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Double Cross Playing Diamonds

Understanding interactivity in/between

bigraphs and diamonds

in:

Paradoxes of Interactivity

Perspectives for Media Theory, Human-Computer Interaction, and Artistic Investigations

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Understanding interactivity in/between bigraphs and diamonds

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1. Models of Interactivity between flows and salti¹

"Interactivity is all there is to write about: It is the Paradox and the Horizon of Realization."

Grammatologically, the Western notational system is not offering space in itself to place sameness and otherness necessary to realize interaction/ality. Alphabetism is not prepared to challenge the dynamics of interaction directly. The Chinese writing system in its scriptural structuration is able to place complex differences into itself, necessary for the development and design of formal systems and programming languages of interaction. The challenge of interactionality to Western thinking, modeling and design interactivity has to be confronted with the decline of the scientific power of alphanumeric notational systems as media of living in a complex world. (cf. Kaehr 2006a)

The challenge I see for media artists is not only to develop interactional media constellations but also to *intervene* between the structures and dynamics of interactional systems as international corporations, governments, military and academia force them on us. (cf. Kaehr 2003a, b)

1.1 Comparison of two approaches to interactivity

This paper takes the risk to compare two fundamentally different approaches to interaction and reflection in computational systems: Milner's bigraphs and diamond theory. Milner's bigraph model and theory of interaction is highly developed, while the diamond model applied to this interactional scenario and confronted with the bigraphs model is presented here for the first time.

The Milner model is presupposing a world-view (ontology, epistemology) of homogeneity and openness. Its basic operation is composition in the sense of category theory. Composition is associative and open for infinite iterability. Milner's model is a model of interaction in a global sense but it is not thematizing formally the chiastic interplay of local and global aspects of interaction. Its merits is to have developed a strict separation of topography (locality) and connectivity for a unifying theory of global and mobile interaction (ubiquitous computing) surpassing, in principle, the limits of Turing computability.

In contrast, the diamond model, which is just emerging, (cf. Kaehr1996), is based on an antidromic and parallactic structure of combination of events in an open/closed world of a multitude of discontextural universes. In such a pluriversal world model, each composition is having its complementary combination. With that, iterability for diamonds is not an abstract iterativity but interwoven in the concrete situations to be thematized, and determined by iterative and accretive repetitions, involving their complementary counterparts, without a privileged conceptual initial/final object.

This leads to a theory of diamonds as a complementary interplay of categories and saltatories (jumpoids) with the main rules, globally, of complementarity and locally, of bridging. Diamonds are involving bi-objects belonging at once to categories and to saltatories, ruled by composition and saltisition (jump-operation). (Kaehr 2007a)

¹ Thanks to Marianne Dickson, Edinburgh, for bridging the corrections and correcting the bridges of this composition.

1.2 Interactionality as interplays between categories and saltatories

In less technical terms, the polycontextural approach of diamond theory is supporting three new features:

First, it supports the idea of irreducible multi-medial contextures and their qualitative incomparability. That is, different media like sound, video, picture, text, graphics, etc., are conceived as logically different and as organized and distributed conceptually in a heterarchical sense. To thematize media as a digital contexture is not more than to emphasize their informatical and physical aspect, which is as such a contexture, too.

Second, it supports the possibility of mapping the (outer) environment of a contexture (media) in itself, i.e., to offer an inner environment for reflectionality. Contextures, to be different from systems, have to reflect their environment into their own domain. Hence, a contexture has to be understood as being involved into interplays of inner and outer environments.

Third, it supports the possibility of realizing simultaneously movements (actions) and complementary counter-movements on a basic level of conceptualization and formalization. If composition of events inside a contexture and mediation of different contextures to a compound contexture, polycontexturality, are characterized by the rules of combination, i.e., identity, commutativity and associativity, a new feature of composition is discovered by the diamond approach, which is antidromic and parallax, corresponding structurally the otherness of the categorical system.

Therefore, the questions of interactionality in a diamond framework are not primarily, how do we globally move, physically and informatically, from one topographic place to another, but how do we move by interaction from one medium to another medium of a complex knowledge space. With the appearance of the semantic web and knowledge grid (cf. Kaehr 2004b) such developments are unavoidable. Obviously, the polycontextural diamond approach is not opting for a principally homogeneous global field of informatical and physical events but for a discontexturality of different media, situations, contexts of meaning.

The Milner Model is well based, principally, on category theory, the diamond model has to develop its own new formalism, risked here as a diamondization of category theory. Hence, both theories are in a constellation, which offers a reasonable possibility for comparisons.

Because the bigraph model is based on category theory and its concept of composition with its abstract iterability, the diamond model has to develop a distinct concept of composition (combination), one which involves a complementarity of, at least, two different concepts of composition; technically, the categorical and the saltatorical composition, and which is opening up the operativity of an open/closed concept of iter/alterability.

Even if only metaphorically and still vague, what is common to both models is there dichotomous, dual, complementary and orthogonal approach to interaction and interactionality. The Milner model is focused on message passing, flow of informatic objects, the diamond model on agents and their reflectional/interactional activities with an emphasis on intervention.

2 Milner's bi-graph model of interaction

Out of his cloud of keywords to ubiquitous computing and interactivity, Milner chooses at his Beijing 2005 performance 3 leading features: locality, mobility and connectivity. (Milner 2005: 49)

2.1 Locality and connectivity

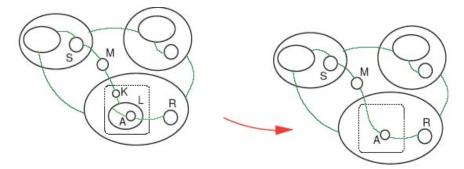
locality

"Programming the digital computer ramifies the use of space and spatial metaphor, both for writing programs and for explaining why they work. This shows up in our vocabulary: flow chart, location, send and fetch, pointer, nesting, tree, etc. Concurrent computing expands the vocabulary further: distributed system, remote procedure call, network, routing, etc.

We are living with a striking phenomenon: the *metaphorical* space of algorithms qgraph, array, and so on q is mixed with the space of *physical* reality." (Milner 2007:1)

physical and virtual space

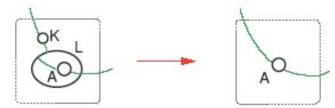
"Informatic objects flow in physical space; physical objects such as mobile telephones manipulate their informatic space."



