Memristics: Memristors, again? - Part II

How to transform wired ‘translations’ between crossbars into interactions?

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Abstract

The idea behind this patchwork of conceptual interventions is to show the possibility of a “buffer-free” modeling of the crossbar architecture for memristive systems on the base of a purely difference-theoretical approach. It is considered that on a nano-electronic level principles of interpretation appears as mechanisms of complementarity. The most basic conceptual approach to such a complementarity is introduced as an interchangeability of operators and operands of an operation. Therefore, the architecture of crossbars gets an interpretation as complementarity between crossbar functionality and “buffering” translation functionality. That is, the same matter functions as operator and at once, as operand - and vice versa. Hence, the construction of an additional devise for translations between crossbars is conceptually an inheritance of the old paradigm of (mirco)electronic computation, and probably or hopefully, superfluous.
The exercise is concretized by a an example of the conceptual architecture of a multi-crossbar arithmetic processor in the sense of Blaise Laurent Mouttet’s patent.
Aditionally, the concept of a *complementary antiserial resistive switch* (Rainer Waser) is discussed.
Nevertheless, a new challenge arises, how to realize on a nano-technological level functional interchangeability of any complexity and complication?

Keywords
memristics, crossbar, memristor, interchangeability, chiasm, multi-layer, complementarity, polycontexturality, category theory, antiserial, complementarity

1. Mediation of multi-layered systems

1.1. Towards a new significance of crossbar architectures

How are layers of a multi-layered crossbar system connected?

Are there new strategies available supporting interactivity between crossbars without rejecting the possibilities of the new conditions of the nanosphere? Is the metaphor of “crossbar” as it was used over decades as “crossbar switch” in information processing, not misleading the attempts of new memristive technologies? Are they still well understood in terms of parallel information processing?

Speculations are risked in this proposal to overcome the limitations of entity-ontology, which is still governing scientific and engineering achievements by the concept of a simultaneous “upwards” and “downwards interaction” in crossbar architectures, which is constituting the functionality of three-dimensional crossbars constructions.

The aim, again, is to dissolve retarding ‘buffers’, called Glue, into direct actions of interactivity.
http://www.thinkartlab.com/pkl/lola/Category%20Glue%20II/Category%20Glue%20II.html

The concept of interactivity in the sense of diamond category theory is proposed in the paper *“Double Cross Playing Diamonds”* .
http://works.bepress.com/thinkartlab/2/

[**0003**] Crossbar interconnect technology has been developed in recent years with a primary focus in applications in information storage and retrieval. A crossbar array basically comprises a first set of conductive parallel wires and a second set of conductive parallel wires formed so as to intersect the first set of conductive wires. The intersections between the two sets of wires are separated by a thin film material or molecular component. A property of the material, such as the material’s resistance, may be altered by controlling the voltages applied between individual wires from the first and second set of wires. Alteration of the materials resistance at an intersection may be performed so as to achieve a high resistance or low resistance state and thus store digital data. It is noted that crossbar arrays are occasionally referred to as cross point or crosswire arrays.

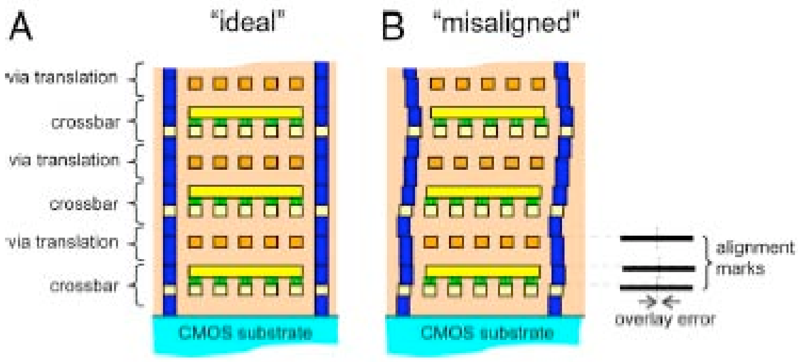


Fig. 4. Cross-section of three-dimensional circuit illustrating (*A*) "ideal" alignment between layers and (*B*) more realistic scenario with overlay error.

> "We present a topological framework that provides a simple yet powerful electronic circuit architecture for constructing and using multilayer crossbar arrays, allowing a significantly increased integration density of memristive crossbar devices beyond the scaling limits of lateral feature sizes. The truly remarkable feature of such circuits, which is an extension of the CMOL (CmosMOLecular-scale devices) concept for an area-like interface to a three-dimensional system, is that a large-feature-size complimentary metal-oxide-semiconductor (CMOS) substrate can provide high-density interconnects to multiple crossbar layers through a single set of vertical vias."
> http://www.pnas.org/content/106/48/20155.full

"Memristics: Memristors, again?", as Part I, is collecting material for an understanding of memristors and memristive systems and is organized under the focus of the concept of a simultaneous interchangeability of memory and computing.
http://www.scribd.com/doc/30166581/Memristics-Memristors-again
http://www.thinkartlab.com/pkl/lola/Memristics/Memristics:Memristors, again.pdf

1.1.1. Cross-section of a three-dimensional circuit

Different polycontextural modeling approaches are possible for three-dimensional crossbar circuits.

Crossbars with translation
crossbar: contexture, with logic+memory
via translation: mediation between contextures

This modeling allows clean separation of crossbars with the help of a "buffering" translation layer.

Crossbars with memristors
crossbar: contexture, with logic+memory
memristors: mediation between contextures

This modeling without translation allows a clean mediation of crossbars with the help of 'translational' mediative memristive layers. It seems that this approach is more close to the characteristics of memristive systems which are able, as it is mentioned again and again, to realize *simultaneously* different functionalities, say memory and computing.

