}_\alpha) \\ \tilde{O}_\alpha \equiv \text{diff}(O_\alpha) \\ \tilde{O}_\omega \equiv \text{diff}(O_\omega). \end{array}}$$

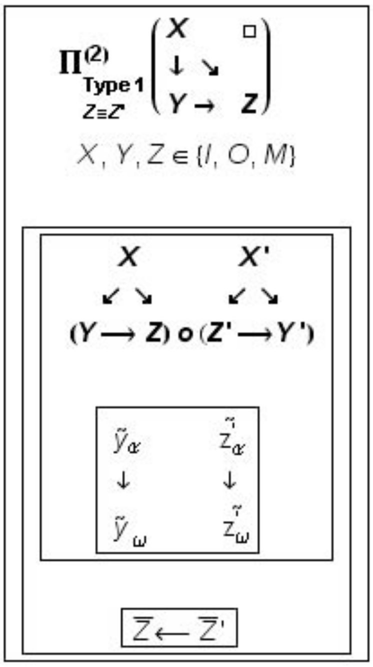
Macro – analysis of Type 1, MC = *M* ≡ *M'*

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M\equiv M'}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O \rightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix} \mid (\overline{M} \leftarrow \overline{M'})}$$

Micro / Macro – analysis of Type 1, MC = *M* ≡ *M'*

$$\frac{\Pi^{(2)}_{\substack{\text{Type 1}\\ M\equiv M'}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O \rightarrow & & M \end{pmatrix}}{\begin{bmatrix} I & \square & I' \\ O & MM' & O' \end{bmatrix} \left| \begin{array}{l} \tilde{O}_\omega \longleftarrow \tilde{O}_\alpha \parallel \\ \tilde{\tilde{O}}_\omega \longleftarrow \tilde{\tilde{O}}_\alpha \end{array} \right| \overline{M} \leftarrow \overline{M}'}$$

Generalized Notations: Boxes and brackets

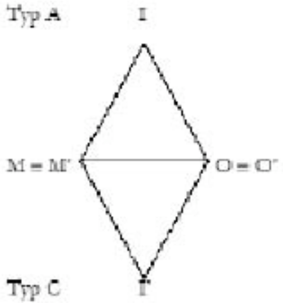


$$\Pi^{(2)}_{\text{Type 1}, Z \equiv Z'} = \text{Diam}\Big(\text{diam}\big(X, Y, Z\big), \text{diam}\big(X', Z', Y'\big)\Big)$$

$$\left[\begin{array}{l}\textbf{Matrix} = \\ \Pi^{(2)}_{\text{Type 1}, Z \equiv Z'} \\ \left[\begin{array}{l}\text{Diamond} \\ \left[\begin{array}{l}\text{diamond} \\ \big[(X, Y, Z) \\ \left[\begin{array}{l}\text{diamond} \\ \big[(X', Z', Y')\end{array}\right.\end{array}\right.\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \left[\begin{array}{l}\text{Diamond} \\ \big[\big[(X, Y, Z) \,\big|\, (y_\alpha, y_\omega) \\ \big[\big[(X', Z', Y') \,\big|\, (\dot{z}_\alpha, \dot{z}_\omega)\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \big[\big[(X, Y, Z) \,\big|\, (y_\alpha, y_\omega) \\ \big[\big[(X', Z', Y') \,\big|\, (\dot{z}_\alpha, \dot{z}_\omega)\end{array} \;\Bigg|\; (\mathbf{z}_\omega \longleftarrow \mathbf{z}_\alpha)\right]$$