"The picture illustrates how physical and virtual space are mixed. It represents how a message M might move one step closer to its destination. The three largest nodes may represent countries, or buildings, or software agents. In each case the sender S of the message is in one, and the receiver R in another. The message is en route; the link from M back to S indicates that the messages carries the sender's address. M handles a key K that unlocks a lock L, reaching an agent A that will forward the message to R; this unlocking is represented by a reaction rule that will reconfigure the pattern in the dashed box as shown, whenever and wherever this patterns arises." (ibid: 1)

"Bigraphical reactive systems are a model of information flow in which both *locality* and *connectivity* are prominent. In the graphical presentation these are seen directly; in the mathematical presentation they are the subject of a theory that uses a modest amount of algebra and category theory. A bigraph may reconfigure both its locality and its connectivity. The example pictured above shows how reconfiguration is defined by reaction rules; in that case, the rule may be pictured thus:

The mathematical structure of bigraphs allows concepts to be treated somewhat independently; for example, connectivity and locality are treated orthogonally." (ibid: 2)



"So the challenge to bigraphs is to provide a uniform behavioural theory, allowing many process calculi to be expressed in the same frame while preserving their treatment of behaviour." (ibid: 2)

The aim of a new design

"The challenge for global ubiquitous computing is to devise theories and design principles in close collaboration, . . ." (Milner 2005: 64)

"The long-term aim of this work is to provide a model of computation on a global scale, as represented by the Internet and the Worldwide Web. The aim is not just to build a mathematical model in which we can analyse systems that already exist. Beyond that, we seek a theory to guide the specification, design and programming of these systems, to guide future adaptations of them, and not to deteriorate when these adaptations are implemented. [...]

This will only be achieved if we can reverse the typical order of events, in which design and implementation come first, modelling later (or never). For example, a programming language is rarely based thoroughly upon a theoretical model. This has inevitably meant that our initial understanding of designed systems is brittle, and deteriorates seriously as they are adapted.

We believe that the only acceptable solution, in the long run, is for system designs to be expressed with the concepts and notations of a theory rich enough to admit all that the designers wish." (Milner 2004b: 7)

2.2 Strategies of orthogonal simultaneity

"So our strategy here is to tackle just two aspects – mobile connectivity and mobile locality – simultaneously. In fact this combination contains a novel challenge: to what extent in a model should connectivity and locality be interdependent? In plain words, does where you are affect whom you can talk to? To a user of the Internet there is total independence, and we want to model the Internet at a high level, in the way its connectivity appears to users. But to the engineer these remote communications are not atomic, but represented by chains of interactions between neighbours, and we should also provide a low-level model, which rejects this reality. So we want to have it both ways; furthermore, we want to be able to describe rigorously how the high-level model is realised by the lowlevel one." (Milner 2004b: 7)

Milner's Model of bigraphs (Milner 2006: 21)

$\begin{array}{c} \begin{array}{c} \textbf{bigraph} \\ G: \langle m, X \rangle \rightarrow \langle n, Y \rangle \\ \end{array} \\ \begin{array}{c} \textbf{place graph} \\ G^{p}: m \rightarrow n \\ roots \dots & 0 \\ \end{array} \\ \begin{array}{c} \textbf{place graph} \\ g^{p}: m \rightarrow n \\ roots \dots & 0 \\ \end{array} \\ \end{array} \\ \begin{array}{c} \textbf{place graph} \\ g^{p}: m \rightarrow n \\ \end{array} \\ \begin{array}{c} \textbf{sites} \dots & 0 \\ \end{array} \\ \begin{array}{c} \textbf{place graph} \\ \textbf{graph} \\ \end{array} \\ \begin{array}{c} \textbf{graph} \\ \textbf{graph}$

RESOLVING A BIGRAPH INTO PARTS

2.2.1 Statics of interaction: Categorical framework

"Abstract. This paper axiomatises the structure of *bigraphs*, and proves that the resulting theory is *complete*. Bigraphs are graphs with double structure, representing *locality* and *connectivity*. They have been shown to represent dynamic theories for the pi-calculus, mobile ambients and Petri nets, in a way that is faithful to each of those models of discrete behaviour. While the main purpose of bigraphs is to understand mobile systems, a prerequisite for this understanding is a well-behaved theory of the structure of states in such systems. The algebra of bigraph structure is surprisingly simple, as the paper demonstrates; this is because bigraphs treat *locality* and *connectivity* orthogonally." (Milner 2004a: 1)

2.2.2 Dynamics of interaction: Labeled process calculi

"Let us repeat: in a pure bigraph G : <m, X > -> <n, Y > we admit no association between its outer names Y and the roots (regions) n, nor between the inner names X and the sites m. It is this dissociation that enables us to treat locality and connectivity independently, yielding a tractable theory." (Milner 2004b: 20)

The dynamics of bigraphs is are formalized by labeled process calculi:

"The challenge from process calculi is to provide a uniform behavioural theory, so that many process calculi can be expressed in the same frame without seriously affecting their treatment of behaviour. We now outline how research leading up to the bigraphical model has addressed this challenge.

It is common to present the dynamics of processes by means of reactions (also known as rewriting rules) of the form $r \rightarrow r'$, meaning that r can change its state to r' in suitable contexts. In process calculi this treatment is typically refined into labelled transitions of the form $a \rightarrow^l a'$, where the label *I* is drawn from some vocabulary expressing the possible interactions between an agent *a* and its environment. These transitions have the great advantage that they support the definition of behavioural preorders and equivalences, such as traces, failures and bisimilarity. But the definition of those transitions tends to be tailored for each calculus." (Milner 2005: 8)

2.2.3 Formalization of interaction: Bigraphs as tensor categories

"This chapter establishes place graphs, link graphs and bigraphs as arrows in certain kinds of category. Any kind of category is concerned with operations upon arrows, especially composition." (Milner 2007: 13)

"Note that this combination is quite distinct from the categorical composition used to insert one bigraph into another (e.g. an agent into a context). But it is simply related to them; to compose two bigraphs categorically, we first resolve them into their respective place graphs and link graphs, then compose these, and finally combine the results into a new bigraph." (Milner 2004b: 19)

2.2.4 Axiomatics of bigraphs

"The topic of this paper is to axiomatise the resulting structure of bigraphs. The justication for such a specific topic is threefold.