Hence, the crossbar concept for memristive systems gets involved into the fragile but biomorph behavior of strict functionality or actionality, eliminating any substantialistic relicts from other older paradigms. That is, the crossbar is the place of memory and calculation, and is functioning at once as a mediative translator between different crossbars in the topology of distributed crossbars, here, the three-dimensional grid. Therefore, what is basic, is not a new entity, like a translation bar, but the interchangeing difference of both movements of interpretation, i.e. the difference of the upwards and the downwards movement, which is co-creating the functioning of the crossbar system. Hence, the primary feature of the crossbar whith its memristors, the dynamics of the differences between its own constuting and co-creating parts.

The crossbar as a nano-physical *entity* is not yet the crossbar of a memristive system. Only its *intractivity* between different layers of different crossbars is building the memristive crossbar system.

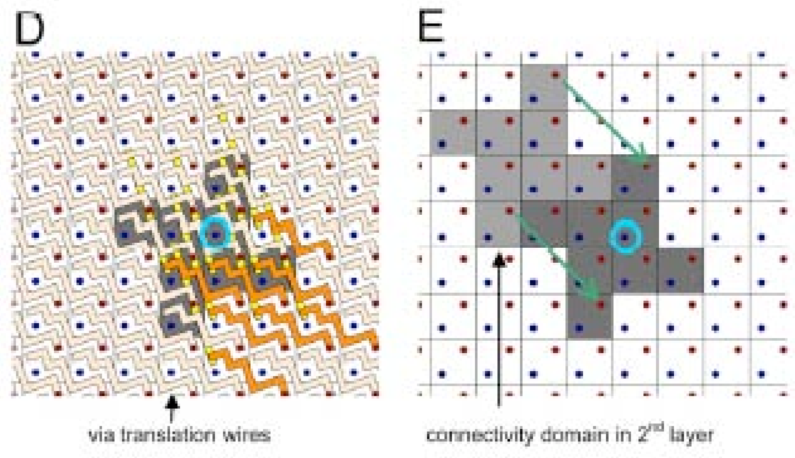
Single crossbar?
What happens now with a single crossbar? Is it still a crossbar or a non-

dynamic entity?
There is still an important dynamics to observe: the dynamics between the 'horizontal' and the 'vertical' wires of the single crossbar, mediated by memristors.
Again, this construct is not positively given as a physical system or entity. It depends on the dynamics of 'upward' and 'downward' activities.

Quantum physics and non-Boolean logics
I guess, the speculation proposed goes well together with the results of modern quantum physics and is in concordance with the features (complementarity, non-Boolean logics, complexity), as they are conceived by the theoreticians of quantum mechanics.

This paper is not intending to give a full description of the co-created situation encountered by the new challenges of a possible memristive systems theory.

> *"Modern quantum mechanics put an end to atomism and hence to reductionism: The so-called "elementary particles" (such as electrons, quarks, or gluons) are patterns of reality, not building blocks of reality. They are not primary, but arise as secondary manifestations, for example as field excitations, in the same sense as solitons are localized excitations of water, and not building blocks of water."*
>
> *"A complementary description refers always to a contextually chosen decomposition of the universe of discourse. Note that these formulations do not make any reference to physics."*
>
> *"The fact that every single experiment allows a description in terms of classical Boolean logic does not imply that the family of all feasible experiments can be combined into a single Boolean context." (Hans Primas)*
>
> Dmitri B. Strukov, R. Stanley Williams, Four-dimensional address topology for circuits with stacked multilayer crossbar arrays



"Similarly, D highlights the wires implementing the translation of red vias within the considered domain, whereas E shows the corresponding connectivity domains of a given blue via for the first and second layers."

"The problem of stacking multiple layers becomes one of geometry to ensure that only one crosspoint device in all of the arrays can be addressed by any allowed set of four address labels (or pair of vias). For example, one algorithm (out of many different possibilities) to place the next crossbar in a sequence is to *translate* it with respect to the fixed locations of one kind of via (e.g., red vias in Fig. 3) by a distance such that the contacted wire fragments in the new layer are connected to a set of cells that is different from any preceding layer."

"Fig. 3 D and E shows how a crossbar can be translated with respect to red vias by $\approx\beta$ (2 for r = 3) cells to the left and down with respect to blue vias (translation indicated with green arrows) using the via-translation wiring layer placed between crossbar arrays. Clearly, the connectivity domains in the first and second layers in Fig. 3E do not overlap, and unique access to each memristive device is possible. For instance, the shift of the red vias ensures that they have wire-memristive device-wire connections to the highlighted blue via only in one (the first) crossbar layer, whereas there is no such connection in the second layer." http://www.pnas.org/content/106/48/20155.full

1.1.2. Transforming ‘translation’ into interactivity

The argumentation of a ‘dissolution’ of the ‘via-translation’ part (D) in the crossbar construction shall be risked with the help of a simple scheme for the cross-section of a three-dimensional circuit. A kind of an ‘overlapping’ combinatorics, instead of concatenation, is applied. In technical terms, this construction is not based on sign-sequences but on the composition an decomposition of morphograms, based on kenograms.
Some hints about kenogrammatics might be found at:
Gotthard Gunther, Natural Numbers in Trans-classic Systems, Cybernetics and Systems, Vol. 1, Issue 3 1971 , pp. 50 - 62
in: http://www.thinkartlab.com/pkl/archive/Cyberphilosophy.pdf

orange	**orange**	D
yellow		C
green	green	B
white	white	A
orange	**orange**	D
yellow		C
green	green	B
white	white	A

(ABC): crossbar
D: via-translation

Transformation of the via-translation into interwoven crossbars
The question is: What is the conceptual and operative character of the *“via translation”* between crossbars?
In which sense is it different from crossbars?
Can it be replaced by the functionality of crossbars and therefore reducing redundancy?
How much is *“via-translation”* buffering, i.e. retarding operativity?

In short: Is there a transformation from D to (ABC)?

$$[(ABC)\ D\ (ABC)\ ] \Rightarrow ?\ \ [(ABC)_1\ (ABC)_3\ (ABC)_2\ ]$$

D operates (computes) between the operands (data, memory) $(ABC)_1$ and $(ABC)_2$.
But D might operate as a memristive crossbar operand (ABC) too, hence D

becomes $(ABC)_3$.