Diamond micro / macro – analysis of Type *A*, MC = *M* ≡ *M*', *O* ≡ *O*' (*p*.63)

$$\frac{\Pi^{(2)}_{\substack{\text{Type } A \\ \text{MC } M \equiv M', \\ O \equiv O'}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M & \longrightarrow & O \end{pmatrix},}{\begin{bmatrix} I & \square \\ MM' & OO' \\ I' & \square \end{bmatrix} \Bigg| \big(M \longleftarrow M'\big) \big\| \big(O \longleftarrow O'\big)}$$



$$\left[\begin{array}{l}\textbf{Matrix} = \\ \Pi^{(2)}_{\text{Type } A,\, M\equiv M',\, O\equiv O'} \\ \left[\begin{array}{l}\text{Diamond} \\ \left[\begin{array}{l}\text{diamond} \\ \left[(I, M, O)\right. \\ \left[\begin{array}{l}\text{diamond} \\ (M', I', O')\end{array}\right.\end{array}\right.\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \left[\begin{array}{l}\text{Diamond} \\ \left[\begin{array}{l}\left[(I, M, O) \mid (M_\alpha, M_\omega)\right) \\ \left[(M', I', O') \mid (I_\alpha, I_\omega)\right)\end{array}\right.\end{array}\right.\end{array}\right] = \left[\begin{array}{l}\textbf{Matrix} \\ \left[\begin{array}{l}\left[(I, M, O) \mid (M_\alpha, M_\omega)\right) \\ \left[(M', I', O') \mid (I_\alpha\ I_\omega)\right)\end{array}\right.\end{array} \middle| \ \mathbf{(M,\ M') \parallel (O,\ O')}\right]$$

Diamond micro / macro – analysis of Type 4 = 4 (iter, acc)

Type 4 = 4 : (iter)

MC: $I' \equiv O''$, $M \equiv M'$, $M' \equiv M''$, $O \equiv I'$ (Toth, 2008 *a*, *p*. 65)

$$\Pi^{(4)}_{\textbf{Type 4=4}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M \longrightarrow & & O \end{pmatrix}, \text{ macro}$$

$$\begin{bmatrix} O' & I'O'' & I'' \\ MM' & \square & M'M'' \\ I & OI' & O' \end{bmatrix} \left| \begin{array}{l} (I' \longleftarrow O'') \parallel \\ (M \longleftarrow M') \parallel \\ (M' \longleftarrow M'') \parallel \\ (O \longleftarrow I') \end{array} \right.$$

Type 4 = 4 is ***iterating*** system (I', M', O') as

$op_{mirr}(I', M', O') = (M', I', O')$ •

Accretive distribution of system (I^1, M^1, O^1) as

$op_{acc}(op_{mirr}(I^1, M^1, O^1)) = (M^2, I^2, O^2)$

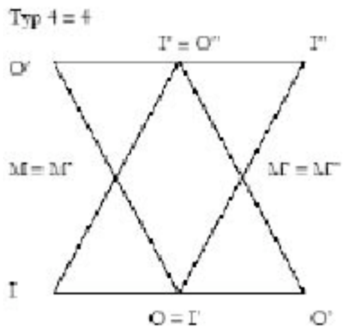
$X' = X^1,\ X'' = X^2,\ X''' = X^3,\ X \in \{M, I, O\}$

Type 4 = 4 : **(acc)**

MC : $I^2 \equiv O^3,\ M \equiv M^2,\ M^1 \equiv M^3,\ O \equiv I^1$

$$\Pi^{(4)}_{\textbf{Type 4=4}} \begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ M & \longrightarrow & O \end{pmatrix}, \text{ macro}$$

$$\left[\begin{matrix} O^2 & I^2\,O^3 & I^3 \\ MM^2 & \square & M^1\,M^3 \\ I & OI^1 & O^1 \end{matrix}\right] \left| \begin{matrix} (I^2 \longleftarrow O^3) \ \| \\ (M \longleftarrow M^2) \ \| \\ (M^1 \longleftarrow M^3) \| \\ (O \longleftarrow I^1) \end{matrix} \right.$$

