From Dialogues to Polylogues

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ThinkArt Lab Glasgow 2007

Monopolism always leads to censorship

From Dialogues to Polylogues

1 An interpretation of the model

1.1 How does it work?

System S3 (C), see below, is giving place to reflect/interact for System1 (A) and systemS2 (B) about their common goals and rules. Thus, system C is playing the part of a super-visor enabling S1 and S2 to realize a kind of self-reflection about their common actions. Without S3, the goals and rules would be *implicit* for S1 and S2 and pre-given for their game. And thus, not changeable *during* the game. If they would to change the game, they would have to stop, to change and then to restart a new game. Start and end of an interactional/reflectional game between system1 and system2 is placed in system3. The negotiation about the goals and the rules and decision or even contract to accept the situation is outside the actual actions between A and B and is therefore localized, systematically, at the place C.

1.2 Metaphor of an application

My	Your	Our
calculation interaction reflection comparision	calculation interaction reflection comparision	calculation interaction reflection comparision

calculation (goal)
reflection (modeling)
interaction (action, realization)
comparison (correction).
Comparision can be seen as computation.

Calculations

```
My calculations are my-calculations,
Your calculations are your-calculations,
Our calculations are our-calculations.
```

Interaction

```
My interactions are accepted by your-modeling as my-interactions,
Your interactions are accepted by my-modeling as your-interactions,
Our interactions are accepted by our-modeling as our-interactions.
```

Modeling

```
I am reflecting/modeling your calculation in my-reflection,
You are reflecting/modeling my-calculations in your-reflection,
We are reflecting/modeling our-calculations in our-reflection.
```

Comparision

```
I am comparing (reflecting, modeling) your-interaction
with my-reflection on your-calculation in my-comparison.
You are comparing my-interaction
with your-reflection on my-calculation in your-comparison.
We are comparing (super-vision) our-interactions
with our-reflections on our-calculations in our-comparison.
```

Leibniz-Monads

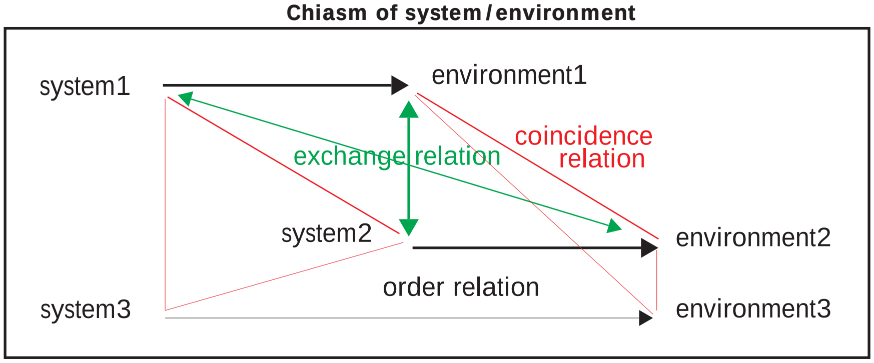
Each agent is able to give structural space to himself and to the neighbor agents to *model* all his neighbor agents' *interactions*; *comparising* and correcting his model about the others *calculations* and *interactions*, and being able to be interacted by all his neighbor agents.

This is the case of a *harmonized* agent system, called the *Leibniz-Monads*.

1.2.1 System environment distinction

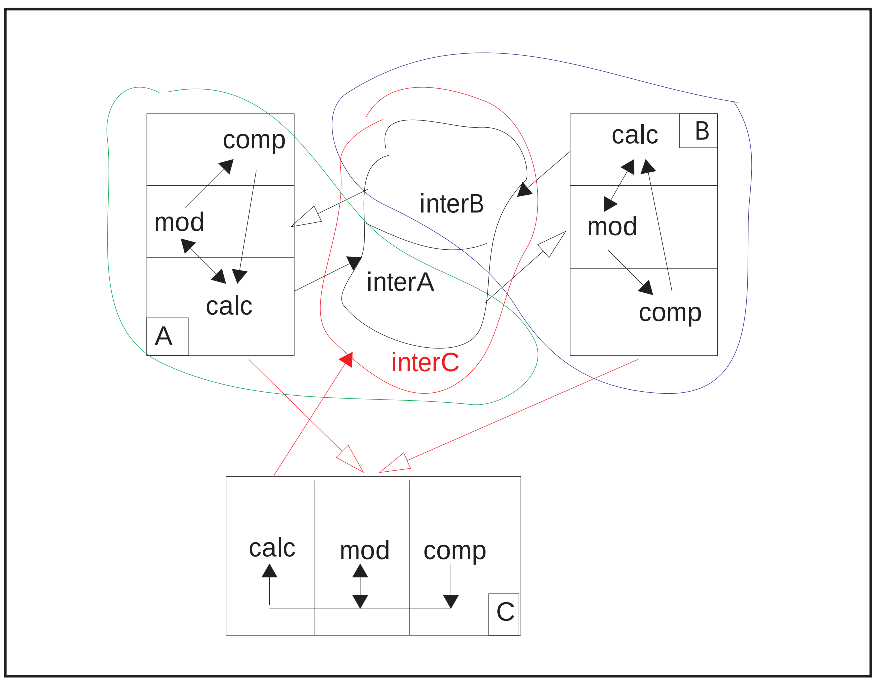
```
What's my environment is your system,
What's your environment is my system,
What's our environments and our systems is
the environment of our-system.
```

Chiasm of system/environment



Chiastic interdependency

Interactions are based on computations and reflections.
Computations are based on interactions and reflections.
Reflections are based on interactions and computations.
Comparisions are based on reflections and interactions.



http://www.thinkartlab.com/pkl/lola/Abacus.pdf

1.3 Negations, permutations and allocations for 3-player games

We can connect the general model of interactional/reflectional games with the concept of games with winning strategies between opponents and proponents. From the point of view of my-game the aim is to win the game to my favor. If I would lose the game against your-game, your-game would win the game. Thus, the aim of each winning strategy is to win the game.

Between winning and losing a *negation* operation happens. The negation of winning is losing and the negation of losing is winning. This is quite trivial and natural. But it isn't as simple in a 3-player game. Because in a 3-player game negation is connected with *permutation*. This is a fact which was emphasized by Gunther all the time. Interestingly, Abramsky has found a similar formulation for negations in respect to permutations for n-person games.

But, what is the meaning of a permutation in a negational context?

The negative as the positive

In an isolated situation of a 2-player game to lose is to lose and to win is to win. And if I win you lose and if you win I lose.

In a 3-person game it could make sense that I change my position and my interest is not in winning the game for my position but in winningly the game for your position. This still means that I am losing and you are winning the game. But it could be of my interest that you win the game. Thus, if you win the game because of my interest that you win the game, I have won the game. Not from my position but from your position. That is, I didn't only negate and change my interests from winning to losing but I also made a permutation between the positions of interest.

It could be of good reasons from my position for the benefit of the whole system, our-game, that I lose and you win. Obviously, such kind of polycontextural m-player games are not simply zero-sum games between players. The intricate interplay between local and global thematizations of m-player games has to be considered.

Nevertheless, in such a game not both positions, my- and your-game, can win or lose at once. The reading still is, if my-game lose, your-game wins. But this situation is distributed and mediated over different positions. Thus, the permutations. The situation is different in m-player games, with m≥4 where commutativity of negations enter the game, enabling independent negations.

In other words, the negative situation of losing in a classical game which has no own status is turned into a positive situation in a 3-player game with its own status or relevancy. Logically speaking it's about the semantic difference of positivity/negativity and designation/non-designation which doesn't coincide in a 3-player game.

$$\begin{array}{llllll}
3\text{-}person\ game & & & & & \\
my-game: & opponent & \longrightarrow & proponent & & \\
 & & & \Updownarrow & & \\
your-game: & & & opponent & \longrightarrow & proponent \\
 & \cong & & & & \cong \\
our-game: & opponent & \multicolumn{3}{c}{\longrightarrow\!\!\longrightarrow\!\!\longrightarrow} & proponent
\end{array}$$

Mediation relations: order, exchange and coincidence relations over 3 positions (my-, your-, our-game).

Even for a simple situation like a 3-player game a new kind of a play can be introduced. Permutations, produced by negations, can be realized in two directions: right or left wards. That's not much, but at least one level more than the classic limitations. A game, based on four basic players, a 4-player game, which has 6 positions to distribute 2-player games, has an interesting network of permutations. Also these permutations are quite regular, a new kind of games can be invented: the permutation game of basic players. All that happens even before the internal strategies have entered the game.

Negations and permutations

Classic games start with atomic expressions. They can not be involved into a heterarchic game of permutations. Atomic expressions can not be attacked. Because they are atomic by introduction. The complexity of polycontextural m-player games are demanding complexity at the very systematic beginning of the game.

Negations (neg_1) and (neg2)

$$\begin{bmatrix} & our-game & \\ & \swarrow\ \searrow & \\ my-game & & your-game \\ \swarrow\ \searrow & & \swarrow\ \searrow \\ win_{1.3}\quad lose_1 & & win_2\quad lose_{2.3} \end{bmatrix} \xrightarrow[neg_1]{} \begin{bmatrix} & your-game & \\ & \swarrow\ \searrow & \\ my-game & & our-game \\ \swarrow\ \searrow & & \swarrow\ \searrow \\ lose_{2.1}\quad win_1 & & win_3\quad lose_{3.2} \end{bmatrix}$$

$$\begin{matrix} lose_1 \Updownarrow win_2 \\ win_1 \cong win_3 \\ lose_2 \cong lose_3 \\ win_i \rightarrow lose_i \end{matrix} \qquad \xrightarrow[neg_2]{} \begin{bmatrix} & my-game & \\ & \swarrow\ \searrow & \\ our-game & & your-game \\ \swarrow\ \searrow & & \swarrow\ \searrow \\ win_{3.1}\quad lose_3 & & lose_2\quad win_{2.1} \end{bmatrix}$$

Stripped down from the roles, the structure which remains are negations/permutations with simple rules.

Formal rules of negation for 3-players

$$neg_i(X) \neq (X),\ i = 1,2$$
$$neg_i\big(neg_i(X)\big) = (X)$$
$$neg_1\Big(neg_2\big(neg_1(X)\big)\Big) = neg_2\Big(neg_1\big(neg_2(X)\big)\Big)$$

A 3-player game has 2 independent negations. (In fact it is a real 2-person game.) A 4-player game has 3 independent negations. The other negations are composed negations.

Formal rules of negation for 4-players

$$neg_i(X) \neq (X),\ i = 1,2,3$$
$$neg_i\big(neg_i(X)\big) = (X)$$
$$neg_1\big(neg_3(X)\big) = neg_3\big(neg_1(X)\big)$$
$$neg_1\Big(neg_2\big(neg_1(X)\big)\Big) = neg_2\Big(neg_1\big(neg_2(X)\big)\Big)$$
$$neg_2\Big(neg_3\big(neg_2(X)\big)\Big) = neg_3\Big(neg_2\big(neg_3(X)\big)\Big)$$

Role allocations, short

pos	neg_1	neg_2
my	my	our
$your$	our	$your$
our	$your$	my

Hence, negations are playing a twofold game: they are changing the *win/lose states* and they are responsible for complex *role allocations* of the players in the game. For m-player games, role allocations are systematically primary to the negational change of the win/lose states.
All that gets its practical value in complex distributed computer games. Today, such meta-games of role allocations are not implemented in computer games. The human/machine players have to allocate their roles outside of the play while stopping it.

role allocations and as-abstractions

According to the general polycontextural matrix I introduced in earlier texts we can write the negational situation and its role allocations for the involved agents more explicit by emphasizing their behavior as a change of roles. The only change in a 2-player game are the changes between attack and defense inside the framework of the stable distinction of opponent and proponent.