Therefore, D($(ABC)_1$, $(ABC)_2$) gets represented by $(ABC)_3$($(ABC)_1$, $(ABC)_2$), which is relationally represented by the triple: ($(ABC)_1$, $(ABC)_3$,$(ABC)_2$).

But this *substitution* is not giving any hints how a *mechanism* would work to avoid fixed conditions for translation.

A step closer to a solution might be achieved with the support of an additional memristive layer B':

[$(ABC)_1$, B', $(ABC)_3$, B '',$(ABC)_2$B '''].

Hence D might be represented concretely by: (B', $(ABC)_3$, B '').

Thus the whole construction becomes more intriguing with:
[$(ABC)_1$, B', $(ABC)_3$, B '',$(ABC)_2$ B'''].

But how is it working? This not yet a mechanism but still an augmented substitution only. Conceptually, there is no dynamics involved. No metamorphosis of the parts between the layers is involved.

The next steps shall demonstrate how we get a more explicit and dynamic, interactional, transformation and dissolution of D.
Again, D is responsible for a connection between different crossbar layers, but it is not involved in any productive activity, therefore such a reduction would be a step to reduce complication and use of resources.

1.1.3. Exemplification of the exercise

Problem: Transforming D into a functionality of (ABC).

D= | orange | orange | ⇒ (ABC) =

yellow	
green	green
white	white

Strict concatenative substitution

yellow	
green	green
white	white

= (ABC)2

yellow	
green	green
white	white

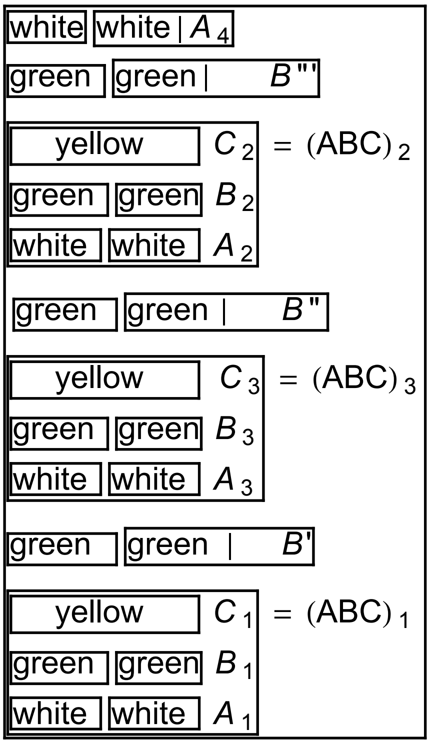
= $(ABC)_3$

yellow	
green	green
white	white

= $(ABC)_1$

Again, it seems not to be reasonable to simply replacing D by a new crossbar $(ABC)_3$. There is no mechanism offered to change from a crossbar construct (ABC) to a via-translation construct (D), like [(ABC) D (ABC)] ⇒? [$(ABC)_1$ $(ABC)_3$ $(ABC)_2$].

Interwoven and mediated concatenation



A new approach to a functional implementation of the *interaction* between the functionality of crossbars and via-translations seems to be accessible by the introduction of the concept of a *double functionality* of the memristive component B. This double functionality might be achieved in two steps. One is to duplicate the memristive element (B) , the other step is to take the characterization of nanoscale phenomena as being *complementary* seriously and trying to implement it. This happens with the strategy of a *double reading* of the phenomenon (crossbar construction) as being *up*- and *downwards* characterized in the systems of layers. In other words, complementary movements as *dromic* and *antidromic* are involved into an interplay of the realization of crossbar and ’translation’ functions.

Again, what has to be done.

> *The Miraculous Memristor - Logic And Memory Plus Going Beyond Moore's Law...*
> "- This gets interesting. We have now discovered that memristors can be used for logic -- they can be used as processors. This is very significant because instead of *shuttling data to the processor and then back again*, which takes time and energy, we could shuttle the processing code to the data -- which is smaller and quicker.” Tom Foremski - April 19, 2010
> http://www.siliconvalleywatcher.com/mt/archives/2010/04/the_miraculous.php

This approach of interplay is demonstrate in extenso in my papers to *Diamond Theory.*
hhttp://www.thinkartlab.com/pkl/lola/Diamond-Category-Theory.pdf

Balanced version with symmetry
upwards: white-green-yellow + green :
$((ABC)_1, B')$, i.e $\overrightarrow{((ABC)_1, B')}$.
downwards: white+green-yellow-green :
$((A_3B'C_1), B_1)$, in reverse order, i.e $\overleftarrow{(B_1(C_1 B' A_3))}$.

L_1	L_3

$A_1 \leftrightarrows A_3$
$B' \leftrightarrows B_1$
$C_1 \leftrightarrows C_1$
$B' \leftrightarrows B_1$

Between A_1 and A_3 : order relation
Between C_1 and C_1 : coincidence relation
Between B_1 and B' : exchange relation relation

How could the interactivity of $\overrightarrow{((ABC)_1, B')}$ and $\overleftarrow{(B_1(C_1 B' A_3))}$ replacing the entity D?

Both activities, $\overrightarrow{((ABC)_1, B')}$ and $\overleftarrow{(B_1(C_1 B' A_3))}$, are localized at two different loci, i.e. at two different crossbars of the multi-layer system. D plays the role of a fixed translator between the levels of those crossbar systems. The task is to transform this *passive* translation into mediating *activity* between crossbar domains as a part of other activities of the crossbars. The possible logical cirularity implied, that domains of crossbars are mediating between crossbars, is resolved in the concept of mutual interchangeability of actions at different loci.