First, the work already cited gives ample evidence that a graphical structure combining topography with connectivity has wide application in computer science; for as we have seen it brings unity to at least three models of discrete dynamics, each of which has already many applications.

Second, it appears that the algebraic treatment of such dual structures has not been previously addressed; yet the behaviour of systems whose connectivity and topography are both reconfigurable may be so complex that their dynamics cannot be properly understood without a complete and rigorous treatment of their statics. Bigraphs are just one possible treatment of such dual structure, but it is likely that their static theory can be modified for other treatments.

Third, as we shall see, dual structures seem to require a novel kind of normal form which is essential to a proof of axiomatic completeness." (Milner 2004a: 4)

Axiomatics (Table 1)

CATEGORICAL AXIOMS:

$$\begin{array}{rcl}
A & \mathrm{id} &=& A &=& \mathrm{id} \, A \\
A(BC) &=& (AB)C \\
A \otimes \mathrm{id}_{\epsilon} &=& A &=& \mathrm{id}_{\epsilon} \otimes A \\
A \otimes (B \otimes C) &=& (A \otimes B) \otimes C \\
(A_1 \otimes B_1)(A_0 \otimes B_0) &=& (A_1A_0) \otimes (B_1B_0) \\
\gamma_{I,\epsilon} &=& \mathrm{id}_I \\
\gamma_{J,I}\gamma_{I,J} &=& \mathrm{id}_{I \otimes J} \\
\gamma_{I,K}(A \otimes B) &=& (B \otimes A)\gamma_{H,J} \\
\end{array}$$
(A: $H \to I, B: J \to K$)

LINK AXIOMS:

 $\begin{array}{rcl} & /y \circ {}^y/x & = & /x \\ & /y \circ y & = & \operatorname{id}_{\epsilon} \\ z/(Y \uplus y) \circ \left(\operatorname{id}_Y \otimes {}^y/X\right) & = & z/(Y \uplus X) \end{array}$

PLACE AXIOMS:

 $\begin{array}{rcl} merge(1 \otimes id_1) &=& id_1 & (unit) \\ merge(merge \otimes id_1) &=& merge(id_1 \otimes merge) & (associative) \\ merge\gamma_{1,1} &=& merge & (commutative) \end{array}$

NODE AXIOMS:

 $(\mathsf{id}_1 \otimes \alpha)\mathsf{K}_{\vec{x}} = \mathsf{K}_{\alpha(\vec{x})}.$

"In other words, the axioms are both sound and complete. They say simple things: The place axioms say that join is commutative, has a unit and is associative; the link axioms say that the formation of links obeys obvious rules; the node axiom says that we can name ports arbitrarily." (ibid: 23)

2.2.5 Completeness of the axiom system

"The completeness of the axiom system in Table 1 depends primarily on two things:

first, that all linking can be exposed at the outermost level of an expression;

second, that we have a strict symmetric monoidal category of bigraphs, with a tensor that is partial on objects.

Crucial to the tensor is that it is bifunctorial, i.e.

 $(A_1 \times B_1)(A_0 \times B_0) = (A_1A_0) \times (B_1B_0)$; this axiom underlies most of our manipulations.

Thus the discrete normal form, DNF, has been crucial for the proof of completeness." (ibid: 21)

2.3 Orthogonality of topography and connectivity

2.3.1 Underlying world model

The bi-graph model of interaction is highly flexible and is liberating from unnecessary fixations. Bigraphical reactive (re-writing) systems as models of information flow are dealing *locality* and *connectivity* as *orthogonal* events, distributed over two dimensions. Such a separation of structural locality and behavioral connectivity enables a clear modeling and an effective formalization as a bigraph or bipartide system. Spaciality is conceived as static, formalized by category theory and behavior as dynamic, formalized by process calculi (pi-calculus).

The bigraph model of interaction seems to belong to a world model with the characteristics of: "Everything in this world is changing but the world in which everything is changing doesn't change." Ubiquitous and global computing is presupposing an epistemologically uniform, homogeneous world of physical and informatical events. (cf. Kaehr 2007d)

Diamond theory can be set in some kind of a correspondence with a bipartide model but it is turning to a world model where there are many worlds in which things are changing and in which worlds themselves are changing too, being involved, not in a new super-stable world, but in the game of interactionality/reflectionality between worlds and events, hence, enabling system designers and media artists to *intervene* in and between those worlds guided by the metamorphic dynamics of polycontextural diamonds.

Hence, messages, in the diamond model, are conceived as polycontextural and are belonging simultaneously to different contextures of irreducible kinds of meaning. Message passing in such a model is not done by the metaphor of key/lock/unlock/agent in a location/connectivity setting because a key in this pluriversal world-model appears always as necessarily polysemic and its acceptance has to be negotiated by reflectional and interactional activities. If such complex transactions are becoming stable in their usage, a reduction to the mono-contextural key-model can be introduced by reducing complexity.

2.3.2 Chiastic transition metaphor

Hence, in a chiastic metaphor, we can state, that statics in the bigraph model becomes dynamics in the diamond model; and dynamics becomes statics in the diamond setting because its dynamics is bracket and moved into a multitude of process-structures wherein the dynamics of the different behavioral systems are getting an arena to act. Therefore, category theory as formalism for interaction has to be dynamized towards diamond theory. That is, category theory has to be diamondized towards a dynamic structural formalism, which is an operational structuration.

2.3.3 Opting for an interventional design

The British Grand Challenge project for computing is not touching the principle hierarchy between mathematics and informatics. Time since the Greeks has changed and a reversion and displacement of this hierarchy might be the grand challenge of a new understanding of global computing. (cf. Kaehr 2003a)

From a model of interactions to a design of interactionality, the transitions to be risked might be:

From the global, ubiquitous and universal web of computation, to the kenomic grid of pluriversal contexturality, containing the chiasm of global/local scenarios.

From the locality in the Actor model of informatical events to the positionality of contextures in the kenomic grid, positioning informatic localities.