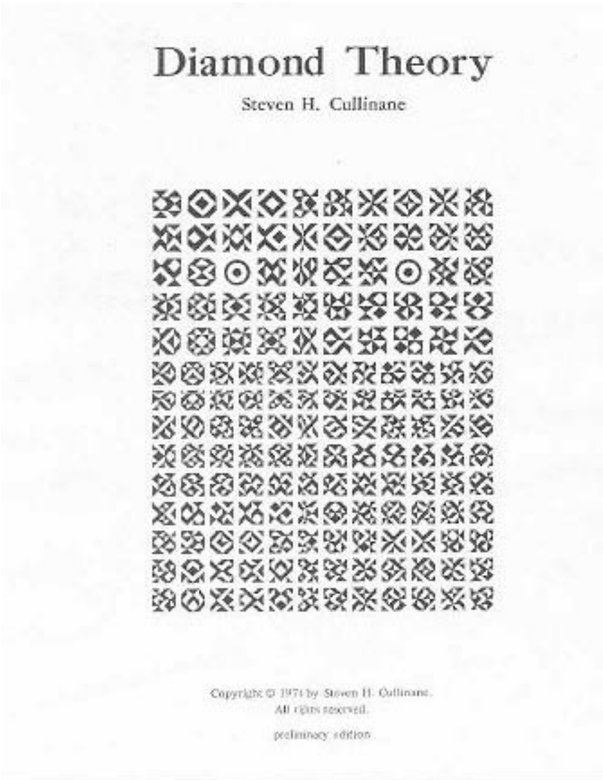


$$\left[\begin{matrix} \textbf{Matrix} = \\ \Pi^{(4)}_{\textbf{Type 4=4 (iter)}} \\ \left[\begin{matrix} \text{Diamond} \\ \left[\begin{matrix} \text{diamond}_1 \\ [(I, M, O)] \\ \left[\begin{matrix}\text{diamond}_2 \\ (I', M', O')\end{matrix}\right] \\ \left[\begin{matrix}\text{diamond}_3 \\ (M', O', I')\end{matrix}\right] \\ \left[\begin{matrix}\text{diamond}_4 \\ (M'', O'', I'')\end{matrix}\right] \end{matrix}\right] \end{matrix}\right] \end{matrix}\right] = \left[\begin{matrix} \textbf{Matrix} \\ \left[\begin{matrix} \text{Diamond} \\ \left[\begin{matrix} [(I, M, O) & | (M_\alpha, M_\omega) \\ [(I', M', O') & | (M'_\alpha, M'_\omega) \\ [(M', O', I') & | (O'_\alpha, O'_\omega) \\ [(M'', O'', I'') & | O''_\alpha, O''_\alpha) \end{matrix}\right] \end{matrix}\right] \end{matrix}\right] = \left[\begin{matrix} \textbf{Matrix} \\ \left[\begin{matrix} [(I, M, O) & | (M_\alpha, M_\omega) \\ [(I', M', O') & | (M'_\alpha, M'_\omega) \\ [(M', O', I') & | (O'_\alpha, O'_\omega) \\ [(M'', O'', I'') & | O''_\alpha, O''_\alpha) \end{matrix}\right] \left| \begin{matrix} (I' \longleftarrow O'') \ \| \\ (M \longleftarrow M') \ \| \\ (M' \longleftarrow M'') \| \\ (O \longleftarrow I') \end{matrix} \right. \end{matrix}\right]$$

Generalizations of the matrix or bracket notation should easily be accessible to develop a general notational system for semiotic diamonds.

Diamond Theory (Steven H. Cullinane) has many faces:

http://finitegeometry.org/sc/gen/dth/DiamondTheory.html
http://diamondtheorem.com/



2.7. Composition of semiotic diamonds

A *first* analysis of the composition operation for semiotic diamonds results into an iterative and accretive or serial and parallel form of “2-dimensional” composition. These operators might correspond to the semiotic operations “*adjunction*”, “*superisation*” and ”*iteration*”. As a further operation, a general reflection or mirror operation is necessary. Hence, a figure like *Type 1≡A* might be written as a combination of iteration, ”op_{iter}" accretion, op_{acc}, and mirroring, op_{mir}.