Role allocations in the polycontextural matrix

$$\begin{array}{c|ccc} PM^{(3)} & O_1 & O_2 & O_3 \\ \hline M_1 & my & - & - \\ M_2 & - & your & - \\ M_3 & - & - & our \end{array} \xrightarrow[neg1]{} \begin{array}{c|ccc} neg_1 & O_1 & O_2 & O_3 \\ \hline M_1 & \overline{my} & - & - \\ M_2 & - & - & your \\ M_3 & - & our & - \end{array}$$

$$\xrightarrow[neg2]{} \begin{array}{c|ccc} neg_2 & O_1 & O_2 & O_3 \\ \hline M_1 & - & - & my \\ M_2 & - & \overline{your} & - \\ M_3 & our & - & - \end{array}$$

This notation is making explicit the *as-abstractions* of the role allocations.

The wording for the starting standard situation is:

```
my as my,
your as your,
our as our,
all together in the matrix.
```

But this identifying function of the starting situation has to be seen as positioned in the matrix. It is only an abbreviation to isolate the agents and omit their environment.

The wordings for the two negations, neg_1 and neg_2, of the start situation are:

Negation neg_1 is allocating my as my to my as negated my.
Negation neg_1 is allocating your as your to your as our.
Negation neg_1 is allocating our as our to our as your.

Negation neg_2 is allocating my as my to my as our.
Negation neg_2 is allocating your as your to your as negated your.
Negation neg_2 is allocating our as our to our as my.

Negators are producing inversion of values, win and lose, and permutations of other role of actors. But this has to be seen as a changes of allocations in the epistemic characterizations of actors in the as-mode of roles. Hence, changes of distinctions ruled by the as-abstraction are not simply permutations. Permutations of objects or actors are not changing their role allocations in the sense of as-abstractions. To change place is not yet to change the role an actor is playing.

This kind of negation is not yet involving any reflectionality and interactionality, neither cloning or other cognitive possibilities except of being embedded in the epistemic as-abstraction or as-thematization. That is, the activities of the agents as *calculation, interaction, reflection* and *comparison*, are not yet modeled in this thematization of the role allocations of the actors. Hence, the full polycontextural modeling of the activities of a 3-player game as sketched is much more complex than the mechanism of winning strategies of abstract agents in their roles as my-, your- and our-agents alone are suggesting.

1.4 To calculate means to take part in the culture of calculation

What are we doing if we are using an Abacus?

The common answer is: Buy an Abacus, follow the instructions and then use it for your business calculations. What you are doing while using an Abacus is to calculate with the physical devise Abacus according to the rules you learned from your buckled. You have not to understand that your Abacus is based on a positional system to organize your calculations.

This might not be totally wrong. But this explanation is presupposing a lot more.

Even in the solitaire use of an Abacus the complexity of the game always happens. Even if my-calculation are my-calculation, they are not reasonable in isolation. I learned the rules from a teacher. He represents your-calculation. And our-calculation happens as a result of my-calculation and your-calculation, that is, if my-calculations correspond to the calculation I learned from your-calculation. This gives my-calculation the guarantee that my-calculations are correct. The correctness of my-calculations are represented in our-calculations, i.e., in the accordance with the general rules. Thus, there is never something like my-calculation in a solitaire isolation.

To know about these intricate relationships is a first step to implement them in a physical mechanism, i.e., to *objectify* the mental processes of learning and using an Abacus.

Now we can leave the metaphor of the Abacus and turn to better funded research programs for cognitive systems in robotics and game development.

And all the rest is the work to be done by a plumper. But as we know, there are no plumpers left.

http://www.thinkartlab.com/pkl/lola/Abacus.pdf

Other wordings

Interaction is based on *inquiries* and not on calls (send, receive). Inquiries can be *rejected* or *accepted*. The inquiring forms an internal model of the inquirer, only if this succeeds, can it step into a communication process. In the communication model, defined through (process, send, receive, buffer), additionally to the non-interactive structure of the algorithms, this basic encounter structure of the agents must be pre-given by the designer.

2 Game theoretic foundations of formal systems

In a 3-agent system the basic agent systems are founding the rules and existence of the required formal system which will be represented and realized in the third agent system. The third agent system is formulating the requirements and receiving the results produced by the other two agent systems.

Thus, the theory of natural numbers can be founded in a 3-agent system. If the requirements are of higher complexity, higher order agent systems can be introduced.

2.1 Lorenzen's dialogical foundation of arithmetic

A nice, and technically simple application of our 3-agent game could be a *re-construction* and *de-construction* of Paul Lorenzen's concept of a dialogical introduction of logic and arithmetic.

From anti-psychologism and platonism to graphemic pragmatism

> **M. Y. Bar-Hillel.** "It seems to me that our discussion has shown signs of a well known confusion between two different, though related, meanings of "proof": the syntactical (or semantical) one, in which a proof is a certain series of sentences, and the pragmatical one, in which a proof is a certain action by a human being, taken in order to convince another such being of something or other. Proof, in its first sense, has no connection whatsoever with such psychological notions as evidence."
>
> **M. Ch. Perelman.** "Pour qu'on puisse parler d'une conception syntaxique de la preuve, il faudrait que cette forme de preuve soit convaincante pour quelqu'un; une transformation syntaxique n'est donc une preuve que gr^ace a ses proprietes pragmatiques."
> Paul Bernays,
> On judging the situation in proof theoretical research (with discussion) (1954),
> http://www.phil.cmu.edu/projects/bernays/Pdf/bernays20_2003-05-17.pdf

Opponent, Propponent, Structural Rules. This is the trinity of Lorenzen's dialogical constructivism. Obviously, it fits well into a 3-agent game.

S1 = (Opp, Prop) :: Dialogue position1 :: Local Rules
S2 = (Opp, Prop) :: Dialogue position2 :: Local Rules
S3 = (Opp, Prop) :: Dialogue position3 :: Global (Structural) Rules

A Lorenzen dialogue logic is based on system1 and a *mental* representation of system2 and the structural rules. Both, system2 and the Structural Rules are written down, explicitly, only in a successive way. Also they hold at once conceptionally, they are studied in a successive way. This is establishing a hierarchy between system1, system2 and the Structural Rules on top. In contrast, a 3-agent game tries to develop the interplay of the three constituents at once, concurrently, in a heterarchic way.

Position1 is representing the Lorenzen dialogue games as we know them. The same for the structural rules. What is the meaning of a dialogue game at the position2 in a 3-agent game? And how are the structural rules involved into the dynamics of the game?

The game at position one, system1, happens for a single player which is playing an inner monologue between himself and his oppositional part. The steps of this game follows sequentially, one after the other. There is no need for two partners or parts to play the game. But nevertheless, Lorenzen and his school are making a big thing in emphasizing the dialogicity of their 2-person dialogue games. What happens is a kind of a coincidence, i.e., a sedimentation of two games. The opponent and the proponent game, played from two positions are superimposed to each other. Because the game is played in a succession of moves from one virtual player, changing positions between proponent and opponent, realizing attack and defence, this sedimentative superposition is not harming the liberty of this monological dialogue between opponent and proponent played according the structural rules.

Lorenzen 2-person games are played by one person. There is no need for a second person to be involved, structurally, into the game. The aim of the game is anyway to find the dialogical truth of a statement which holds independently from the players. The dialogue is a method, a way to do proofs. The statements are not dialogical per se. They are not involving any points of view. From the point of view of rhetorics, Lorenzen dialogues are not dialogues but disputations. Dialogues, in a Socratic sense, don't need a win strategy.

Only if we want to make the mental activities of the virtual player *explicit* the superposition has to be de-sedimented. The virtual player is switching from his position as a proponent the his position as an opponent without reflecting the architectonic or choreography of his jumps. The rationality of his moves are guaranteed by the local rules of the moves, i.e., the rules for the operators, like logical connectives or arithmetical operations, and the global structural rules which are determining the kind of the game, pre-given to any move of the player.

Instead of a mentally represented interaction between two systematic positions of a player, in a polycontextural thematization an *explicit* notation of their positions and their rules has to be realized.

It is well known too, that dialogue systems can be represented by non-dialogical sequence calculi a la Gentzen, i.e., there is a translation from 2-person games into 0-person games. Rule systems, like Gentzen systems, can be seen as 1- respectively 0-person games. This is indicating clearly enough that the dialogicity of the dialogue logics is merely a question of philosophical interpretation and proof methods but not of genuine logical conception, structure and paradigm. In other words, nothing can be proven by dialogues what can not be proven by rule or axiomatic systems. Thus, there is a profound coincidence or correspondence between syntactical, semantical and pragmatic paradigms or versions of logic.

Both player are inherently bi-polar.

All that should be well known since Immanuel Hermann von Fichte (1797 – 1879): *Die Duplizität des Ich.*

To play the game of the opponent, the opponent needs a model of the proponent.

To play the game of the proponent, the proponent needs a model of the opponent.

Thus, a single 2-person game, played by one person is not enough.

In each move of a player the other player's position is in the game as a reflection.

A 2-person game played by two persons which is aware about the global rules can change these rules during the game. Thus, interaction alone is not enough it needs reflections, too. This is in a main contrast to the interactional approach in computer science of Peter Wegner et al. for which a change of logic seems to be taboo.

2.2 Some dialogue logic mantra

Dialogue Logic: two person, perfect information games

"In Dialogue Logic the validity of some given formula F is examined in *two person*, *perfect information* games. There are two players moving alternatively, both having complete information of the current situation of the game. The player who claims being able to justify F is called the *proponent*, his adversary the *opponent*.

The *initial* state, the dialogue setting, is determined by a (possibly empty) set of hypotheses (brought into the game by the opponent) and a thesis F, the formula to be shown valid (brought into the game by the proponent). The *consecutive* moves of the dialogue game are attacks upon formulae set earlier or defences against previous attacks. Some moves include the setting of formulae that might be subject to subsequent attacks.

The legal moves that a player can perform are defined by so-called *particle rules* and *frame rules*. For each logical connective ϕ a particle rule is given which specifies how attacks upon moves setting formulae that have ϕ as main connective and defences against such attacks have to be performed. The frame rules order the exchange of arguments, i.e. they impose restrictions on when attacks and defences may take place in the dialogue.

The game has two possible outcomes: *win and loss*. A dialogue game is won by a player, if the other player cannot perform any action that is conform to the dialogue rules. The proponent is said to have a *winning strategy* for a formula F, if he is able to win any game with formula F as thesis (and a given set of hypotheses) by appropriate choices of his statements. A formula F is *valid*, if the proponent has a winning strategy for F. The winning strategy is not unique."

"To the author's [Claus Zinn] knowledge, Dialogue Games were never considered from the automated theorem proving viewpoint. Lorenzen et al. were motivated by philosophical ideas (foundations of logic).
The work described here has been inspired by the insight that
theorem proving == game playing." (Claus Zinn)
http://www-ags.dfki.uni-sb.de/~zinn/Colosseum/MetaDL.html

Agents are strategies

"In this setting, we model our agents or processes as *strategies* for playing the game. These strategies *interact* by playing against each other. We obtain a notion of *correctness* which is logical in character in terms of the idea of winning strategy—one which is guaranteed to reach a successful outcome however the environment behaves. This in a sense replaces (or better, refines) the logical notion of “truth”: winning strategies are our *dynamic* version of tautologies (more accurately, of proofs)." (Abramsky)
http://www.illc.uva.nl/HPI/Draft_Information_Processes_and_Games.pdf

Another Overview

"This paper is essentially a survey of some logical approaches to dialogue. We start with Dialogical Logic, which was initiated by Lorenzen and has mainly been explored as a new foundation for logics. It continues with Hintikka’s Game Theoretical Semantics, which has been more developed in contact with Natural Language. For instance, we show how to deal with generalized quantifiers by using games, after ideas taken from Ahti Pietarinen. The two perspectives, if different in their objectives, could be mixed for applicative purposes like the treatment of argumentative dialogues: this requires that they be recast in a neutral form, which consists in Dialogue Games in Extensive form. Nevertheless, to stay at one level of elementary language games is not sufficient: in every day life, games are combined. At this point, it seems that the Game-Theoretic interpretation of Linear Logic provides us with the appropriate tool for *combining* elementary games of various kinds." (Alain Lecomte)
http://brassens.upmf-grenoble.fr/~alecomte/LogicDial.pdf

2.3 Conceptual and procedural graphs for dialogues

2.3.1 Dialogue skeleton

$$dialogue = \begin{bmatrix} opponent \\ \begin{bmatrix} proponent \\ \begin{bmatrix} dialogue \\ \begin{bmatrix} uniquenes \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

Dialogue games are founded and based in uniqueness. They are unique and closed under the concept of dialogueness and are not split or schizophrenic. Multiplicity enters the game only as formal or material applications of the one and only one dialogue game.