Triple crossbar scheme

Interpretation of the scheme $\left(ABC, B^*\right)$ as up – and down – wards implementation :

1. $\overrightarrow{\left(\left(ABC\right)_1, B'\right)}, \overleftarrow{\left(B_1\left(C_1 B' A_3\right)\right)},$

$\overrightarrow{\left(\left(ABC\right)_3, B''\right)}\ \overleftarrow{\left(\left(B_2\left(C_3 B'' A_2\right)\right.\right.},\quad \overrightarrow{\left(\left(ABC\right)_2, B'''\right)}\ \overleftarrow{\left(B_3\left(C_2 B''' A_4\right)\right)}$

2. $\overrightarrow{\left(\left(ABC\right)_{1}, B'\right)}$, $\overleftarrow{\left(B_{1}\left(C_{1}B'A_{3}\right)\right)}$,

$\overrightarrow{\left(\left(ABC\right)_{3}, B''\right)}$, $\overleftarrow{\left(B_{3}\left(C_{3}B''A_{2}\right)\right)}$,

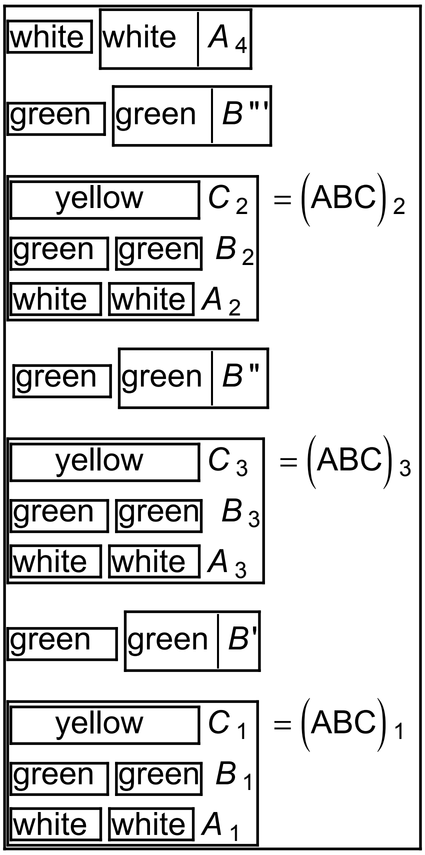
$\overrightarrow{\left(\left(ABC\right)_{2}, B'''\right)}$ $\overleftarrow{\left(B_{3}\left(C_{2}B'''A_{4}\right)\right)}$

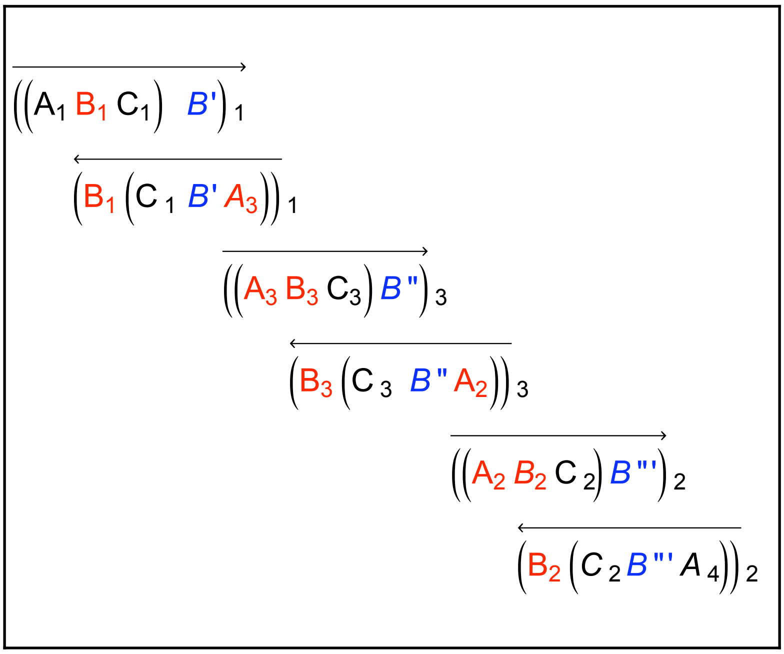
3. $\overrightarrow{\left(\left(ABC\right)_{1}, B'\right)}$, $\overrightarrow{\left(\left(ABC\right)_{3}, B''\right)}$, $\overrightarrow{\left(\left(ABC\right)_{2}, B'''\right)}$

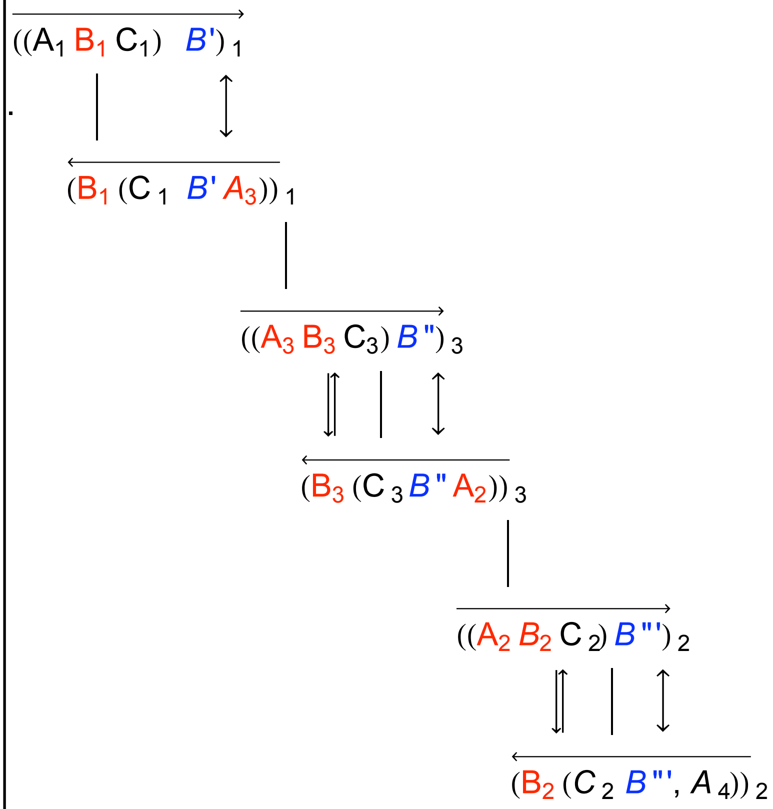
$\overleftarrow{\left(B_{1}\left(C_{1}B'A_{3}\right)\right)}$, $\overleftarrow{\left(B_{3}\left(C_{3}B''A_{2}\right)\right)}$, $\overleftarrow{\left(B_{3}\left(C_{2}B'''A_{4}\right)\right)}$