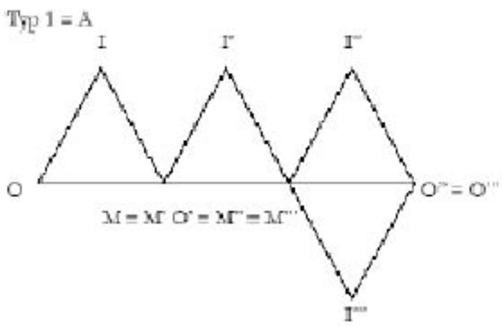
Each operation has to fulfill the matching conditions, MC, and is prepared to be involved into diamondization. Matching conditions are reflecting, externally, the composability of compositions, therefore they are represented as hetero-morphisms of saltatories. Both together, the categorical compositions with ”op_{iter}", op_{acc}, and op_{mir} and the heteromorphic representations of their combinations are building the diamond structure of the semiotic composition.

Semiotic Figure Type 1 ≡ A (Toth, 2008 a, p.79)

$(op_{acc}(op_{mir}(op_{iter}(op_{iter}(op_{iter}([O, I, M])))))),$

reflecting the matching conditions:

$MC = \{M \equiv M', O' \equiv M'' \equiv M''', O'' \equiv O'''\}$



Diamondization of Figure Type 1 ≡ *A* :

Type 1 ≡ *A* :
MC: $M \equiv M', O' \equiv M'' \equiv M''', O'' \equiv O'''$

$$\Pi^{(4)}_{\mathbf{Type\,1\equiv A}}\begin{pmatrix} I & & \square \\ \downarrow & \searrow & \\ O & \longrightarrow & M \end{pmatrix}, \text{ macro}$$

$$\left[\begin{matrix} I & I' & I'' & \square \\ O & MM' & O'M''M''' & O''O''' \\ \square & \square & \square & I''' \end{matrix}\right] \left| \begin{matrix} (O'' \longleftarrow O''') \,\| \\ (M'' \longleftarrow M''') \,\| \\ (O' \longleftarrow M'') \,\| \\ (M \longleftarrow M') \end{matrix} \right.$$

Matching conditions

op_{iter} : MC = $(M \equiv M', O' \equiv M'' \equiv M''', O'' \equiv O''')$

op_{acc}: MC = $(M'' \equiv M''', O'' \equiv O''')$

op_{mir}: MC := $(O'' \equiv O''', M'' \equiv M''')$ •

3. Goguen's semiotics: From Peirce to Pierce

A closer connection between *trichotomic semiotics* and mathematical *category theory* is accessible with Joseph Goguen's work to the semiotics of Human-Computer-Interface (HCI) theory. This chapter is a preliminary hint to a new *Short Story* about the interplay of semiotics, category theory and *diamond theory* in respect to a theory of presentation, representation and evocation. It might then be seen as a late contribution to my old (German) project **"Kalkül und Kreativität" (1998-2002).**

Goguen, Semiotic Morphism, 1996/2004
Definition 1
A **sign system**, or **semiotic system** or **semiotic theory**, consists of:
1. a **signature**, which declares sorts, subsorts and operations (including constructors and selectors);
2. a subsignature of **data sorts** and **data functions**;
3. **axioms** (e.g. equations) as constraints;
4. a **level ordering** on sorts, including a maximum element called top; and
5. a **priority ordering** on constructors at the same.

The non-data sorts classify signs and their parts, just as in grammar, the "parts of speech" classify sentences and their parts. There are two kinds of operation: constructors build new signs from old signs as parts, while selectors pull out parts from compound signs. Data sorts classify a special kind of sign that provides values serving as attributes of signs. Axioms act as constraints on what count as allowable signs for this system. Levels indicate the whole/part hierarchy of a sign, with the top sort being the level of the whole; priorities indicate the relative significance of subsigns at a given level; social issues play a dominant role in determining these.

Definition 2
A **semiotic morphism** $M : S_1 \text{-->} S_2$ from a semiotic system S_1 to another S_2 consists of the following partial mappings:
1. from sorts of S_2 to sorts of S_2, so as to preserve the subsort relations,
2. from operations of S_2 to operations of S_2, so as to preserve their source and target sorts,
3. from levels of S_1 sorts to levels of S_2, so as to preserve the ordering relation, and
4. from priorities of S_1 constructors to priorities of S_2 constructors, so as to preserve their ordering relations,

so as to strictly preserve all data elements and their functions.