Between the two players, *opponent* and *proponent*, there is not a neutral symmetry but an asymmetric order relation. The player "opponent" is dominating the player "proponent". Both together are defining the dialogueness of the *dialogue* which itself is anchored in *uniqueness*. This conceptual asymmetry is the base for the *duality* of the two dialogical functionalities: opponent and proponent.

For the conception of a classic dialogue the uniqueness of the conception and its contexture are not distinguishable because they are in a coincidence with the uniqueness of the conception which itself is not included in the common definition of a dialogue. To add uniqueness to the definition of the classical dialogue logic would imply a questioning of the uniqueness which can not be justified with the means of dialogue logic itself. Traditionally it would lead to a concept which got banned from logic in the advent of mathematical logic: *the principle of legitimate reason* (Satz vom zureichenden Grund).

General concept

$$\begin{array}{l} dialogue\ \ logic \\ \begin{bmatrix} scenario \\ \left\{ \begin{matrix} classic \\ intuit \\ free \end{matrix} \right\} \\ \begin{bmatrix} global\ \ rules \\ \begin{bmatrix} local\ \ rules \\ \begin{bmatrix} (opp, prop) \\ \begin{bmatrix} \begin{pmatrix} A \\ D \end{pmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{array}$$

A dialogue logic is determined by the choice of its *scenario* which is determining the type of logic: classic, intuitionistic or free logic, etc.

The *global* rules are incorporating the general rules of the logic types and the main distinctions for winnning and losing the dialogue.

The *local* rules are, again, depending on the scenario and the global rules, determining the use of the specific logical connectiveres: negation, conjunction, implication, quantors, modalities, etc.

The whole system of distinctions is running in the limits of the *framework*: opponent and proponent.

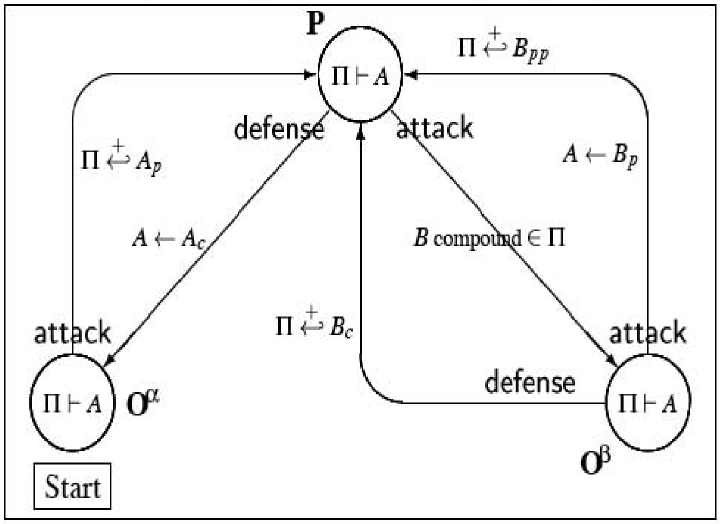
Both, opponent and proponent, are realizing their positions by the functions of *attack* and *defence* ruled by the previous distinctions. Especially, *winning* strategies and rules.

2.3.2 Procedural modelling of dialogue games

Complementary to the conceptual graph presentation of dialogue logic a more procedural modelling is possible. If there would be some plumbers around a modelling could be done along the guidelines of Gurevich's Abstract State Machines (ASM) or as shown below as state transition model.

Christian G. Fermüller, Agata Ciabattoni's state transition model

Fig. 1. Dialogue as state transitions (⊃-fragment)



From Intuitionistic Logic to Gödel-Dummett Logic via Parallel Dialogue Games
www.logic.at/people/chrisf/ismvl03.ps

"A dialogue tree t for P ded C is a rooted, directed and labelled tree with nodes labelled by dialogue sequents and edges corresponding to moves, such that each branch1 of t is a dialogue with initially granted formulas P and initial formula C. We thus identify the nodes of a dialogue tree with *states* of a dialogue.
We distinguish P-nodes and O-nodes, according to whether it is P's or O's turn to move at the corresponding state.
A finite dialogue tree is called winning strategy (for P) if the following conditions are satisfied:
1. Every P-node has at most one successor node.
2. If a P-node is a leaf node, then the winning conditions for P are fulfilled at this node.
3. Every O-node has a successor node for each move by O that is a permissible continuation of the dialogue at this stage." (Fernmüller, Ciabattoni)

Guidelines to model dialogue logics with ASM could be taken from the modelling of tableaux based logics.
Peter H. Schmitt, Egon Börger
A Description of the Tableau Method Using Abstract State Maschines
ftp://www.eecs.umich.edu/groups/gasm/tableau.pdf

Overview (most literature about Dialoglogik is in German and the books are out of print):

- http://faui80.informatik.uni-erlangen.de/IMMD8/Lectures/KRR/05a-ConstructiveIntroDiaLogic-4.pdf
- http://autofocus.informatik.tu-muenchen.de/~juerjens/papers/gamesrevd.pdf
- Thiel, Chr.: Paul Lorenzen (1915-1994). - Bibliographie der Schriften von Paul Lorenzen. in: Journal for General Philosophy of Science 27 (1996),

2.4 Lorenzen's Own Goal in the Günther-Lorenzen Correspondence

From a Letter of Lorenzen to Günther, 23/1/1964

Bewertungen offen ist) — — selbstverständlich kann
ich Sie nicht hindern, weiterhin in unkontrollierbarer
Weise deutsche Wörter zu verwenden (das mag ja auch
zweckmäßig sein, um die Leute zu beeindrucken), aber
daß ein *Abweichen* vom Sprechen in pragmalogisch geordneter
Reihenfolge (was ich "methodisch" nenne) für die Zwecke
der Mathematik, der Computer und auch der Reflexion
über das Sprechen der Menschen irgendwann einmal
notwendig ist, das bestreite ich. Und wie wollten
Sie diese Behauptung: "Abweichen vom pragmalogischen Sprechen
ist notwendig" mir gegenüber verteidigen? Etwa
dadurch, daß Sie bei Ihrem Verteidigungs*versuch* vom
pragmalogischen (d.h. methodischen, wohl auch "vernünftig"
genannten) Sprechen *abweichen*?

Ich vermute (und hoffe), daß Sie — trotz des
Fiebers — schon jetzt an dieser Stelle einsehen, daß
ein solcher Verteidigungsversuch aussichtslos ist.

"Selbstverständlich kann ich Sie nicht hindern, weiterhin in unkontrollierbarer Weise deutsche Wörter zu verwenden (das mag ja auch zweckmässig sein, um Leute zu beeindrucken), aber dass ein Abweichen vom Sprechen in pragmalogisch geordneter Reihenfolge (was ich "methodisch" nennne) für die Zwecke der Mathematik, der Logik, der Computer und auch der Reflexion über das Sprechen der Menschen irgendeinmal *notwendig* ist, das bestreite ich. Und wie wollten Sie die Behauptung: "Abweichen von pragmalogischen Sprachen ist notwendig" mir gegenüber verteidigen? Etwa dadurch, dass Sie bei Ihrem Verteidigungs*versuch* von pragmalogischen (d.h. methodischen, wohl auch "vernünftig" genannten) Sprechen *abweichen*?

Ich vermute (und hoffe), dass Sie – trotz des Fiebers – schon jetzt an dieser Stelle einsehen, dass ein solcher Verteidigungsversuch aussichtslos ist." (Lorenzen to Günther, 23/1/1964)

"Natural I cannot prevent you to use further in an uncontrollable way German words (that also may be appropriate to impress over people), but that a deviation from speaking in a pragmalogically ordered sequence (which I call "methodically") for the purposes of mathematics, logic, computers and also the reflection over the speaking of humans will be someday *necessary*, I deny. And how do you want to defend against me the statement: "Deviating from pragmalogical languages is necessary"? About by the fact that you *deviate* with your defense attempt from pragmalogical (i.e. methodical, probably also called "reasonable") speaking?
I assume (and hope) that you - despite the fever - understand already at this point that such a defense *attempt* is futile."(Lorenzen, 23/1/1964), [kae]

Despite the neo-rationalistic arrogance of Paul Lorenzen which is obvious even in this small part of argumentation in his letter to Gotthard Günther, the madness which is guiding his argumentation is simply this: Lorenzen knows the structure or logic of rationality and Vernunft; to question his kind of rationality can only be mad. OK, Günther is not declared mad by Lorenzen, but despite his fever of 100F he should be able to agree with Lorenzen that he (Günther) has no chance to defend his thesis against Lorenzen without losing contact to human rationality. Without doubt Lorenzen is in perfect possession of rationality and the dialogical rules of it; and he is not suffering from any fever. Who is deviant of Lorenzen's understanding of rationality and its rules of argumentation has to be mad. Or he lost his brain in the fever of 100F. Hence, he will never succeed to justify rationally his thesis. Günther's thesis is that it is necessary to go beyond pragmalogical languages. Pragmalogical languages, i.e. Lorenzen's dialogue logic, are based in logical atomism.

There are no following letters of Günther to my knowledge. Thus, the argument is not (directly) responded by Günther. But there may be – and I guess there are – indirect replies to it in some later papers (cf. Günther's Habermas paper, 1968). And the argument was developed *in extenso* by Günther in earlier papers and books.

Günther's counter argument might be: give me a rational proof of the necessity and completeness of the decision for a dialogical 2-player game to codify rational behavior. If all reasonable argumentations have to be proceeded by a 2-player game how can the decision for the two players be justified? By a two-player game?! The figure is well known in history and is called *petitio in principii*.

But that doesn't matter. Not even for philosophers. Lorenzen's approach was a relief for German philosophers and sociologists not educated in mathematical logic. His students and the influence of Lorenzen to people like Habermas did their best to deny any fundational support for Günther's project. This has to be mentioned because the correspondence could suggest another view than the science political facts had been proving.