From the mobility in the Actor model of informatical flows between ambients (context, locality) of the same contextural (ontological, logical, semiotic) structure to a metamorphosis between contextures, augmenting complexity/complication of contextural scenarios implementing clusters of informatical ambients and mobility.

From the operations between actional ambients to the operationality in polycontextural situations realized by the *super-operators* (identity, replication, permutation, reduction, bifurcation) placing ambient operations into the grid.

From the connectivity of actions at a locality of message passing, using a key to unlock a lock of an agent, to different kinds of *mediation* between contextures containing informatical connectivity.

These transitions seem to record a catalogue of *minimal* conditions to be fulfilled to realize interactionality/reflectionality and interventionality in such complex constellations as the emerging knowledge grid. (cf. Kaehr 2006b)

3 Diamond theory of interactionality

3.1 Diamond Strategy

Encounter

Diamond strategies are sketching transitions from the mail model of interaction in bigraphs to the *encounter* model of interactionality/reflectionality and intervention.

Before we can play the bipartide game of locking and unlocking by passing a key in a structure of orthogonal locality and connectivity to reach an agent, capable of passing the message to another agent, the otherness of the actors involved has to be acknowledged and accepted by the interactional activities of the actors involved.

It can be described as the action of addressing an addressee, which is able to accept the addressing by offering its own addressable structure. After having been addressed and the addressing is accepted by the addressed and the addresser has recognized the acceptance of being addressed and the addressing is thus established, information can be exchanged between agents in the sense of communication. (cf. Kaehr 2004)

Interactivity in the encounter-model, therefore, is conceived as a *mutual* action of acceptance and rejection between different agents. Only on the base of this interactional agreement information exchange can happen. (Kaehr 2004a)

Therefore, the structure of interaction is always complex: at once realizing the addresser and the inner environment of the addressee. This simultaneity of inner and outer environments of agents is involving a kind of structural bifurcation and mutual actions of acceptance and/or rejection of the involved agents based on the complexity of their architectonics. That is, the addressee has to give space (einräumen) to the addresser to be addressed. To address and to accept to be addressed is a *mutual* action of at least two agents in a common co-created environment. Hence, the actional structure of interactionality is not only bipartide but antidromic, too. This phenomenon is forcing a formalization paradigm beyond mathematical category theory, which finds a very first attempt to a realization in the proposed diamond theory. (cf. Kaehr 2007c)

Intervention

An interaction of an agent, including reflections on the behavior of a partner agent, which is intended to change the meta-rules of the partner-agent can be called an *intervention*. An agent is intervening into an interaction in attempting to change the meta-rules of the agent. An intervention takes place if an agent is interacting with another agent in a way that the agent is forced to change his metarules to stay in the game of computation and interaction. (cf. Kaehr 2005, 2006c)

"The aim is not just to build a mathematical model in which we can analyse systems that already exist. Beyond that, we seek a theory to guide the specification, design and programming of these systems, to guide future adaptations of them, and not to deteriorate when these adaptations are implemented. There is much talk of the vanishing ubiquitous computer of the future, which will obtrude less and less visibly in our lives, but will pervade them more and more. Technology will enable us to create this. To speak crudely, we must make sure that we understand it before it *vanishes*." (Milner 2004b: 7)

Diamond strategies are not only asking for an understanding of such trends, like the vanishing of computational challenges for users by ubiquitous computing, but for the possibility of intervention by computer designers, scientists and users into such trends. Thus, opening up interplays between users and general computation, avoiding any kind of regression into euphorism, criticism and luddism of humanistic self-defence.

3.2 Towards Diamond Theory

3.2.1 From categorical composition of morphisms to diamonds

Actions from A to B can be considered as morphisms, symbolized by an arrow from A to B, A -> B. In this sense, morphisms are universal, they occur everywhere. But morphisms don't occur in isolation, they are composed together to interesting complexions. The composition of morphisms (arrows) is defined by the coincidence of codomain (cod) and domain (dom) of the morphisms to be composed, called the matching conditions (MC). That is, (f, g) is composed (f o g) iff cod(f) = dom(g). This highly general notion of morphism and composition of morphisms is studied in Category Theory. (cf. Kaehr 2007a)

A general descriptive explication of the concept of composition of morphisms is given by the following diagram. It contains the table of the matching conditions. Here, the distinction between objects, A, B as domain and codomain properties of morphisms, and the alpha (a) and omega (w) functionality of morphisms are included.

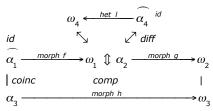
Matching Conditions

$$\begin{array}{c} \textbf{Descriptive Composition} \\ (A^{1}, \alpha_{1}) \xrightarrow{f} (B^{1}, \omega_{1}) \ o \ (A^{2}, \alpha_{2}) \xrightarrow{g} (B^{2}, \omega_{2}) \\ & \swarrow \\ (A^{3}, \alpha_{3}) \xrightarrow{fq} (B^{3}, \omega_{3}) \end{array} \qquad \begin{bmatrix} \omega_{1} \updownarrow \alpha_{2} \\ A^{2} \triangleq B^{1} \\ (A^{1}, \alpha_{1}) = (A^{3}, \alpha_{3}) \\ (B^{2}, \omega_{2}) = (B^{3}, \omega_{3}) \end{bmatrix}$$

Hence, not only the codomain B^1 and the domain A^2 as objects have to coincide, but also the actional domain "alpha₂" (a₂) and the actional codomain "omega₁" (w₁) as functional properties of the morphisms f and g, have to match. Obviously, the commutativity of the diagram has to fulfill, additionally, the matching conditions for (A¹, a₁) with (A³, a₃) and (B², w₂) with (B³, w₃), defining the composition (f o g).

First, without the actional alpha/omega-notation we get the matching conditions, coincidences, for categorical composition based on the objectional distinction of domains and codomains.

Second, stripped off of the set-theoretical or objectional content of the domains and codomains of morphism, the functionality of beginnings (a) and endings (w) remain. Composition then means an exchange relation between the ending of a morphism and the beginning of another morphism, i.e., between (w₁) and (a₂). Both founded in the coincidence relation between the actional domain of the first and the actional codomain of the second morphism, establishing the commutativity of "object-free" categorical composition, i.e., the morphism between (a_3) and (w_3) , i.e., $(a_3) - >(w_3)$.