It is easy to prove that this definition of composition obeys the following identity and associative laws, in which A : R --> S, B : S --> T and C : T --> U,

$A ; 1_S = A$

$1_S ; B = B$

$A ; (B ; C) = (A ; B) ; C$

where 1_S denotes the identity morphism on S. These three laws are perhaps the most fundamental for a calculus of representation, since they imply that semiotic theories and their morphisms form what is called a "category" in the relatively new branch of mathematics called category theory [Mac Lane, 1998].

The basic ingredients of a **category** are *objects*, *morphisms*, and a *composition* operation that satisfies the above three laws, and that is defined on two morphisms if and only if they have *matching* source and target.

Three of the simplest categorical concepts are isomorphism, sum and product.
A morphism A : R --> S is an **isomorphism** if and only if there is another morphism B : S --> R such that $A ;B = 1_R$ and $B ;A = 1_S$, in which case B is called the inverse of A and denoted A-1; it can be proved that the inverse of a morphism is unique if it exists. The following laws can also be proved, assuming that A : R --> S and B : S --> T are both isomorphisms (and no longer assuming that B is the inverse of A).

$1_R{}^{-1} = 1_R$
$(A^{-1})^{-1} = A$
$(A ; B)^{-1} = B^{-1} ; A^{-1}$

Because sign systems and their morphisms form a category, these three laws apply to representations.
http://www.cs.ucsd.edu/users/goguen/4mari/4mari.html

"The creative process is to some extent *unpredictable* and *uncontrollable*; this is more true of artistic creation than of design, but it holds for both. The best designs often seem both surprising and obvious, and they also often seem to come suddenly out of nowhere, but usually after a lot of hard work."
http://www-cse.ucsd.edu/users/goguen/papers/sm/node5.html#SECTION3-2

A sign theory of representation, as it is the case for the Peirceian semiotics, might well be conceptualized in its basic structure by the concepts and laws of mathematical category theory.

Also signs are introduced by Peirce as '*representamen',* sign events haven't to be restricted to the process of *representation*. Innovative and creative sign events are not representational. They are not re-presenting something existing but are creating something, which has not to be in any sense an ontological "*something*", which isn't yet existing.
Qualitative categories like "*surprise*", "*suddenly of nowhere*", "*unpredictable*", etc. are not covered by category theory and category based semiotics. Categorical and semiotic compositions are conservative and save, not leaving the framework of their definition.

Neither is the *New* covered by the category of Becoming (Hegel, Lawvere).
"Thus I believe to have demonstrated the plausibility my thesis that category theory will be a necessary tool in the construction of an adequately explicit science of knowledge." (Lawvere, 1994, p. 55)
F. William Lawvere: Tools for the Advancement of Objective Logic: Closed Categories and Toposes, in: (Eds.) John McNammara, Gonzales E. Reyes, the logic foundation of cognition, Oxford 1994, pp. 43-56

A new approach beyond the *magic of inspiration* and the *mechanization of creation* is proposed by the general diamond strategies.

"The possibility of being surprised is exhibited as the main argument for the existence of protentions. It is an observable fact, Husserl says, that we all can be at every moment suddenly surprised; now, in order one to be surprised, she must experience an unexpected event; therefore we have at every moment some expectation about the most immediate future. We anticipate the future whenever we are having intentional acts." (Julio Ostalé)
http://staff.science.uva.nl/~michiell/docs/Corrected%20Handout.pdf

"The harmonic My-Your-Our-Class conceptualization has to be augmented by a class which is incorporating the place for the other, the unknown, the difference to the harmonic system. That is, the NotOurClass is thematized positively as such as the class for others, called the OthersClass. Hence, the OthersClass can serve as the place where intruders, attacks, disturbance, etc. can be observed and defended. But also, it is the place where the new, inspiration, *surprise* and challenge can be localized and welcomed." (Kaehr, 2007)

4. Notes and References

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