A possible *direct* reference to Lorenzen could be found in this paragraph:

"Wir nehmen an, dass die von S1 und S2 produzierten Weltereignisse U1 und U2 in irgendeiner Form als Repräsentationen und symbolische Darstellungen gewisser Eigenschaften von U aufgefasst werden müssen. D.h., S1 "sagt" oder *"schreibt"* etwas, in dem es seine Erkenntnis von U manifestiert. Diese Aktivität haben wir U1 genannt. Wir nehmen außerdem an, dass S2 genau dasselbe tut und sich über dieselben Welteigenschaften durch U2 ausdrückt. Es ist evident, dass wir hier ein *Kommunikations-Problem* haben. D.h., wir müssen uns fragen: wie können sich S1 und S2 darüber verständigen, dass sie über dieselbe Sache reden?" [emphasis, rk]
G. Günther, Das Problem einer trans-klassischen Logik
in: Sprache im technischen Zeitalter, 1965, Heft 16, pp. 1287-1308

Dialogue=[S1, S2, U1, U2, S, U]

2.5 There is no end in sight: Search for the simple and intuitive

Ong:

> "Game semantics is a way to give meanings to computation (and to proofs) using simple and intuitively ideas of game playing.
>
> Two players:
>
> P "Proponent" "Verifier" 0
>
> O "Opponent" "Refuter" 1
>
> (Terminology from Logic: P asserts a thesis, O seeks to demolish it.)
>
> *Basic idea of game semantics:*
>
> The meaning of a system is given in terms of all its potential interaction with its environment."
>
> https://www.newton.cam.ac.uk/webseminars/pg+ws/2006/laa/0206/ong/part1/all.pdf

simple: simple means 2-person games.

intuitively: it is presumed that n-person games are constructible/reducible by/to 2-person games.

games: the idea of games has to be reconsidered. Games are not reducible to winning strategies

"simple and intuitively" is still the jargon of foundational studies.

Polycontexturality is supposing that there are no intuitive and simple constructions for poly-games on the base of 2-person games. Thus, 2-person games are not intuitive but useful reductions of poly-games. Poly-games are complex games consisting of computation, reflection and interaction.

In general, everything simple and intuitive, or as it is called natural, universal and ultimate, is a product of serious cultural fights, its merits had to be discovered and defended and the transition had to be established. Take the historic transition from Pythagorean to Aristotelian world-view and mathematics.

The distinction between opponent/proponent or system/environment is not an absolute one. It depends on a point of view from which such a distinction is drawn. Hence, the point of view should be part of the game. There is no reason to be restricted by one and only one point of view. A multitude of viewpoints can enter the game. But it also should be clear that such a multitude is not simply an addition of viewpoints ad infinitum but has a structure to be explored.

Statement

The justification of the Induction Principle for arithmetic is not intuitively realized by Dialogue Logic and dialogically founded arithmetic of 2-player games.

On the other hand, without the Induction Principle not much is working properly.

Hence, 2-player games are handling only very simple and highly reduced structures. But to deal with the recursivity of the syntactical system of dialogue logic arithmetic is needed.

> Introduction Dialogue Game and Game Semantics for Programming Languages
> http://autofocus.informatik.tu-muenchen.de/~juerjens/papers/gamesrevd.pdf

Logical atomism

Absurdity of the examples. Use: *Cheese, please!* Cheese as Käse and cheese as smiling, alternatively and at once and in an interplay. Consequence: no atomic expression without a context.

3 Frameworks vs. strategies

Because the structure of the framework of a classic dialogue game is stable and not changing by decision–it is always simply the duality of opponent and proponent–, there is no need to focus on it in classical studies. There, the focus is on what is changing, and that are the different sequences inside the framework of the game. Depending on different rules and meta-rules defining different strategies.

To emphasize on the structure of the framework is only producing irritation in the stable world of dialogue games. And it is even mentioned that such a thematization is itself irritated and misled. The irony for dialogue logicians like Lorenzen is, that not to listen to a dialogue partner is not against the rules of his dialogues. Computer scientists have other aims. And again, I'm not on the way to criticize such aims in any way. Nevertheless, we have not to give up our knowledge and experiences.

3.1 Dialogue Games in Computer Science

The Abramsky/Ong school of Games Semantics for computer science purposes is giving a good starting point for the exposition and deconstruction of the framework/strategy distinction of formal games.

"Game Semantics is a way of giving meanings to computation (and to proofs) using the intuitive idea of game playing." (Ong)

Luke Ong is giving a concise introduction to the subjet.

To model a first-order function, we have:

	Input	Output
System (P)	*consumes*	*produces*
Environment (O)	*produces*	*consumes*

Example. A typical interaction of the successor function:

$$\mathrm{succ} : \mathbb{N} \to \mathbb{N}$$

1. OQ: "What is the output of this function?"
2. PQ: "What is the input to this function?"
3. OA: "The input is 5."
4. PA: "The output is 6."

Systematic table of system/environment interactions

Players	Point-of-view	Functional	Imperative	Concurrent
P	System	term	procedure	process
O	Environment	program context	context & store	rest-of-system

"The meaning of a system is given in terms of all its *potential interaction* with its environment." (Ong)

https://www.newton.cam.ac.uk/webseminars/pg+ws/2006/laa/0206/ong/part1/all.pdf

We have to ask: What does "meaning" mean in such a context of a system/environment context?

It becomes not only clear with Ong's exposition that the structure of the system, its system / environment distinction, is not only stable but also that the internal dynamics of the structure in contrast to the successions of the strategic moves, is not in focus and is in fact not existing in the expllication. From a system theoretic point of view it is an *external* description of a 0-level distinction system(P / O).

> "Who is the System? Who is the Environment? This depends on point of view. We may designate some agent or group of agents as the System currently under consideration, with everything else as the Environment; but it is always possible to contemplate a rˆole interchange, in which the Environment becomes the System and vice versa. (This is, of course, one of the great devices, and imaginative functions, of creative literature). This symmetry between System and Environment carries a first clue that there is some structure here; it will lead us to a key *duality*, and a deep connection to logic." (Abramsky)

"We may designate some agent", but this is depending on an external observer / designer and not on the game itself. Neither is it including a plurality of points of view. Also there is no doubt that there is no environment of an environment, no second-order distinctions are involved. Othewise the nice *duality* and its *"deep connection to logic"* would be lost.

3.2 Second-order analysis of the system / environment relationship

As we can know from Second-Order Cybernetics the system / environment relationship has other features, too.

First, the system / environment-distinction is depending on a point of view from which it is drawn. Thus in contrast, an internal description of the distinction has to be drawn from all intrinsic elements of the system (system, environment), i.e., from both, the point of view of the system: *(system, (system / environment)* and from the point of view of the environment: *(environment, (environment / system)*.

Second, both internal descriptions from the point of view of the system and of the environment holds *at once*. There is no environment without system, and no system without environment. It is presupposed that we are not doing naive physics.

Third, the system / environment-distinction or opponent / proponent-distinction is coupled with the activities: (questioning, answering) or in more "dialogical" terms: (defense, attack) of the proponent and the opponent.

As a result we get the as-abstractions of (PP, PO, OP, OO). We can freeze this dynamics into an is-abstraction mode with (OQ, OA, PQ, PA) where we have four fixed entities. This is reasonable if we don't consider the possibility of changing the complexity of dialogues towards polylogues. But nevertheless, there are here too 4 elements and not only two as we would like to belief for a dialogue game.

Again, I'm considering *frameworks* of games and not games as *strategies* which can have different compositions inside their common framework.

This is still not the full description but it should make the difference between the external and the internal description of dialogue games clear enough.

The whole manoeuvre can be mapped onto the diagrams:

$$\begin{tikzcd} PQ \arrow[r] \arrow[d, phantom, "\cong"] & PA \Updownarrow OQ \arrow[r] & OA \arrow[d, phantom, "\cong"] \\ PQ \arrow[rr] & & OA \end{tikzcd} \qquad \begin{tikzcd} PP \arrow[r] \arrow[d, phantom, "\cong"] & PO \Updownarrow OP \arrow[r] & OO \arrow[d, phantom, "\cong"] \\ PP \arrow[rr] & & OO \end{tikzcd}$$

is-abstractions:
OQ=O-question
OA=O-answer
PQ=P-question
PA=P-answer

as-abstractions
PP= Proponent as proponent, i.e., proponent as answer,
PO= Proponent as opponent, i.e., proponent as question,
OP= Opponent as proponent, i.e., opponent as answer,
OO= Opponent as opponent, i.e., proponent as question.

3.3 Towards a Semantics for logical n-player games

It should be easy now to translate the *"value-talk"* of multi-valued logics in the sense of polycontexturality into a *"game-talk"* for n-player logical games.

> "Since in classic logic only one value is available for designation, all objects must—as far as their logical structure is concerned—belong to the same ontological category (one-valued ontology). But if we assume that the universe contains at least two basic categories of potential logical objects, namely systems without self-reference and systems with self-reference, two-valued logic has no means to distinguish them by designation. It is safe to assume that an object with the capacity of self-reference displays a higher structural complexity than one without this capacity. But in order to designate a difference in the complexity of ontological structure, the designational process itself has to show a corresponding difference. This, however, can only be effected—as we noted before—by a difference in the number of values employed for designation. On the other hand, we have shown that the number of positive valued in any m-valued system always remains 1. It follows that the coincidence of the distinction between assertion and negation with the distinction between designation and non-designation can only hold in a two-valued system, of logic. If we proceed to systems where m > 2, two cases may occur: either there is no excess of non-designational values or there is such an excess. An m-valued system which does not show an excess of non-designational values cannot be interpreted as a logic. It must be considered an ontology followed by logics which refer to it. The logics, of course, represent all those cases where trans-classical systems show a distinction between designative and non-designative values."

> "Differences in *information* are structurally equivalent to the distinction between positive and negative values. Differences in *meaning*, on the other hand, are related to the distinction between designation and non-designation. To put it differently: *the non-coincidence of the negational and the designational function of values is formally equivalent to the difference between information and meaning*. Where both coincide, as in two-valued logic, structural characteristics interpretable as meaning are unavailable. If a computer produces a map and the map represents only information, two-valued logic is all that is required. This is beyond dispute. Information theory therefore states quite rightly that the meaningful aspect of the informational input into a system may be ignored by its computations."

> "In self-referential systems a map serves a double function: a) relative to the environment, and b) relative to the self-referential organization of the system. In case a) the relation is purely *informational*; in case b) it is *hermeneutical*. The relation of the map to the mappable object (ignoring the "subject" for which it *is* a map) is fully expressible in terms of two-valued logic. Self-reference, however, requires an "outside" observer who does not identify himself with either the map or the object, but is capable of comparing them. The concept of the object at which the map points belongs to the traditional one-valued classic ontology and requires therefore only a single value for designational purposes. The self-referential function of the observer, however, requires two distinctions: one between himself and map-and-object; and second between the map on one side and the object on the other side. The functional role of the observer as that which is excluded from the domain of the observables is represented by a non-designational value. Designated are only map and object. This requires, in order to keep object and map apart, and to indicate that the map *means* the object, two values for designation. But designation by more than one value is (as we know) only available in trans-classic many-valued systems of logic. The m-valued non-coincidence of informational and hermeneutic structure for all cases where m > 2 is a necessary but not sufficient prerequisite of any cybernetic theory of self-referential systems." (Gunther 1965)

> G. Günther, Cybernetics and the Transition from Classical to Trans-Classical Logic
> http://www.vordenker.de/ggphilosophy/gg_bcl-report-no-30.pdf
> http://www.vordenker.de/ggphilosophy/gg_many-val-desig-hierarch.pdf

Information vs.meaning

What could the interaction between n players, n>2, of a non-reductional n-player game be if not being involved in meaning, i.e., hermeneutical interactions in contrast to information passing?

Meaning is depending on interpretations. Interpretations are involved in points of view or positions of agents. Meaning is thus engaged in difference. Logical games, like dialogue logics, know only one difference, the difference between winning or losing the game based on the difference of opponent and proponent. Both differences, winning / losing and opponent / proponent, are coinciding in the search for truth or provability. But the objective is only one: the end of the game.

In 2-player games there is one and only one (dialogical) truth. This one-truth situation is distributed in n-player polylogical games over different positions. Depending on the mediation rules, different positions can produce different dialogical truth constellations, alternatively or in parallel. This is the *local* thematization of the agents in a multi-agent system. Because of the mediated plurality of agents a global thematization has to be considered. What is globally the logical n-player system producing?

Such *global* thematizations had been in the main focus of Gunther's studies of the semantic / meontic / thematic properties of polycontextural logical systems.