Such a chiastic approach, emphasizing the pure functionality of composition, is discovering the possibility of a new relationship involved in the definition of actional composition: the complementarity of the commutative morphism between the beginning (a_2) and the ending (w_1) involved in the categorical composition, building the "antidromic and parallax" hetero-morphism between (a_4) and (w_4) , i.e., $(w_4) < q(a_4)$.

Hence, functional composition of morphisms, which are represented by order relations, is based on the functional matching conditions, MC, of two types of relations: exchange and coincidence relation building together with the order relations, a chiastic pattern in form of a diamond. Obviously, this singular diamond is occupying a place and is localized in a grid of diamonds and thus ready to be disseminated.

Third, both thematizations together, the objectional and the actional, with morphisms and hetero-morphisms, are defining the *diamond* composition of morphisms.

3.2.2 Diamond model of system/environment

Some wordings to the diamond system/environment relationship might be listed:

What's my environment is your system.

What's your environment is my system.

What's both at once, my-system and your-system, is our-system.

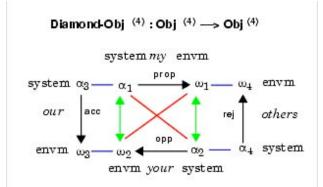
What's both at once, my-environment and your-environment, is ourenvironment.

What are our environments and our systems is the environment of our-system.

What's our-system is the environment of others-system.

What's neither my-system nor your-system is others-system.

What's neither my-environment nor your-environment is others-environment.



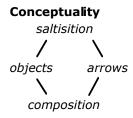
The diamond modeling of the otherness of the others is incorporating the otherness into its own system. An external modeling of the others would have to put them into a different additional contexture. With that, the otherness would be secondary to the system/environment complexion under consideration. The diamond modeling is accepting the otherness of others as a "first-class object", and as belonging genuinely to the complexion as such.

In another setting, without the "antropomorphic" metaphors, we are distinguishing between a system, its internal and its external environment. The external environment corresponds the rejectional part, the internal to the acceptional part of the diamond. Applied to the diamond scheme of diamondized morphisms we are getting directly the diamond system scheme out of the diamond-object model.

Much work had been done about interactionality/reflectionality and interventionality/interlocutionality on the base of *polycontextural* notions and formalisms (cf. Kaehr 2005&2006d). Despite its chiastic and proemial approach, this work did not yet include the others-system of the diamond model.

3.3 Diamond Structuration

Diamonds in this sketch are conceived as interplays between categories and saltatories based on morphisms and hetero-morphisms with their compositions, saltisitions and bridgings. Saltatories are the complementary concept of categories.



The conceptuality of diamond theory is introduced by an application of the diamond strategies to the basic concepts of category theory: objects and morphisms (arrows). Objects are understood in this setting as propositions, arrows as oppositions. Compositions appear as the both-at-once of objects and arrows, and saltisitions as the neither-nor of objects and arrows. Composition and saltisitions, hence, are complementary concepts.

saltisition, saltatory

salto mortale: jump from the apriori to the empirical (Imannuel Kant).

diamond strategies: double salto mortale from the theoretical to the hypertheoretical.

Diamond Theory	Diamond duality			
Category : $\mathbf{A} = (Obj^{A}, hom, id, o)$ Saltatory : $\mathbf{a} = (Obj^{a}, het, id,)$ DTh = $([\mathbf{A}; \mathbf{a}], compl, diff, \bullet)$	Category		Saltatory	
	Cat	Cat ^{op}	Salt	Salt ^{op}

Categories are dealing with composition of morphisms and their laws. Saltatories are dealing with the jump-operation (saltisitions) of hetero-morphisms and their laws. Diamonds are dealing with the *interplay* of categories and saltatories. Their operation is interaction realized by the *bridging* operations.

The laws of *identity* and associativity are ruling compositions, as well as saltisitions. Complementarity between categories and saltatories, i.e., between acceptional and rejectional domains of diamonds, are ruled by *difference* operations. Duality operations are applicable to both, categories and saltatories.

Commutativity and associativity

Commutativity Condition

$$\begin{array}{ll} If f, g \in MC, \ l \in MC : \\ then \\ g \Diamond f = (g \circ f) \parallel l \\ \\ with \begin{pmatrix} \omega(f) \updownarrow \alpha(g) \\ diff(\alpha(g)) = \alpha(l) \\ diff(\omega(f)) = \omega(l) \end{pmatrix} \\ with \begin{pmatrix} \omega(f) \updownarrow \alpha(g) \\ diff(\alpha(g)) = \alpha(l) \\ diff(\omega(f)) = \omega(l) \end{pmatrix} \\ \\ such that \\ A \xrightarrow{f} \rightarrow B \\ gf \searrow \downarrow g \\ C \end{pmatrix} \left| b_1 \xleftarrow{l} b_2 \\ b_1 \leftarrow b_2 \\ b_2 \leftarrow b_2 \\ b_1 \leftarrow b_2 \\ b_2 \leftarrow$$

3.3.1 Identity and difference

"This shift becomes even more apparent if one examines the foundational concepts Nishida develops later in his career, the "self-identity of the absolute contradiction" and the "many in one, one in many" (tasokuitsu, issokuta); the former can be paraphrased as the "identity of absolute difference" and the latter as "plurality in oneness, oneness in plurality." (Kopf 2004: 80)

Identity and difference morphisms

-

$$\begin{array}{c} \mathbf{bi - Object} \begin{bmatrix} \mathbf{X}, \mathbf{x} \end{bmatrix} \\ \widehat{id} \\ \mathbf{x} \in Salt \\ \uparrow diff \\ \mathbf{X} \in Cat \\ \widehat{id} \end{array} \right| \in Diam$$

Identity is a mapping onto-itself as itself.

For each object X of a category an identity morphism, $ID_{[X_r X]_r}$ which has domain X in the category and codomain X in the same category exists. Called ID_X or id_X for $ID_{[X_r X]_r}$.