Null

Interwoven and mediated combination







$$CR^{(m)} = \left(\overrightarrow{computation_{1.2}}\ ,\ \overleftarrow{memory_{1,2}}\right),$$

$$mediation = \left(\overrightarrow{computation_{3}}\ ,\ \overleftarrow{memory_{3}}\right)$$

What does it mean?

The way up for $(ABC)_{i=1,2,3}$ is involved with an additive replication of B to B',

hence $\left((ABC)_{i=1,2,3},\ B^{i}\right)$.

The way down for $(ABC)_{i=1,2,3}$ is conserving $(ABC)_{i=1,2,3}$ but in a reverse order, overlapping crossbar levels and involving the replicative B',

i.e. $(B_1 (A_3 B' C_1))$ and $(B_2 (C_3 B'' A_2))$.

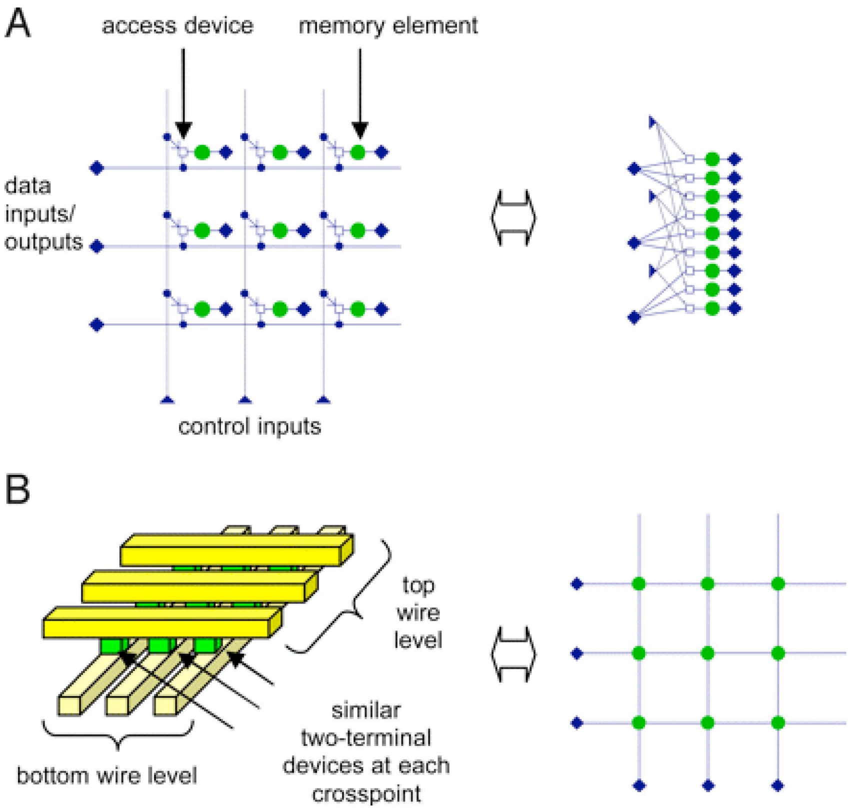
The mechanism of the *interplay* between the upwards and downwards parts of the construction is demonstrated by the polycontextural category-theoretical conceptions of *interchangeability* between the operations of combination (composition, inter-change, replication) and the operations of mediation.

Epistemological it might be argued that a quantum phenomenon is not given (read) as such, in a single reading, i.e. observation, but has to be read (observed) in all directions possible, here, at least, up- and downwards. And only its full ‘holistic’ description of the complementary and mutually excluding observations is characterizing the phenomenon as such properly.
Obviously, connections to the problems of endo/exo-physics and observer theory would be a further step necessary for the understanding of the proposed construction. But this exercise is working conceptually without such further connections with quantum mechanics.

As a consequence of the construction, all retarding buffering of the three-dimensional crossbar, i.e. via-translation, is dissolved and eliminated in favor of a purely functional ambiguity and simultaneity of the *behavior* of a complex memristive crossbar construction.

What are the conceptual costs?

Interwoven mediated implementations are involving *view-points* of interpretation, i.e. different directions of reading a chain, here as ”upwards and downwards”. This sounds not familiar to hardware implementation strategies but seems to be natural for biological and bio-morph phenomena and should therefore be emulated and realized in technical artifacts of artificial living systems.
The main feature of this transformation mechanism, from a buffer to a pattern of interactivity, is played by the double role of the memristive part (B). But that is exactly what is pretended throughout the papers about memristors: the possibility of a *simultaneity* of opposing functions, say memory and computation.
Therefore, new methods of modeling and programming are needed to realize nanotechnological machines.