There is no strict logical need for m-players if their logical aim is the same as for 2-player logical games.

Local vs. global n-player semantics

Internal or local semantics:
agent-oriented winning strategies

External or global semantics:
group-oriented winning strategies

3.4 Wordings for the is-abstraction mode

A proponent question PQ can be introduced. A proponent answer PA can be given. A proponent answer PA is the inverse or *dual* of an opponent question OQ. Both are localized at different structural positions and connected by the *inversion* of their functionality. The two distinctions are mirrored at a third place. We can assume that this third distinction of PQ and OA is representing the external description of the game as a result. In other words, dialogues as we know them are localized at this third position omitting their construction by the two other distinctions.

Ong's typical example of the successor function for natural numbers and its *deconstruction*:

succ: N –> N:

1. OQ: "What is the output of this function?"
2. PQ: "What is the input of this function?"
3. OA: "The input is 5."
4. PA: "The output is 6."

First step deconstruction

$$\mathbb{N}_0 \rightarrow \left\{ \begin{array}{l} succ_1 : \mathbb{N} \rightarrow \mathbb{N} \\ succ_2 : \mathbb{N} \rightarrow \mathbb{N} \\ succ_3 : \mathbb{N} \rightarrow \mathbb{N} \end{array} \right\} \rightarrow \mathbb{N}_0$$

The decomposition of the numeric function and the mediation of its parts can be symbolized by the *"magic box"* as applied for polycontextural parallel computation. The result of the interplay between $frame_1$ and $frame_2$ is mapped on $frame_3$ and fixed as *such* in $frame_0$.

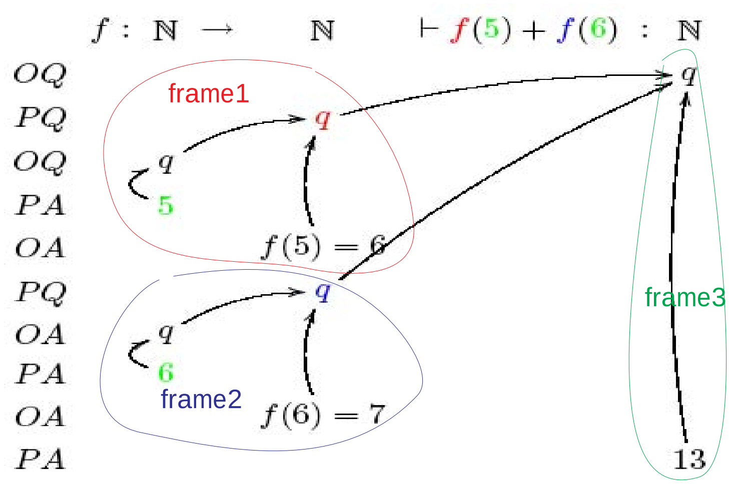
frame1
1 OQ: "What is the output of this function?"
3.OA: "The input is 5."
----

frame2
2. PQ: "What is the input of this function?"
----
4. PA: "The output is 6."

frame3
5. OQ/OA: answer accepted.

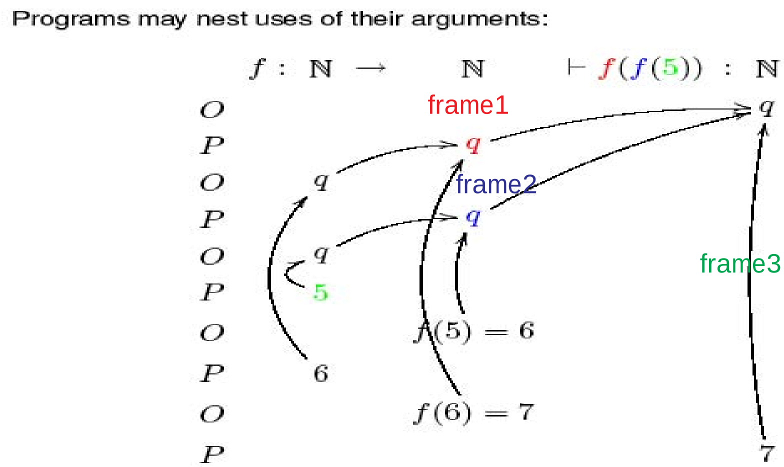
frame3
6. PA/PQ: OK.

Ong's example for multiple use of argument and its deconstruction



Instead of painting even more pictures I simply encircle Ong's subsystems.

The given arguments are not braking down if we can omit the quadruple notation in favor of the dual P/O-notation. Again, the illustration should do the job.



$$\mathbb{N}_0 \rightarrow \left\{ \begin{array}{l} num_1 : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N} \\ num_2 : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N} \\ num_3 : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{N} \end{array} \right\} \rightarrow \mathbb{N}_0$$

Here again, a first step deconstruction of the function "f: N –> N –> N: f(f(5))" can be modeled by the dissolvement of the uniqueness of N into a triple of (N_1, N_2, N_3) and its final representation in N_0 which is, back home, the reality of the unique natural number series symbolized by N.

If a game gives a meaning to computations by playing the game (Wittgenstein, Ong) we can stubbornly insist on the experience that in fact we are not playing a single and simple but a highly complex game composed by the different steps defined by the general game (formula to compute). Thus, the uniqueness of this game is dissolved into different sub-games which are mediated, i.e., interacting together, to build the general game. Thus, the uniqueness of natural numbers is a construct and is not intuitively and pre-given to the players.

Such a thematization may be exaggerated if we take it not as an example and exercise for the introduction of a different kind of thematization. This new kind of thematizing computation is not to confuse with parallel or concurrent versions of dialogue games. Parallelism in dialogue games is focussed on parallel strategies inside the dualistic structure of the framework of 2- or n-player games.

Example for parallelism of strategies:

Christian G. Fermuller, Agata Ciabattoni
From Intuitionistic Logic to Godel-Dummett Logic via Parallel Dialogue Games
http://www.logic.at/people/chrisf/ismvl03.ps

Or n-player games by Samson Abramsky:

"What kind of logic has a natural semantics in multi-player (rather than 2-player) games?"
"The whole question of proof theory for the logic LA has been left open. In a sense, we have given a semantics of proofs without having given a syntax! How multi-agent proof theory should look is conceptually both challenging and intriguing."
"The logic and semantics we have developed appears to flow from very natural intuitions." (Abramsky)

Socially Responsive, Environmentally Friendly Logic
http://web.comlab.ox.ac.uk/oucl/work/samson.abramsky/sandu.pdf

3.5 What is the exercise telling us?

The internal description of the game is producing a de-sedimentation of the overlapping descriptions realized by the external thematization. The focus of this study is not on the development of strategic moves inside a given framework of dialogical argumentation but on the structure of the framework in which such moves can happen. Interestingly, such frameworks have not to be frozen structures but show structural dynamics of their multiple constituents, i.e., systems and environments, connected with their local functions of attack and defence.

As long as the game is restricted to a 2-player game this distinction between internal and external description may remain redundant. Especially if it turns out that the 2-player game doesn't need any two players but is played by one player taking successively one or the other position.

De-sedimentation becomes crucial if combined with the *dissemination* of games into real multi-player games. Dissemination is interwoven with the history of existing systems, it is not an ad hoc decision to distribute and mediate systems by the free will of a designer. In contrary, the designer has to serve the structures given. That's the only way to surpass them – by means of their own history and composition.

The classic strategy to deal with multitude is to *apply* the same unique and universal system to different concrete situations, again and again, without changing the singularity and hegemony of it over the application to such empirical diversity.

De-sedimentation and inscription

De-sedimentation is a first step to move mental images of logical constructions into inscriptions. De-sedimentation is reversing the process of mentalization towards inscriptive realization.

This is best demonstrated by the dialogical introduction of the arithmetical Principle of Induction (IP). Are dialogue models giving a more acceptable justification of IP than Peano's axiomatics? It seems, that computer science is not concerned about such meta-mathematical questions.

3.5.1 The stroke calculus

Stroke Calculus

$$Rules:\begin{cases} R_0 : \longrightarrow \mid \\ R_1 : n \longrightarrow n\mid \end{cases}$$

$$Meta-Rules:\begin{cases} n \in Var \\ R_2 : R_1 \in \mathbb{N}, \end{cases}$$

The stroke calculus is ruling the way how to produce as many strokes as you want. To do that, it starts with the introduction rule R_0 which allows to introduce one stroke as a start stroke. The second rule R_1 rules how to produce from n strokes n+1 stroke. This is managed by an object variable n which doesn't belong to the production calculus but to its conditions. Thus, it is placed a Meta-Rules. But the real point of the game is another rule which is mostly not mentioned at all: it is the indefinite iteration rule R_2 which states that the production rule R_1 can be applied as often as desired, i.e., potentially infinitely often. This is working together with the object variable which can deal with a set of potentially indefinitely many strokes.

Further analysis is omitted here. For example, the uniqueness of rule R_0.

All in all, the whole little constructivist calculus, a basic form of mathematical reasoning and construction, looks highly circular. In a happy contradiction to its introduction rules.

David Isle puts it into words and delivers a profound criticism to such evidences.

R1) Write down a stroke |;
R2) Given a set of strokes (call it X) write down X|;
R3) Now apply R1 once and then apply R2 again and again.

Interpretation
Set | as 1
Set || as 2
Set ||| as 3, and so on.

"An understanding of the "structure of the natural numbers" thus consists in an understanding of these rules. But what has actually been presented here? Rules R1 and R2 are fairly unambiguous, in fact, one could easily use them to write down a few numerals.
But rule R3 is in a different category. It does not determine a unique method of proceeding because that determination is contained in the words "apply R2 again and again". But these words make use of the very conception of natural number and indefinite repetition whose explanation is being attempted: in other words, this description is circular." (Isle, p. 133), Epstein, Carnielli, Computability, p. 265/66
http://www.tufts.edu/as/math/isles.html

Even if we accept this criticism of the rules, we have to accept, that rule R2 demands some preconditions, at least, we have to add the new stroke in line with the other strokes, and not somewhere else, e.g. behind the blackboard. If we use the quite misleading terminology of sets in rule R2, the new stroke has to be written in the domain of the set and not outside of it.

But why should we accept all that, if it is not explicitly asked? Therefore, the game is not so clear as it should be. The presupositions of the stroke calculus is linearity of repetition and atomizity of its strokes, short: semiotic identity.

All these presuppositions may not be very natural, they are not pre-given, we simply have to learn them, that is, to produce a semiotic abstraction and to internalize it by education.

http://www.thinkartlab.com/pkl/media/SKIZZE-0.9.5-medium.pdf
http://www.thinkartlab.com/pkl/media/DERRIDA'S MACHINES.pdf

Parallactic situation?

Again, the proposed *linearity* of the construction can not be justified and the hidden *circularity* of the construction can not be accepted. And the attempted *constructivity* can not be abandoned. What to do? The common characteristics of the approaches is mono-contexturality. Why not try *poly-contexturality*?

3.6 Justification of IP

Again, Lorenzen's dialogical approach.

$$\begin{array}{r||l} A(0) & \\ \forall x.A(x) \rightarrow A(x'). & \forall y(Ay) \end{array}$$

As far as I remember the whole thing is nothing else than what Ludwig Wittgenstein (or Poincare) proposed long before and as Waismann kept over the wars alive. That is, a cascade of arithmetical *modi ponens* up to the presumed number *n* as limit.

$$\begin{array}{r||l} ?n & \end{array}$$

Oponnent is O is attacking the statement of Proponent P with a choosen number n: ?n.

$$\begin{array}{r||l} & ?0 \\ A(0) \rightarrow A(0') & A(0) \\ A(0') & ?0' \\ A(0') \rightarrow A(0'') & A(0') \\ A(0'') & ?0'' \\ \circ\circ\circ & \circ\circ\circ \end{array}$$

Proponent P is continuing to attack Opponent O as long the opponent reaches his statement A(n).