For each object x of a saltatory an identity morphism, $ID_{[x, x]}$, which has domain x in the saltatory and codomain x in the same saltatory exists. Called ID_x or id_x for $ID_{[x, x]}$.

Difference is a mapping onto-itself as other.

For each object X of a category, a difference-morphism,

 $\mathsf{DIFF}_{[X,\,\,x]}$, which has domain X in the category and codomain x in the saltatory exists.

For each object x of a saltatory a difference morphism, $DIFF_{[x, X]}$, which has domain x in the saltatory and codomain X in the category exists.

This wording is a strict paraphrase of the common wordings of category theory. It also emulates its architectonics: from objects to morphisms to isomorphisms and to natural transformation, etc. Nevertheless it is not yet reflecting the reversed architectonics of the diamond way of thinking, where objects occur last and not first.

Identity and difference composition

ID and DIFF composition

Identity $\forall f, X, Y, o \in Cat: \\ f \circ_{XXY} ID_X = f = ID_Y \circ_{XYY} f. \\ \forall I, x, y, \| \in Salt: \\ I \parallel_{XXY} ID_X = I = ID_Y \parallel_{XYY} I. \\ = [f, I] = \\ DIFF_{[y, Y]} (\|, o)_{[XYY, XYY]} [I, f].$

3.3.2 Diamond concepts between iso- and xenomorphism

"One philosophical reason for categorification is that it refines our concept of 'sameness' by allowing us to distinguish between isomorphism and equality." (Baez/Dolan1998: 7)

Isomorphisms

Isomorphism in Cat: Cat_{Iso}

$$\begin{array}{l} \forall f, g \in Cat: \\ X \xleftarrow{f}{g} Y \quad iff \begin{cases} g \ o \ f = id_{\chi} \\ f \ o \ g = id_{\chi}. \end{cases}$$

Isomorphism in Salt: Salt

$$\begin{array}{l} \forall l, \ m \in Salt : \\ x \xleftarrow[m]{}{}{} x \xrightarrow[m]{}{} y \ iff \ \begin{bmatrix} m \parallel \ l = id_x \\ l \parallel m = id_y \end{bmatrix} \end{array}$$

Diamond Isomorphism: Diam_{Iso}

Om Cat , *Salt* \in *Diam* :

right - domain - ISO :

$$\begin{pmatrix} x \\ \uparrow diff \\ X \xleftarrow{f}{g} Y \end{pmatrix} iff \begin{bmatrix} (g \circ f) \bullet id_x = id_{[X, x]} \\ id_x \bullet (f \circ g) = id_{[Y, x]} \end{bmatrix}$$

left - codomain - ISO :

$$\begin{pmatrix} Y \\ \uparrow diff \\ g & Y \end{pmatrix} iff \begin{bmatrix} (g \circ f) \bullet id_y = id_{[x, y]} \\ id_y \bullet (f \circ g) = id_{[Y, y]} \end{bmatrix}$$

Hetero-ISO

right - domain - ISO :

$$\begin{pmatrix} x \xleftarrow[m]{m} y \\ \updownarrow diff \\ \widehat{X} \end{pmatrix} iff \begin{bmatrix} (I \parallel m) \bullet id_x = id_{[x, x]} \\ id_x \bullet (m \parallel I) = id_{[x, y]} \end{bmatrix}$$

left - codomain - ISO :

 $\begin{pmatrix} X \xleftarrow{l} & Y \\ & \uparrow & M \\ & \uparrow & M \\ & & \uparrow & Y \\ & & \uparrow & Y \end{pmatrix} iff \begin{bmatrix} (m \parallel l) \bullet id_{Y} = id_{[Y, Y]} \\ id_{Y} \bullet (l \parallel m) = id_{[Y, Y]} \end{bmatrix}$

Category theory is studying, at first, isomorphisms between objects as domains and codomains of morphisms, then the trip goes on with functors, natural transformations and so on. Their basic element, thus, is an elementary, single morphism and their basic operation is a single identity morphism. Diamond theory is dealing with the interplay between categories and saltatories, hence, the elementary situation is not a single morphism but the interaction of the selected morphism and its two corresponding, i.e., interacting hetero-morphism based on identity and difference operations. That is, the domain and the codomain of the selected morphism has to consider the corresponding domain and codomain of the hetero-morphisms involved. This is ruled by the difference operation. Hence, the isolated objects as domains and codomains have to be supplemented by their own counter-parts, codomain and domain, to build their heteromorphisms. In other words, the full interplay of morphisms, identity and difference mappings, have to be involved to realize proper diamond iso- and xenomorphisms.

Full combined isomorphisms between morphisms and hetero-morphisms are naturally constructed out of the partial iso- and xenomorphisms. (cf. Kaehr 2007a)

3.3.3 Diamond concept of transversality

A difference-philosophical interpretation of transversal isomorphisms could be found in the classical formulations of "The identity of oppositions, i.e., the identity of difference and identity." and "The difference of identity and difference". Both formulations are in some sense dual.

Further, more complex isomorphisms are easily composed by a combination of right- and left-isomorphisms.

Transversality ISO

$$trsv_{A}: diff(A) \longrightarrow (B)$$

$$trsv_{B}: A \longrightarrow diff(B).$$

right - transversal - ISO

\

$$\begin{array}{c} X \\ \uparrow diff \searrow trsv \\ X \xleftarrow{f}{g} Y \end{array} \right) iff \begin{bmatrix} trsv_{[x, y]} \bullet diff_{[x, x]} = id_{[y]} \\ trsv_{[x, y]} \bullet (f \ o \ g) = diff_{[y, x]} \end{bmatrix}$$

left - transversal - ISO :

$$\begin{pmatrix} Y \\ \checkmark trsv \ddagger diff \\ X \xleftarrow{f}{g} Y \end{pmatrix} iff \begin{bmatrix} (g \ o \ f) \bullet trsv_{[\gamma, X]} = diff_{[\gamma, X]} \\ trsv_{[X, Y]} \bullet diff_{[\gamma, Y]} = id_{[X]} \end{bmatrix}$$

3.3.4 Facets of diamond isomorphisms

The concept of diamond isomorphisms is not solely dynamizing the realm of sameness, as it is the aim of category theory, but it is also inert-wined with the differentness and strangeness of otherness.