"Fig. 1. Typical structures for (A) arrays with each cell having a dedicated access element (transistor) and (B) crossbar arrays with equivalent circuit representations used in the following discussion. The specific case n = 3 is used for illustration, but practical arrays are much larger (e.g., to reduce peripheral overhead in memory applications)."
"By integrating the access function into the crosspoint memory device, one can implement crossbar memory circuits (Fig. 1B)."
http://www.pnas.org/content/106/48/20155.full

Towards "Complementary resistive switches"

What ever it means, there is some further stuff which sounds interesting, added 14.05.2010:

Complementary resistive switches for passive nanocrossbar memories
Eike Linn, Roland Rosezin, Carsten Kügeler & Rainer Waser
Abstract
On the road towards higher memory density and computer performance, a significant improvement in energy efficiency constitutes the dominant goal in future information technology. Passive crossbar arrays of memristive elements were suggested a decade ago as non-volatile

> *random access memories (RAM) and can also be used for reconfigurable logic circuits.*
> *As such they represent an interesting alternative to the conventional von Neumann based computer chip architectures. Crossbar architectures hold the promise of a significant reduction in energy consumption because of their ultimate scaling potential and because they allow for a* local fusion *of logic and memory, thus avoiding energy consumption by data transfer on the chip.*
> *However, the expected paradigm change has not yet taken place because the general problem of selecting a designated cell within a passive crossbar array without interference from* sneak-path *currents through neighbouring cells has not yet been solved satisfactorily. Here we introduce a* complementary resistive switch*. It consists of two antiserial memristive elements and allows for the construction of large passive crossbar arrays by solving the sneak path problem in combination with a drastic reduction of the power consumption.”* Nature Materials 9, 403 - 406 (2010), Published online: 18 April 2010 | doi:10.1038/nmat2748
> http://www.nature.com/nmat/journal/v9/n5/pdf/nmat2748.pdf

Again, it is not easy to guess what is meant by a “*local fusion of logic and memory*”. What is the mechanism of “fusion"? If it means that the difference between “logic” and “memory” is leveled, two questions are still open. First, what is this "fusion" (or combination) of both, logic and memory, in the new domain of “fusion”? It must be something different of both, hence, a third distinction? Second, if the difference is leveled, how is it nevertheless possible that this fusion ‘device’ is able to process (logic) and to store (memory) data?
The solution seems to be found in the CRS, the *“complementary resistive switch”.* Unfortunately, I couldn’t find any (free) access to it.
Nevertheless, the CRS construct sounds, conceptually, quite familiar!?

> *"It has been known for few years that memristor chips may play an important part in alternative architectures for future computers. Memristive cells have the special property that their resistance can be programmed (resistor) and subsequently remains stored (memory)."*
> http://www.nanowerk.com/news/newsid=15859.php

Again, a memristive cell = $\binom{\text{programmed}}{\text{stored}} \longrightarrow$

$\binom{\text{stored}}{\text{programmed}}$, functions as a chiasm, *i.e.* as a " double device " or Janus – face – device :

$$\chi\begin{pmatrix} \text{progr}_1 & \text{mem}_2 \\ \text{mem}_1 & \text{prog}_2 \end{pmatrix} = \begin{pmatrix} \text{progr}_1 & \longrightarrow & \text{mem}_2 \\ \updownarrow & X & \updownarrow \\ \text{mem}_1 & \longleftarrow & \text{prog}_2 \end{pmatrix}.$$

Or in general terms, as a proemial operation: $PR\begin{pmatrix} \text{operator}_1 & \text{operand}_2 \\ \text{operand}_1 & \text{operator}_2 \end{pmatrix} =$

$$\begin{pmatrix} \text{operator}_1 & \longrightarrow & \text{operand}_1 \\ \updownarrow & X & \updownarrow \\ \text{operand}_2 & \longleftarrow & \text{operator}_2 \end{pmatrix}.$$

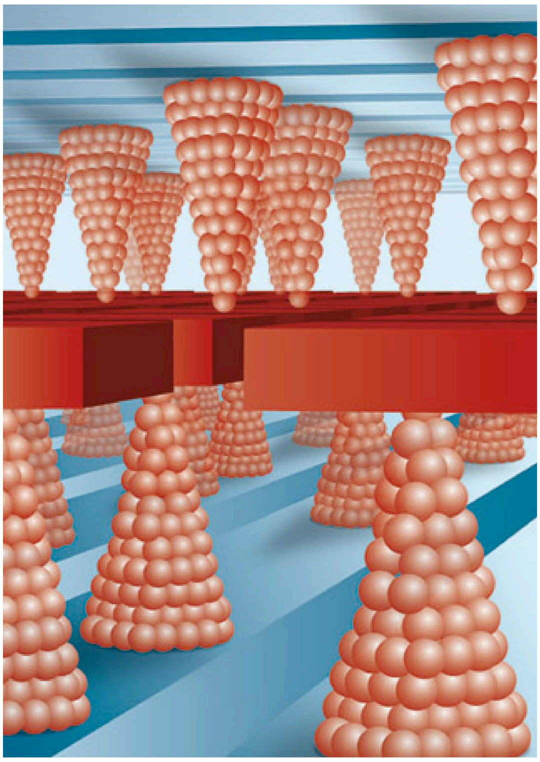
In other words, the manipulation of an operand by an operator (logic, processor) is changing the operand (memory of another history). This change is stored in the new state of the operand (memory). The new operand (memory) is functioning as an operator (logic) manipulating an operand (memory) which itself becomes an operator (logic) for an operand (memory). This is the double and Janus-face character of a dromic/antidromic device. Such a device is not a duality but a complementarity in the sense of diamond category theory, its functionality of time-structures and operativity are strictly complementary, i.e. at least, at once, opposites. Hence realizing a chiastic interplay. In this sense, the concept of a memristor as a singular electronic element is misleading the attempts of memristive systems, hence an *"antiserial switching of* two *memristive cells"* is required.

It might be that the terminology of "*complementary*", "*switching*", "*antiserial*", "*two*" device gets a conceptual clarification or obstruction by diamond-theoretical considerations. (cf. Diamond Category Theory)

Complementary Resistive Switch

"[...] completely new switching concept. This concept is based on the antiserial switching of two memristive cells*. Together, these cells form a novel unit, which the scientists termed a CRS cell (complementary resistive switch). No undesirable superimposition of information takes place between CRS cells."*

http://www.jara.org/de/research/jara-fit/nachrichten/details/green-chips/



"View of a CRS structure with nanometre resolution. Electrochemically produced filaments of copper atoms can be seen, which are either formed in the upper or lower part of a cell and thus represent the state "0" or "1". Due to the small dimensions of these "nanobatteries" the cells can be switched in a few nanoseconds with extremely low expenditure of energy."