1. $A(0)$
2. $\forall x.\ A(x) \rightarrow A(x')$
3. $\forall x(Ax)$

Because the statements (1) and (2) are arithmetically true, the statement (3) is implied as arithmetically true either.

Hence, IP is dialogue arithmetically true.

$$IP: A(0) \wedge (\forall x.\ A(x) \rightarrow A(x')) \rightarrow \forall x(Ax)$$

As long as we don't ask where from the guy has his number *n* in the attacking business we should calm down and accept the proposed construction. As anyone does. It does the house hold of a clean constructivist business. But how do we know that *n* is a number? And if it might be a number, how do we know that it is in a way an accessible or even constructively accessible number? Thus, we would like to have to opportunity to *construct* inside the same manoeuvre, at an own contextural place, the legitimacy of our attacking number in parallel to the *use* of such a number as in an attack. Thus, a *double strategy* – plus their mediation – is necessary to proof the IP.

Double Strategy

If our number *n* is not falling from the sky it has a double function: it is *used* to justify numbers and is itself a *justified* number. For a mentalist approach this circularity is not disturbing at all because, as we are told at school, we shouldn't confuse *use* and *mention* of a number. But for mathematically grown-ups, things are not as comfortable. We have to introduce a new contexture to put things together.

But this is a more creative challenge than to ask for well 100 years if the one or other number really is a (finite) number at all.

D. van Dantzig, "Is 10^10^10 a finite number?", and:
David Isles, What Evidence is There That 2^65536 is a Natural Number?
in: NOTRE DAME JOURNAL OF FORMAL LOGIC, Vol 33 #4, pp 465-480, fall 1992

3.6.1 Induction Principle based on Insight

A new argument enters the game if the number *n* is too big to be realized between two cigarettes. We then are recommended by Lorenzen to use our reason and to produce the *insight* that how long it ever takes in principle it will happen. That is there are no obstacles in the sequence of steps which will lead to the final justification by realizing the postulated number as an attack.

Put the Insight onto the table!

Opponent^1	*Proponent*^{1} / / *Opponent*^{2}	*Proponent*^{2}
$\circ\circ\circ$	$?0$	–
$A(0) \to A(0')$	$A(0)$	–
$A(0')$	$?0'$	–
$A(0') \to A(0'')$	$A(0')$	–
$A(0'')$	$?0''$	–
$\circ\circ\circ$	$\circ\circ\circ$	$(?0)^{\circ\circ\circ}$
–	$A(0) \to A(0')^{\circ\circ\circ}$	$A(0)^{\circ\circ\circ}$
–	$A(0')^{\circ\circ\circ}$	$(?0')^{\circ\circ\circ}$
–	$A(0') \to A(0'')^{\circ\circ\circ}$	$A(0')^{\circ\circ\circ}$
–	$A(0'')^{\circ\circ\circ}$	$(?0'')^{\circ\circ\circ}$
–	$(\circ\circ\circ)^{\circ\circ\circ}$	$(\circ\circ\circ)^{\circ\circ\circ}$
–	–	–

Opponent^1	*Proponent*^{1} ||	*Opponent*^{2}	*Proponent*^{2}
$\circ\circ\circ$	$?0$	–	–
$A(0) \to A(0')$	$A(0)$	–	–
$A(0')$	$?0'$	–	–
$A(0') \to A(0'')$	$A(0')$	–	–
$A(0'')$	$?0''$	–	–
$\circ\circ\circ$	$\circ\circ\circ$	$(?0)^{\circ\circ\circ}$	–
–	–	$A(0)^{\circ\circ\circ}$	$A(0) \to A(0')^{\circ\circ\circ}$
–	–	$(?0')^{\circ\circ\circ}$	$A(0')^{\circ\circ\circ}$
–	–	$A(0')^{\circ\circ\circ}$	$A(0') \to A(0'')^{\circ\circ\circ}$
–	–	$(?0'')^{\circ\circ\circ}$	$A(0'')^{\circ\circ\circ}$
–	–	$(\circ\circ\circ)^{\circ\circ\circ}$	$(\circ\circ\circ)^{\circ\circ\circ}$
–	– –	– –	–

There is even a reassurance given. If the player looses confidence in his insight or into the rationality of the game he is allowed to go back to it and have some exercise with more steps in the cascade of attacks and defenses until he feels well again and his insight is nurtured enough by new experiences. As usual, the most important part of the construction are the famous 3-points ($^{\circ\circ\circ}$). Within the new experience the 3 points are moved a little bit more to infinity... Back in the abstractness of his mental insight the polycontextural demon is asking: Where did he put it? Why not to put the insight into a contexture where the game can go on whoever is playing it or not?

Neither the switch between levels of abstraction nor the production of insight into its rationality are formalized in the proposed dialogue calculus Lorenzen is selling.

The two examples are sketching different ways of reflecting on the Induction Principle to restore the insight into its first initialization by the dialogical instructor. The first is reflecting the inversion of the roles of the opponent and proponent and is gluing the approaches together. The second is demanding for more separation and independence between the levels of reflection about the realization of the IP.

Same level reflection with interuption.

$Opponent^1$	$Proponent^1$
$\circ\circ\circ$	$?0$
$A(0) \to A(0')$	$A(0)$
$A(0')$	$?0'$
$A(0') \to A(0'')$	$A(0')$
$A(0'')$	$?0''$
$\circ\circ\circ$	$\circ\circ\circ$
$A(n-\alpha)$	$?(n-\alpha)$
$A(0) \to A(0')(n-\alpha)$	$A(0)(n-\alpha)$
$A(0')(n-\alpha)$	$?(0')(n-\alpha)$
$A(0') \to A(0'')(n-\alpha)$	$A(0')(n-\alpha)$
$A(0'')(n-\alpha)$	$?(0'')(n-\alpha)$
$(\circ\circ\circ)(n-\alpha)$	$(\circ\circ\circ)(n-\alpha)$

The third example simply continues at the point where it left the development and goes on with it as a new starting point in the very same game. This can be seen as an *application* of the same logic to a different situation, thus the 2-player dialogue is iterated as such. There is, therefore, no epistemic space given by the architectonic of the game to reflect this action.

Because there are obviously many different ways to realize owns insight this term is mystical as long it is not realized into a calculus of its own. Such a calculus is opening up new forms of negotiations and even insights between players of a ultra-logical insight-calculus because not everyone is accepting the other ones strategy of re-establishing his insight(s).

Some will insist to go back to the old game; others will insist on the fact that time has changed and their is no simple way to go back to the beginnings. But nevertheless, there is family resemblance, similarity, enough to be loyal to what happened before.

More modern people will delegate such a job to a computer program and trust in the correctness of its execution to restore their confidence into the universality of the IP. But with that, a new distinction of great importance is introduced: the distinction between *mental* and *mechanical* arithmetic and mathematics. This distinction deserves its own right and rationality, therefore it should be placed and placing a new contexture to deal with it. Obviously, only some SF people would think that this still can be done by a 2-player game. Even transhumanists will accept that a technical computing implant would not change the epistemic structure of delegated computation. That is, the crucial difference of mental and mechanical conceptualizations. Also it remains open who of the famous 2 players will get it, and where.

Setbacks for dialogue logic

– It turned out that the dialogue method is not guaranteeing a constructivist approach to logic. Classic logic as well can be characterized by dialogue rules (Kuno Lorenz). Thus, the whole ideological interests to find the *true logic* went up in smoke.

– Arithmetic is not well defined inside the dialogue method alone.

– Technically, the whole approach never worked on a professional level.

3.6.2 Insight as reflectional distance

What can we understand by the term "insight"?

> "In contrast to the traditional viewpoint – which has its own legitimacy not contested an these pages – the theory of many-valued logic suggests that in a different respect the iterative capacity of a thinking subject has certain limits. Without such limits we might be able to think but we would not be able to be *aware of our process* of iterative reflection. Awareness of iteration implies the power to put a stop to it.
> Such capacity to step out of the iterative process making it an arrested object of our reflection might be dependent an the richness of ontological structure a subject faces. Especially as the structure of consciousness itself must be an exact corollary of the ontology it possesses." (Gunther, 1968)
> G. Gunther, ManyValued Designations and a Hierarchy of First Order Ontologies, http://www.vordenker.de/ggphilosophy/gg_many-val-desig-hierarch.pdf

"Awareness of iteration implies the power to stop it." For whom? Obviously not for the iterating or counting agent. This statement also doesn't mean that there is an obstacle or a gap in the concept of iterability which puts an end on it. What it means, as far as I understand it, is the that the power to make a *shift* in the thematization, i.e., to shift focus, to another level of reflection makes a stop to the first thematization of iteration. Without such a shift of thematization *"we might be able to* think *but we would not be able to be* aware *of our process of iterative reflection."*.

"Such capacity to step out of the iterative process" needs an own logico-arithmetic place to where the step can be made. Thus, a shift in focus is producing the possibility of awareness of the fact of unlimited iterativity. In the words of Gunther's analysis, this place from which a stop of the iteration process by making a shift in thematization can be realized is given by *"the richness of ontological structure"*, i.e., by the complexity of first-order ontologies involved. A plurality of first-order ontologies is designed as polycontexturality.

This possibility of a reflectional agent to distance himself from the process of iterativity and to be involved into *"certain limits"* is not to confuse with the idea that iterativity itself has its *"limits"*. This is not about *"Cracks and gaps"* but about the reflectionality of a thinking agent which with his distancing action is producing insight into the unlimited processuality of iteration.

> *"The law which we applied was the principle of numerical induction; and although nobody has ever counted up to 10^{1000}, or ever will, we know perfectly well that it would be the height of* absurdity *to assume that our law will stop being valid at the quoted number and start working again at 10^{10000}.*
> *We know this with absolute certainity because we are* aware *of the fact that the principle of induction is nothing but an expression of the reflective procedure our consciousness employs in order to become aware of a sequence of numbers. The breaking down of the law even for one single number out of the infinity would mean there is no numerical consciousness at all!"* Gotthard Gunther, Cybernetic Ontology, p. 360

Therefore, there is no paradox in the concept of *"limits"* and *"to put a stop"* of iterativity and the *insight* or *awareness* into the unlimited iterativity of a reflecting or counting process.

The limit of a reflecting process is the beginnning of the process of awareness into the unlimited iterativity of the reflectional process.

That's what we might understand by "insight".

After the heroic moves of distancing alienations are accomplished new *interactional* patterns between different levels of reflection are arising.

3.7 Intuitionism vs. Ultra-intuitionism

> *„I ask: why has such entity as 10^{12} to belong to a natural number series? Nobody has counted up to it (10^{12} seconds constituting more than 20000 years) and every attempt to construct the 10^{12} –th member of sequence 0,0',0'',… requires just 10^{12} steps. But the expression 'n steps' presupposes that n is a natural number i.e. a number of a natural number series. So this natural attempt to construct the number 10^{12} in a natural number series involves a vicious circle. This vicious circle is no better than that involved in the impredicaive definitions of set theory: and if we have proscribed these definitions we have to proscribe the belief in existence of a natural number 10^{12}, too.“* (Yessenin–Volpin, 1970, 4–5)
> http://yessenin-volpin.org/ http://english.mn.ru/english/issue.php?2002-22-10

The story doesn't end here. According to Rohit Parikh new *obstacles* are on the way. It may be only a gradual difference between formalism and constructivism in respect of the constructibility of big numbers (van Dantzig) both wouldn't see an obstacle to construct them in principle.