Facets of diamond isomorphisms

1. Sameness (up to isomorphism)

$$ID_{T_{1}} \circ T_{1} \circ \underbrace{\xrightarrow{f}}_{g} \circ T_{2} \circ ID_{T_{2}}$$
2. Differentness (up to xenomorphism)

$$T_{1}^{2} \circ \underbrace{\xrightarrow{g}}_{T_{2}} \circ T_{2}^{2}$$

$$diff_{T_{1}} \Leftrightarrow \qquad \Leftrightarrow diff_{T_{2}}$$

$$T_{1}^{1} \circ \underbrace{\xrightarrow{f}}_{T_{2}} \circ T_{2}^{1}$$
3. Strangeness (up to heteromorphism)

 $\begin{pmatrix} X \xleftarrow{i} \\ m \end{pmatrix} Y \\ \uparrow diff \\ X \xleftarrow{f} Y \end{pmatrix} iff \begin{bmatrix} (g \circ f) \bullet (m \parallel I) = id_{[X, \times]} \\ (f \circ g) \bullet (I \parallel m) = id_{[Y, Y]} \end{bmatrix}$

3.4 Interactionality as interplays in diamonds

-- f

Interactionality of diamonds is studying the interaction between disseminated categories and saltatories of polycontextural diamond systems. Taken contextures in isolation, topics like *duality* and *complementarity* in diamonds are interactional, but they are not yet considering the inert-wining and intervening properties of interactivity as it happens with *bridging*. Thus, interactionality as an intra-contextural interplay occurs in elementary diamonds in forms of *duality*, *complementarity*, *bridging* and *distributivity*.

Duality for Categories: "two for the price of one"

"The Duality Principle for Categories states

Whenever a property P holds for all categories,

then the property P^{op} holds for all categories.

The proof of this (extremely useful) principle follows immediately from the facts that for all categories A and properties P

(1) $(A^{op})^{op} = A$, and

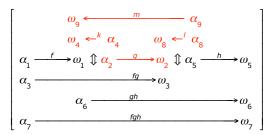
(2) P^{op}(A) holds if and only if P(A^{op}) holds." (Herrlich 2004: 27)

Duality is defined for diamonds as duality of categories and duality of saltatories.

Complementarity of formal languages

"The general principle underlying these limitations was called the *linguistic comple*mentarity by Loefgren. It states that in no language (i.e. a system for generating expressions with a specific meaning) can the process of interpretation of the expressions be completely described within the language itself. In other words, the procedure for determining the meaning of expressions must involve entities from outside the language, i.e. from what we have called the context. The reason is simply that the terms of a language are finite and changeless, whereas their possible interpretations are infinite and changing." (Heylighen: § 6.3) The double meaning of diamond objects, bi-objects, is complementary and in their orientations they are not parallel but *antidromic* (gegenläufig) and *deferred* regarding the complementary system.

Bridging categories and saltatories



Bridging is not an operation of mediation or switching of and between diamonds or acceptional and rejectional actions in diamonds but an operation to knot the two realms together, the categorical and the saltatorical. In the diagram, between the hetero-morphism k, l, the morphism g is offering a bridge, marked in red, and thus interacting between the saltatorical and the categorical domain of the diamond. Complementary, the two bridge pillars of the bridge are offered by the two hetero-morphisms l, k defining the bridgework g. Hence, bridge and bridging are complementary actions, too. Hence, reflecting the complementarity between categories and saltatories.

Distributivity of composition, saltisition and bridging

Because diamonds are based on interplays between categories and saltatories, which are involved with two fundamental operations: composition (o) and saltisition (11) with bridging (\dot{z}) too, it is reasonable to find interactive laws as distributivity between those basic operators inside the very definition of the conception of diamonds.

3.4.1 Duality in diamonds as duality in categories and saltatories

duality in categories	duality in saltatories
$\overline{(g \circ f)} = A \rightarrow C$ $dual(g \circ f) = dual(dual(B \rightarrow C) \circ dual(A \rightarrow B))$ $= dual((B \leftarrow C) \circ (A \leftarrow B))$ $= ((A \leftarrow B) \circ (B \leftarrow C))$ $= (A \leftarrow B \leftarrow C)$ $= A \leftarrow C,$	$ \begin{aligned} u &= \left(\omega_{4} \leftarrow \alpha_{4} \right) = compl\left(g \ o \ f \right) \\ dual\left(compl\left(g \ o \ f \right) \right) = dual\left(u \right) \\ dual\left(u \right) &= dual\left(\omega_{4} \leftarrow \alpha_{4} \right) \\ &= \left(\alpha_{4} \rightarrow \omega_{4} \right). \\ compl\left(dual\left(g \ o \ f \right) \right) = compl\left(f \ o \ g \right) = \left(\alpha_{4} \rightarrow \omega_{4} \right). \end{aligned} $
Hence, $((g \circ f) = A \rightarrow C) \in Cat$ iff $(dual(g \circ f) = A \leftarrow C) \in Cat^{op}$.	$ \begin{array}{l} \textit{Hence, } \left(u = \left(\omega_{_{4}} \leftarrow \alpha_{_{4}} \right) \right) \in \textit{Salt} \\ \textit{iff} \\ \left(\textit{dual} \left(u \right) = \alpha_{_{4}} \rightarrow \omega_{_{4}} \right) \in \textit{Salt}^{op} . \end{array} $
$X = g \Diamond f = [(g \circ f); u]:$ $X \in Cat \text{ iff } dual(X) \in Cat^{op}$	$ X \in Salt iff dual(X) \in Salt^{op}$