1.2. Polycontextural modeling of interwoven crossbars

1.2.1. Interchangeability schemes

The functionality of interwoven crossbars gets some explanation by a logical and categorical representation.
The simplest case of conceptual modeling is the case of a strict *parallelism* between crossbars.
More interesting is the case of *interactivity* between interwoven crossbars. In fact, the construction of interweaving crossbars is an architectural implementation of possible interactions between crossbars.

The double interpretation of the interwoven crossbars in their up- and down-functionality, is an interweaving of operators and operands of the crossbar complexion. Interweaving means, that there is a interchangeability of the operational functionality involved. That is, an object might get a double role: one as an operator, one as a an operand, both at once.

A straightforward formalization is possible in the framework of monoidal categories and a proposed generalization to polycontextural monoidal categories.

What are the pretensions of monoidal categories?

> *"The (Φ,$\otimes$)-logic is a* logic of interaction*. It applies to cooking processes, physical processes, biological processes, logical processes (i.e. proofs), or computer processes (i.e. programs). The theory of* monoidal categories*, the subject of this chapter, is the mathematical framework that accounts for the common structure of each of these* theories of processes*. The framework of monoidal categories moreover enables* modeling *and* axiomatising *(or 'classify') the extra structure which certain families of processes may have."* (Coecke, Paquette, Categories for the practising physicist, p. 3)
> web.comlab.ox.ac.uk/people/Bob.Coecke/ctfwp1_final.pdf

Monoidal categories are based on a single Universe $\mathcal{U}_0$. This concept therefore shall be called *mono* – contextural.

classical scheme with composition and yuxtaposition

BIFUNCT$^{(2)}$:

$$(\mathcal{U}_0),\ \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix} : \begin{pmatrix} f_1 \\ \otimes \\ f_2 \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes \\ g_2 \end{pmatrix} = \begin{pmatrix} (f_1 \circ g_1) \\ \otimes \\ (f_2 \circ g_2) \end{pmatrix}$$

$\forall f_i,\ g_i \in$ Universe $\mathcal{U}_0,\ i \in \mathcal{N}$

$\circ$: composition

$\otimes$: yuxtaposition

The techniques of mono-contextural monoidal category theory are generalized to *polycontextural* categories based on a multitude of autonomous and disjunct interacting universes $\mathcal{U}^{(m,\, n)}$.

Therefore, the interchangeability of composition ($\circ$) as a *sequential* operation and yuxtaposition ($\otimes$) as a *parallel* operation in classical monoidal categories gets radicalized in polycontextural monoidal categories as a (metamorphic) interchangeability between a mediation ($\amalg$) of contextures and different intra- and trans-contextural operations, like composition ($\circ$), replication ($\square$), iteration, metamorphosis ($\diamond$) and bifurcation between different contextures. The classical construct of yuxtaposition is well conserved for each single contexture too. Hence, we get an interactional parallelity of parallelities, i.e. a trans-contextural parallelity of contextures and an intra-contextural parallelity of contexts.

Hence, the inverse parallelism or *antidromic complementarity* (Diamond category theory) gets the chance of an adequate conceptual and operative implementation as a working formalism with the help of polycontextural category theory.

On the other hand, only the *minimal conditions* for a polycontextural category-theoretic modeling is proposed. All other important features have to be omitted to give a first idea about the conceptual construction.

$$\begin{pmatrix} \mathcal{U}_2 \\ \mathcal{U}_1 \end{pmatrix}, \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix}:$$

$$\begin{pmatrix} f_2 \\ \amalg \\ f_1 \end{pmatrix} \circ\diamond \begin{pmatrix} g_2 \\ \amalg \\ g_1 \end{pmatrix} = \begin{pmatrix} f_2 & \circ & g_2 \\ \amalg & \diamond & \amalg \\ f_1 & \circ & g_1 \end{pmatrix}$$

$\amalg$: mediation between contextures
$\circ$: composition of morphisms
$\diamond$: cross – interchange between levels
$=$: equivalence

$\forall\, f_i, g_i \in \text{Universe}\,\mathcal{U}_i,\ i \in \mathcal{N}$ et
$\forall\, i \neq j: \text{Universe}\,\mathcal{U}_i \bigcap \text{Universe}\,\mathcal{U}_j = \emptyset$

$\forall\, f_i \in \mathcal{U}_i,\ g_j \in \mathcal{U}_j,\ i \neq j = 1, 2$
$\text{cod}(f_2) \simeq \text{dom}(g_1)$
$\text{cod}(f_1) \simeq \text{dom}(g_2)$

$\forall\, f_i, g_i \in \mathcal{U}_i,\ i = 1, 2$
$\text{cod}(f_1) \cong \text{dom}(g_1)$
$\text{cod}(f_2) \cong \text{dom}(g_2)$

$m = 3,\ n = 2$

$$\begin{pmatrix} \mathcal{U}_1 \\ \mathcal{U}_2 \\ \mathcal{U}_3 \end{pmatrix}, \begin{bmatrix} g_1 & g_2 & g_3 \\ f_1 & f_2 & f_3 \end{bmatrix}:$$

$$\left(\begin{pmatrix} f_1 \\ \amalg_{1.2\times.0} \\ f_2 \end{pmatrix} \atop {\amalg_{0.2\times.3} \atop f_3}\right) \begin{pmatrix} \circ_1 \circ_2 \circ_3 \\ \diamond_{1.2} \diamond_{2.3} \end{pmatrix} \left(\begin{pmatrix} g_1 \\ \amalg_{1.2\times.0} \\ g_2 \end{pmatrix} \atop {\amalg_{0.2\times.3} \atop g_3}\right) =$$

$$\left(\begin{pmatrix} (f_1 & \circ_{1.0\times.0} & g_1) \\ \amalg_{1.2\times.0} & \diamond_{1.2\times.0} & \amalg_{1.2\times.0} \\ (f_2 & \circ_{0.2\times.0} & g_2) \end{pmatrix} \atop {\begin{matrix} \amalg_{0.2\times.3} & \diamond_{0.2\times.3} & \amalg_{0.2\times.3} \\ (f_3 & \circ_{0.0\times.3} & g_3) \end{matrix}}\right)$$

A formalization of the antidromic structure
of interacting crossbars might now be introduced.