Parikh's paper is posing two surprising questions about concrete (feasible) numbers:

> "a) are the primitive recursive functions concrete?
> e.g. is the function $e(x, y) = x^y$ concrete?
> b) is there a unique set of natural numbers?"
> Rohit J. Parikh, Existence and Feasibility in Arithmetic, J.S.L. 36, 494–508, (1971)

Big numbers are easily defined by exponation. A further radicalization of constructibiliity is given by the difference of potential and feasible numbers. Two numbers may be constructible in a intuitionistic and a ultra-intuitionistic way. But their exponentiation isn't generally feasible in a ultra-intuitionistic setting.

Additionally to the nuisance of the vicious circle inside the principle of induction (IP) another intriguing obstacle occurred. In his research report *„Existence and Feasibility in Arithmetic“* Rohit Parikh proved that exponentiation is not *factual* realizable. Hence denying question a). That is, the attribute "$P(x,y,z)$: $x^y=z$" is factual realizable but not the expression "$\forall(x)\forall(y)\exists(z)(x^y=z)$". Hence, two numbers x, y may be factually constructible but their exponentiation e(x, y) may be not.

http://www.thinkartlab.com/pkl/media/DISSEM-final.pdf

Feasibility of numbers is neither identical with the intuitionistic device of potential infinity nor with a simple finitism of pragmatic order. How to justify constructivism (intuitionism) not only in contrast to platonism and formalism but also to ultra-intuitionism in the framework of a 2-person game? The whole of Lorenzen's endeavour ended in the insight that there is no justification at all but *reconstruction* only.

Polycontexturality vs. monomorphism

As a natural consequence the ideas of this sketch are forcing new formalizations of arithmetic. Reflectional levels are not supporting the *monomorphism* of arithmetic.

> *„Are „the“ intuitive natural numbers categorical? That is, is the description of natural number as clear and definitive as we usually take it to be?*
> *This was no idle question for Frege who in the* Foundation of Arithmetics *attempted to achieve an absolute and clear description of the natural numbers. Any denial of categoricity has importand consequences.*
> *Whenever we define a class of mathematical objects via inductive definition and then proceed to establish results about objects in that class we make tacit use of properties of certain functions.“* (Isle, p.111), in: R. Epstein, W. Carnielli, Computability, 1999

4 Framework of polycontextural n-person games

4.1 Dissemination of dialogue logics over the polycontextural matrix

[Formulae, Opposition, Proposition, Attack, Defence, Rules, Frame, Step]
dialogue=[formulae, O, P, A, D, Rules, Frame, n], n= steps of developments.

Scheme of (m, n)-polylogue games

$$polylogue^{(m,\,n)} = \begin{bmatrix} contextures^{(m,\,n)} \\ \begin{bmatrix} sops \\ \begin{bmatrix} framework \\ \begin{bmatrix} scenario \\ \left\{\begin{matrix} classic \\ intuit \\ free \end{matrix}\right\} \\ \begin{bmatrix} global\ \ rules \\ \begin{bmatrix} local\ \ rules \\ \begin{bmatrix} (opp, prop) \\ \begin{bmatrix}\begin{pmatrix} A \\ D \end{pmatrix}\end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix}$$

contextures: Dissemination of logics over the polycontextural matrix,
sops: Super-operators between contextures= {id, perm, red, repl, bif},
framework: type of mediation of oppponents and proponents,
scenario: type of mixed logics, like classic, intuitionistic, etc.,
global and *local* rules: similar to the rules of dialogues plus transjunctions.

Chiasmus scheme of opponent and proponent

The type of mediation of oppponents and prponents with their roles as attack and defence is ruled by the proemial relationship (chiasmus).

$$\chi\left(opp,\ prop,\ pos_{i,j,k}\right) = \begin{pmatrix} opp_i \longrightarrow prop_i \Updownarrow opp_j \longrightarrow prop_j \\ \cong \qquad\qquad\qquad\qquad\qquad\qquad \cong \\ opp_k \longrightarrow\!\!\longrightarrow\!\!\longrightarrow\!\!\longrightarrow prop_k \end{pmatrix}$$

Polycontextural matrix

poly-Lambda Calculus
Lambda Calculi in polycontextural Situations.
http://www.thinkartlab.com/pkl/lola/poly-Lambda_Calculus.pdf

PolyLogics
Towards a formalization of polycontextural Logics.
http://www.thinkartlab.com/pkl/lola/PolyLogics.pdf

ConTeXtures
Programming Dynamic Complexity.
http://www.thinkartlab.com/pkl/lola/ConTeXtures.pdf

4.2 Correspondence between dialogical and tableaux rules

(O)- *Cases*	(P)- *Cases*
$\dfrac{(\mathbf{O})A \wedge B}{\langle(\mathbf{P})_?\rangle(\mathbf{O})A \mid \langle(\mathbf{P})_?\rangle(\mathbf{O})B}$	$\dfrac{(\mathbf{P})A \wedge B}{\begin{array}{c}\langle(\mathbf{O})_?\rangle(\mathbf{P})A\\ (\langle(\mathbf{O})_?\rangle(\mathbf{P})B)\end{array}}$
$\dfrac{(\mathbf{O})A \wedge B}{\begin{array}{c}\langle(\mathbf{P})_{?\mathrm{left}}\rangle(\mathbf{O})A\\ (\langle(\mathbf{P})_{?\mathrm{right}}\rangle(\mathbf{O})B)\end{array}}$	$\dfrac{(\mathbf{P})A \wedge B}{\langle(\mathbf{O})_{?\mathrm{left}}\rangle(\mathbf{P})A \mid \langle(\mathbf{O})_{?\mathrm{right}}\rangle(\mathbf{P})B}$
$\dfrac{(\mathbf{O})A \to B}{(\mathbf{P})A\ldots \mid \langle(\mathbf{P})A\rangle(\mathbf{O})B}$	$\dfrac{(\mathbf{P})A \to B}{\begin{array}{c}(\mathbf{O})A\\ (\mathbf{P})B\end{array}}$
$\dfrac{(\mathbf{O})\neg A}{(\mathbf{P})A, \otimes}$	$\dfrac{(\mathbf{P})\neg A}{(\mathbf{O})A, \otimes}$
$\dfrac{(\mathbf{O})\bigwedge_x A}{\langle(\mathbf{P})_{?\tau}\rangle(\mathbf{O})A_{[\tau^*/x]}}$ τ has been labelled with a star before	$\dfrac{(\mathbf{P})\bigwedge_x A}{\langle(\mathbf{O})_{?\tau^*}\rangle(\mathbf{P})A_{[\tau^*/x]\tau}}$ τ is new
$\dfrac{(\mathbf{O})\bigvee_x A}{\langle(\mathbf{P})_?\rangle(\mathbf{O})A_{[\tau^*/x]}}$ τ is new	$\dfrac{(\mathbf{P})\bigvee_x A}{\langle(\mathbf{O})_?\rangle(\mathbf{P})A_{[\tau/x]}}$ τ has been labelled with a star before

"The philosophical point of dialogical logic is that this approach does not understand semantics as mapping names and relationships into the real world to obtain an abstract counterpart of it, but as *acting* upon them in a particular way." (Rahman)
http://www.hf.uio.no/filosofi/njpl/vol3no1/symbexis/node2.html

Alain Lecomte, Logics for Dialogs, http://brassens.upmf-grenoble.fr/~alecomte/LogicDial.pdf

4.2.1 Translation of dialogue rules to tableaux rules

Negation scheme for polylogues and negation in 3-polylogic

$$\forall i \in (s(m)):$$

$$\frac{(P_i)\neg_i A}{(O_i)A, \varnothing_i} \qquad \frac{(O_i)\neg_i A}{(P_i)A, \varnothing_i}$$

$\dfrac{t_1\left(\neg_1\ X^{(3)}\right)}{f_1\ X^{(3)}}$	$\dfrac{t_2\left(\neg_1\ X^{(3)}\right)}{t_3\ X^{(3)}}$	$\dfrac{t_3\left(\neg_1\ X^{(3)}\right)}{t_2\ X^{(3)}}$
$\dfrac{f_1\left(\neg_1\ X^{(3)}\right)}{t_1\ X^{(3)}}$	$\dfrac{f_2\left(\neg_1\ X^{(3)}\right)}{f_3\ X^{(3)}}$	$\dfrac{f_3\left(\neg_1\ X^{(3)}\right)}{f_2\ X^{(3)}}$
$\dfrac{t_1\left(\neg_2\ X^{(3)}\right)}{t_3\ X^{(3)}}$	$\dfrac{t_2\left(\neg_2\ X^{(3)}\right)}{f_2\ X^{(3)}}$	$\dfrac{t_3\left(\neg_2\ X^{(3)}\right)}{t_1\ X^{(3)}}$
$\dfrac{f_1\left(\neg_2\ X^{(3)}\right)}{f_3\ X^{(3)}}$	$\dfrac{f_2\left(\neg_2\ X^{(3)}\right)}{t_2\ X^{(3)}}$	$\dfrac{f_3\left(\neg_2\ X^{(3)}\right)}{f_1\ X^{(3)}}$

Translation for conjunction and conjunction schemes

$$\frac{(O)\,A \wedge B}{\langle (P)_? \rangle (O)\,A \mid \langle (P)_? \rangle (O)\,B} \qquad \frac{F\ A \wedge B}{F\ A \mid F\ B}$$

$$\frac{(P)\,A \wedge B}{\begin{array}{l}\langle (P)_? \rangle (P)\,A \\ \langle (P)_? \rangle (P)\,B\end{array}} \qquad \frac{T\ A \wedge B}{\begin{array}{l}T\ B \\ T\ B\end{array}}$$

$\forall i \in (s(m))$:

$$\frac{(O_i)\,A \wedge^i B}{\langle (P_i)_? \rangle (O_i)\,A \mid \langle (P_i)_? \rangle (O_i)\,B} \qquad \frac{F_i\ A \wedge^i B}{F_i\ A \mid F_i\ B}$$

$$\frac{(P_i)\,A \wedge^i B}{\begin{array}{l}\langle (O_i)_? \rangle (P_i)\,A \\ \langle (O_i)_? \rangle (P_i)\,B\end{array}} \qquad \frac{T_i\ A \wedge^i B}{\begin{array}{l}T_i\ B \\ T_i\ B\end{array}}$$

We can *strip down* the additional information about the steps <(..)> and *rename* the signatures O and P into the signaturers T or t and F or f with indices i to get the necessary information for the logical tableaux rules. This can now easily be applied to the common Smullyan Tableaux rules as they are introduced for Poly-Logics.