Duality in Diamonds

3.4.2 Complementarity of categories and saltatories

Complementarity of Acc and Rej

$$\begin{split} X \in Acc \ iff \ compl\left(X\right) \in Rej \\ X = g \ o \ f : \\ \mathbf{1}. \ X \in Acc \ if \ compl\left(X\right) \in Rej \\ compl\left(g \ o \ f\right) = compl\left(compl\left(g\right) \ o \ compl\left(f\right)\right) \\ &= compl\left(diff\left(cod\left(f\right)\right) \ o \ diff\left(dom\left(g\right)\right)\right) \\ &= compl\left(\left(\overline{B_{cod}}\right) \ o \ \left(\overline{B_{dom}}\right)\right) = \omega_4 \leftarrow \alpha_4. \\ (u : \omega_4 \leftarrow \alpha_4) \in Rej \\ Hence, \ (g \ o \ f) \in Acc \ if \ (u : \omega_4 \leftarrow \alpha_4) \in Rej \\ \left(g \ o \ f\right) \in Acc \ if \ \left(\overline{g \ o \ f}\right) \in Rej. \\ \mathbf{2}. \ compl\left(X\right) \in Rej \ if \ X \in Acc \\ compl\left(\omega_4 \leftarrow \alpha_4\right) = compl\left(compl\left(\omega_4\right) \leftarrow compl\left(\alpha_4\right)\right) \\ &= compl\left(\left(A_{dom} \rightarrow B_{cod}\right) \leftarrow \left(B_{dom} \rightarrow C_{cod}\right)\right) \\ &= \left(\left(A_{dom} \rightarrow B_{cod}\right) \ o \ \left(B_{dom} \rightarrow C_{cod}\right)\right) \\ &= \left(f \ o \ g\right). \end{split}$$

3. Hence, $X \in Acc$ iff $compl(X) \in Rej$.

3.4.3 Bridging between categories and saltatories

This new feature of bridge/bridging is ruling concrete intrinsic interactions.

Bridging Conditions and Associativity for Interactions

Bridge and Bridging Conditions BC

1. $\forall k, l, n \in HET, \forall f, g, h \in MORPH$:

 $g \circ f, g \circ h,$ $(h \circ g) \circ f, h \circ (g \circ f) \in MC,$

b. saltisition

 $\begin{array}{l} I \parallel k, \ n \parallel I, \\ n \parallel \left(I \parallel k \right), \ \left(n \parallel I \right) \parallel k \in \overline{MC}, \end{array}$

c. bridges

 $g \perp k, l \perp g,$ $(l \perp g) \perp k, l \perp (g \perp k) \text{ are in } \widehat{BC}.$

d. bridging

 $g \bullet k, \ l \bullet g,$ (l • g) • k, l • (g • k) are in BC. 2. (g • k) \in BC iff dom(k) = diff (dom(g)),

2.
$$(g \bullet k) \in BC$$
 iff $aom(k) = aiff(aom(g))$
 $(I \bullet g) \in BC$ iff $cod(I) = diff(cod(g))$,
 $(I \bullet g \bullet k) \in BC$ iff $(g \bullet k)$, $(I \bullet g) \in BC$.

3.
$$(g \perp k) \in \overrightarrow{BC}$$
 iff $diff(dom(k)) = dom(g)$,
 $(I \perp g) \in \overrightarrow{BC}$ iff $diff(cod(I)) = cod(g)$,
 $(I \perp g \perp k) \in \overrightarrow{BC}$ iff $(g \perp k)$, $(I \perp g) \in \overrightarrow{BC}$.

Bridging

Assoziativity :

If k, g, $l \in BC$, then $(k \bullet g) \bullet l = k \bullet (g \bullet l)$, Bridging : bridging_(g, l, k) : het $(\omega_4, \alpha_4) \bullet hom(\alpha_2, \omega_2) \bullet het (\omega_8, \alpha_8) \to het (\omega_9, \alpha_9)$.

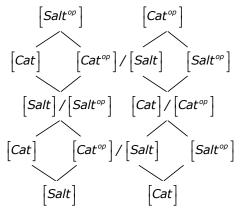
Bridge

Assoziativity : If $k, g, l \in \widehat{BC}$, then $(k \perp g) \perp l = k \perp (g \perp l)$, Bridge : bridge_(g, l, k) : het $(\omega_4, \alpha_4) \perp$ hom $(\alpha_2, \omega_2) \perp$ het $(\omega_8, \alpha_8) \rightarrow$ het (ω_9, α_9) . Bridges vs. Bridging vs. Jumping $(l \perp g \perp k) \triangleq (l \cdot g \cdot k) \triangleq (l \parallel k)$, $(l \perp g \cdot k) \triangleq (l \cdot g \perp k) \triangleq (l \parallel k)$, $(l \cdot g \perp k) \triangleq (l \perp g \cdot k) \triangleq (l \parallel k)$. diff $(\perp) = (\bullet)$, $(\perp) = diff (\bullet)$.

4 Bigraphs in diamond webs

Instead of labelling transitions of the behavioral calculus, the whole system of bigraphs could be labeled (disseminated), i.e., distributed and mediated. Reflectionality between disseminated bigraphs, then might be realized by the "doublecharacter" of diamonds. The possibility to disseminate bigraphs would open up a chiastic chain of connectivity and locality graphs, of statics and dynamics, as a new play of interactionlity/reflectionality between bigraphical systems.

4.1 Disseminated Diamonds



Diamonds, in this possible dissemination, are mapped as categories and saltatories with their dualities.

Mediation between diamonds happens horizontally, by complementarity and accretion from dual-categories to saltatories. An iterative, vertical, mediation is realized by duality and iteration from one diamond to another diamond. (cf. Kaehr 2007c)

4.2 Towards a diamond web of bigraphs

In this setting we would have to introduce first the dual theory of bigraphs, which are themselves incorporating the dual structure of topography and connectivity. The more intriguing step would be to develop the complementary system to bigraphs and its duality being placed in saltatories. Both together are building the diamond of bigraphs, which then could be disseminated to model and design interactionality and reflectionality in a polycontextural system of interaction including the chiasm of global and local situations. Such a diamond web would not be restricted to informatic and physical global interactions like for bigraphs but would be open to offer a framework for knowledge related semantic and pragmatic aspects of pluriversal computation and communication. Dissemination of diamonds might offer a scheme for a distribution and mediation of the orthogonality of connectivity and locality in bigraphs, which are themselves thematized as dualities.

From a more futuristic vision, also with not much theory, Hai Zhuge (Beijing) develops the idea and sketches some steps towards a methodology of a knowledge grid, which is to "foster worldwide knowledge creation, evolution, inheritance, and sharing in a world of humans, roles and machines". (Zhuge 2004:1), (cf. Kaehr 2007e, f)

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