Conjunction and disjunction in 3-polylogic

$$\frac{t_1\ X\ \wedge\wedge\wedge\ Y}{\begin{array}{c}t_1\ X \\ t_1\ Y\end{array}} \qquad \frac{f_1\ X\ \wedge\wedge\wedge\ Y}{f_1\ X \mid f_1\ Y} \qquad \frac{t_1\ X\ \vee\vee\vee\ Y}{t_1\ X \mid t_1\ Y} \qquad \frac{f_1\ X\ \vee\vee\vee\ Y}{\begin{array}{c}f_1\ X \\ f_1\ Y\end{array}}$$

$$\frac{t_2\ X\ \wedge\wedge\wedge\ Y}{\begin{array}{c}t_2\ X \\ t_2\ Y\end{array}} \qquad \frac{f_2\ X\ \wedge\wedge\wedge\ Y}{f_2\ X \mid f_2\ Y} \qquad \frac{t_2\ X\ \vee\vee\vee\ Y}{t_2\ X \mid t_2\ Y} \qquad \frac{f_2\ X\ \vee\vee\vee\ Y}{\begin{array}{c}f_2\ X \\ f_2\ Y\end{array}}$$

$$\frac{t_3\ X\ \wedge\wedge\wedge\ Y}{\begin{array}{c}t_3\ X \\ t_3\ Y\end{array}} \qquad \frac{f_3\ X\ \wedge\wedge\wedge\ Y}{f_3\ X \mid f_3\ Y} \qquad \frac{t_3\ X\ \vee\vee\vee\ Y}{t_3\ X \mid t_3\ Y} \qquad \frac{f_3\ X\ \vee\vee\vee\ Y}{\begin{array}{c}f_3\ X \\ f_3\ Y\end{array}}$$

The game can be played the other way round. Take tableaux rules for polylogics, i.e., polycontextural logics in a Smullyan tableaux setting, and create rules and tableaux rules for polylogue games, i.e., for polycontextural n-player dialogue games.

http://www.thinkartlab.com/pkl/lola/PolyLogics.pdf

Transjunction scheme for polylogues

$$\frac{(P_i)\,A \otimes^i B}{\begin{array}{l}\langle (O_i)_?\rangle (P_i)\,A\\ \langle (O_i)_?\rangle (P_i)\,B\end{array}} \qquad \frac{\forall i,j \in (s(m)):\ (O_i)\,A \otimes^i B}{\begin{array}{l||l|l}\langle (P_i)_?\rangle (O_i)\,A & \langle (P_{i+j})_?\rangle (O_{i+j})\,A & \langle (P_{i+j})_?\rangle (O_{i+j})\,A\\ \langle (P_i)_?\rangle (O_i)\,B & \langle (P_{i+j})_?\rangle (O_{i+j})\,B & \langle (P_{i+j})_?\rangle (O_{i+j})\,B\end{array}}$$

(transjunction, conjunction, disjunction) for 3-polylogue

$$\frac{(P_1)\,A \otimes\wedge\vee B}{\begin{array}{l}\langle (O_1)_?\rangle (P_1)\,A\\ \langle (O_1)_?\rangle (P_1)\,B\end{array}} \qquad \frac{(P_2)\,A \otimes\wedge\vee B}{\begin{array}{l||l}\langle (O_2)_?\rangle (P_2)\,A & \langle (P_1)_?\rangle (O_1)\,A\\ \langle (O_2)_?\rangle (P_2)\,B & \langle (P_1)_?\rangle (O_1)\,A\end{array}}$$

$$\frac{(O_1)\,A \otimes\wedge\vee B}{\begin{array}{l}\langle (P_1)_?\rangle (O_1)\,A\\ \langle (P_1)_?\rangle (O_1)\,B\end{array}} \qquad \frac{(O_2)\,A \otimes\wedge\vee B}{\begin{array}{l||l|l}\langle (P_2)_?\rangle (O_2)\,A \mid \langle (P_2)_?\rangle (O_2)\,B & \langle (P_1)_?\rangle (P_1)\,A & \langle (P_1)_?\rangle (O_1)\,A\\ & \langle (P_1)_?\rangle (O_1)\,B & \langle (P_1)_?\rangle (P_1)\,B\end{array}}$$

$$\frac{(P_3)\,A \otimes\wedge\vee B}{\begin{array}{l||l}\langle (O_3)_?\rangle (P_3)\,A \mid \langle (O_3)_?\rangle (P_3)\,B & \langle (P_1)_?\rangle (P_1)\,A\\ & \langle (P_1)_?\rangle (P_1)\,B\end{array}}$$

$$\frac{(O_3)\,A \otimes\wedge\vee B}{\begin{array}{l||l|l}\langle (P_3)_?\rangle (O_3)\,A & \langle (P_1)_?\rangle (P_1)\,A & \langle (P_1)_?\rangle (O_1)\,A\\ \langle (P_3)_?\rangle (O_3)\,B & \langle (P_1)_?\rangle (O_1)\,B & \langle (P_1)_?\rangle (P_1)\,B\end{array}}$$

Tableaux rules for (trans and or) in polylogic

$$\frac{t_1\ X\ \otimes\wedge\vee\ Y}{\begin{array}{l}t_1\ X\\ t_1\ Y\end{array}} \qquad \frac{f_1\ X\ \otimes\wedge\vee\ Y}{\begin{array}{l}f_1\ X\\ f_1\ Y\end{array}}$$

$$\frac{t_2\ X\ \otimes\wedge\vee\ Y}{\begin{array}{l|l}t_2\ X & f_1\ X\\ t_2\ Y & f_1\ Y\end{array}} \qquad \frac{f_2\ X\ \otimes\wedge\vee\ Y}{\begin{array}{l||l|l}f_2\ X \mid f_2\ Y & f_1\ X & t_1\ X\\ & t_1\ Y & f_1\ Y\end{array}}$$

$$\frac{t_3\ X\ \otimes\wedge\vee\ Y}{\begin{array}{l||l}t_3\ X \mid t_3\ Y & t_1\ X\\ & t_1\ Y\end{array}} \qquad \frac{f_3\ X\ \otimes\wedge\vee\ Y}{\begin{array}{l||l|l}t_3\ X & f_1\ X & t_1\ X\\ t_3\ Y & t_1\ Y & f_1\ Y\end{array}}$$

5 Meta-rule games

5.1 A template of classical and intuitionistic 2-player dialogue logic

Definition 1 ([Felscher, 1986]) A *dialogue* is a couple (δ, η) such that:

- δ is a sequence of signed expressions,
- η is a function on the ranks of the signed expressions in δ such that η(n) is a couple [m, Z] where m ≤ n and Z = A or D, which satisfies the following three conditions (D00), (D01), (D02).

D00 δ(n) is **P**-signed if n is even and **O**-signed if n is odd; δ(n) is a compound formula,

D01 If η(n) = [m, A], then δ(m) is a compound formula and δ(n) is an attack on δ(m) which corresponds to a regular argumentation form,

D02 If η(p) = [n, D], then η(n) = [m, A], and δ(p) is an answer to the attack δ(n) which corresponds to a regular argumentation form.

2.2 Global rules

Global rules are used according to the logic we aim at expressing. Let us envisage the following rules (some of which are not compatible with others):

D10 **P** may assert an atomic formula only if **O** asserted it previously,

D11 If at a position p – 1, several attacks are still not answered, it is only to the last not already defended attack that it can be answered at p,

D12 An attack can have at most one answer,

D12' **P** may repeat an attack or a defence if and only if **O** has introduced a new atomic formula (which can now be used by **P**),

D13 A **P**-signed formula can be attacked at most once,

D13' In any move, each player may attack a complex formula asserted by his partner or he may defend himself against any attack (including those which have already been defended)

Playing this kind of games obviously needs a *winning rule*:

D2 X wins if and only if it is Y's turn but Y cannot move

Definition 2: A formula is said to be valid in a dialogical system (defined by the previous definition + some of the global rules) if and only if **P** has a winning strategy for it, that is, **P** can win whatever moves are done by **O**.

Theorem 1: *Classical theses coincide with formulae valid for the system which includes D10, D12', D13', D2*

Theorem 2: *Intuitionistic theses coincide with formulae valid for the system which includes D10, D11, D12, D13, D2*

What's of interest for the *next game*, the mix of global rules, is the list D10 to D13' and D2. Each contextural dialogue has its own combination of global rules and obviously, D2 will be distributed over the architectonics of polylogues defined by a dissemination of the main definition and the rules D00 to D02.

5.2 Some configurations of global rules for n-person games

Perfect vs. imperfect information games

"Zermelo's theorem says that games of *perfect information*, between two players, in which every play is a win for exactly one player are in some sense trivial, but these are not actually plays we enter into in real life, simply because we play games with imperfect information or games in which every play is not necessarily a win for one of two players, but perhaps a win for both or… a win for none!
We shall therefore suggest further that games can be composed by linear logical operators, say in *protocols*, but that at some stage of the decomposition, we have particular (*imperfect information*) games which have no necessary winning strategy for one player or the other and which are not necessarily of complete information." (Lecomte)

Mixing global rules

Instead of differences on the base of the informational level, perfect or imperfect, about the game, differences in the set and use of meta-rules are proposed.

1. The data-base of the global rules is located in a mediating system. Each local game is mirroring, modeling, reflecting the global rules of the game into its reflectional locus. Depending on the complexity of the game a heterarchy of global rule systems has to be realized at different localities of the complexion of the game.

2. Two players can agree to accept the same global rules and to play the same game together. This is the common case of a stable game.

3. Players are playing together and are not yet aware that they are playing different games. Each player is accepting different global rules. But hasn't negotiated with the other player about the global rules in use. Say, opponent is playing a classic game, proponent is playing an intuitionistic game. This can easily happen because parts of the rules of each game are overlapping. Thus the difference is not explicit in the restricted moves of the game. Obviously, conflicts are emerging, they can lead to an end of the game or to an adjustment of the accepted rules.

4. Players are playing a game and are deciding together to change their global rules during the game. They continue the game under the changed rules and the previous complexity of the frame. Thus, they are not fully restarting a game but are keeping track with the history of the old game and are continuing it with new global rules but keeping the complexity, i.e., the number of players constant. The new meta-rules might be of higher or lower complexity than the previous set of meta-rules.

5. Players are playing a game and are deciding together to change their global rules and to enlarge the complexity of the game, i.e., to move from a n-person to a n+1-person game. They may playing concurrently different games in an interplay between different games.

6. Players can propose to augment their data-base of global rules by adding new rules. That is, a derived formula of a game can by added as a global rule to the set of global rules. Classic examples are to add TND or Double Negation to the rule system (similar in axiomatic theories).

7. Those general characteristics of the dynamics of games have to involve in a further step *reflectional* and *interactional* architectonics opening up new configurations. Hence, a *polylogue* or polylogical game is a mapping of dialogue games onto the polycontextural matrix.

8. These considerations are not restricted to dialogue logics. Every game with first-order and second-order rules can be involved into the dynamics of meta-games if it has a structural space offered like with polycontextural logics.

5.3 Mixed frame-rule games

What could be a reasonable mixed logic?

A polylogue with mixed frame-rules could be reasonable for a situation where one logic is concerned about semantic-ontological environments, like a data-base, and in another logic with the reflection, thematization about the rules of such a semantic system. That is, the data-system may be logically well modeled by a classical two- or multiple-valued logic while the reflections on it may force to use a constructivist approach excluding the TND and pre-given truth-values. Both will run in parallel, i.e., distributed and mediated together to a complex system where in a third logical system the interaction of the two systems is modelled.

To shaken orthodoxy and fundamentalism it is a good opportunity to mix not only different dialogue games together to polylogues but to combine and mediate logics of different methodologies. Thus, a mix of dialogue based logics with axiomatics and tableaux rules based logics, classic or intuitionistic or whatever, is an interesting challenge. And there is no reason why it wouldn't be a success.

Additionally to the internal rules of the logical systems, local rules and global rules, a new kind of rules has to be introduced. Such new rules are ruling the interactivity and reflectionality of mediated logical systems of whatever flavour. Thus, to the local and the structural rules of dialogues, frame-work rules of polylogues have to be considered.

Historically, the conflicts between platonist and constructivist approaches to logic had mostly not been aware that the different positions had been led by different questions and interests, that is, by different modi of thematization of cognitive activities. The unsolvable conflict aroused because both positions declared priority and hegemony over the other position. This was necessary because both lived in the belief that there is one and only one logic. And such a logic must be universal and natural. But none has won the trophy.

Intervention

An interaction of an agent, including reflections on the behavior of a partner agent, which is intended to change the meta-rules of the partner agent can be called an *intervention*. An agent is intervening into an interaction in attempting to change the meta-rules of the agent. In this sense, the intervening agent is taking the meta-position of a super-visor agent. He himself is involved with a different set of meta-rules but including a model of the meta-rules of the partner agent. Because polylogues offer space for different acting agents, interacting at once together, *gender* distinction can be introduced reasonably without being forced to following the empty talk of he / she-correctness.

An intervention takes place if an agent is interacting with another agent in a way that the agent is forced to change his meta-rules to stay in the game.

Interlocution

If two agents are intervening at once at each other an *interlocution* takes place.