With the components of the crossbar interpretation we get :

$$f_1 = \overrightarrow{\left(\left(ABC\right)_1, B'\right)_1}, \quad g_1 = \overleftarrow{\left(B_1\left(C_1 B' A_3\right)\right)_1}$$

$$f_2 = \overrightarrow{\left(\left(ABC\right)_3, B''\right)_2}, \quad g_2 = \overleftarrow{\left(B_3\left(C_3 B'' A_2\right)\right)}$$

$$f_3 = \overrightarrow{\left(\left(ABC_2\right), B'''\right)}, \quad g_3 = \overleftarrow{\left(B_3\left(C_2 B''' A_4\right)\right)_3}.$$

$$\begin{bmatrix} g_1 & g_2 & g_3 \\ f_1 & f_2 & f_3 \end{bmatrix}$$

For simplicity reasons f_3 and g_3 is not yet included in the formula below for the interchanchability of the crossbar components and their antidromic characterication.

The **head** $\begin{bmatrix} g_1 & g_2 \\ f_1 & f_2 \end{bmatrix}$ with $m = 2$,

$n = 2$, gets a substitution with the components :

$$\begin{pmatrix} \overleftarrow{\left(B_1\left(C_1 B' A_3\right)\right)_1} & \overleftarrow{\left(B_3\left(C_3 B'' A_2\right)\right)_2} \\ \overrightarrow{\left(\left(ABC\right)_1, B'\right)_1} & \overrightarrow{\left(\left(ABC\right)_3, B''\right)_2} \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{U}_2 \\ \mathcal{U}_1 \end{pmatrix} \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix} : \begin{pmatrix} \overleftarrow{\left(B_1\left(C_1 B' A_3\right)\right)_1} & \overleftarrow{\left(B_3\left(C_3 B'' A_2\right)\right)_2} \\ \overrightarrow{\left(\left(ABC\right)_1, B'\right)_1} & \overrightarrow{\left(\left(ABC\right)_3, B''\right)_2} \end{pmatrix} :$$

$$\begin{pmatrix} \overrightarrow{\left(\left(ABC\right)_3, B''\right)_2} \\ \parallel \\ \overrightarrow{\left(\left(ABC\right)_1, B'\right)_1} \end{pmatrix} \circ\diamond \begin{pmatrix} \overleftarrow{\left(B_3\left(C_3 B'' A_2\right)\right)_2} \\ \parallel \\ \overleftarrow{\left(B_1\left(C_1 B' A_3\right)\right)_1} \end{pmatrix} =$$

$$\begin{pmatrix} \overrightarrow{\left(\left(ABC\right)_3, B''\right)_2} & \circ & \overleftarrow{\left(B_3\left(C_3 B'' A_2\right)\right)_2} \\ \parallel & \diamond & \parallel \\ \overrightarrow{\left(\left(ABC\right)_1, B'\right)_1} & \circ & \overleftarrow{\left(B_1\left(C_1 B' A_3\right)\right)_1} \end{pmatrix}$$

Explanation

$\overrightarrow{\left(\left(ABC\right)_1, B'\right)_1}$ *as* $\overleftarrow{\left(\mathrm{B}_3\left(\mathrm{C}_3 \mathrm{B}'' \mathrm{A}_2\right)\right)_2}$,

i.e. $\overrightarrow{\left(\left(ABC\right)_1, B'\right)_1} \diamond \overleftarrow{\left(\mathrm{B}_3\left(\mathrm{C}_3 \mathrm{B}'' \mathrm{A}_2\right)\right)_2}$

$$\overleftarrow{\left(B_3\left(C_1 B' A_3\right)\right)_1} \ \textit{as}\ \overrightarrow{\left(\left(\text{ABC}\right)_3, B''\right)_2},$$

$$\text{i.e. } \overleftarrow{\left(B_1\left(C_1 B' A_3\right)\right)_1} \ \diamond\ \overrightarrow{\left(\left(\text{ABC}\right)_3, B''\right)_2}$$

$$\overrightarrow{\left(\left(\text{ABC}\right)_1, B'\right)_1} \ \circ\ \overleftarrow{\left(B_1\left(C_1 B' A_3\right)_1\right.},$$

$$\text{ie. } \overrightarrow{\left(\left(\text{ABC}\right)_1, B'\right)_1} \ \text{ composed with } \ \overleftarrow{\left(B_1\left(C_1 B' A_3\right)_1\right.}$$

$$\begin{pmatrix} \overrightarrow{\left(\left(\text{ABC}\right)_3, B''\right)_2} \\ \coprod \\ \overrightarrow{\left(\left(\text{ABC}\right)_1, B'\right)_1} \end{pmatrix} \text{ i.e. } \overrightarrow{\left(\left(\text{ABC}\right)_1, B'\right)_1} \ \textit{mediated} \text{ with } \overrightarrow{\left(\left(\text{ABC}\right)_3, B''\right)_2}.$$

A composition and crossing of mediated components of different contextures is isomorph (equivalent) to the composition and crossing of the components and their mediation.

In a nutshell, the interplay of crossbar-domains gets a scheme of categorical interchangeability between 2 layers and 2 crossbar functionalities (CR).

$$\begin{pmatrix} \text{Layer}_2 \\ \text{Layer}_1 \end{pmatrix}, \begin{pmatrix} \overleftarrow{\text{CR}}_1 & \overleftarrow{\text{CR}}_2 \\ \overrightarrow{\text{CR}}_1 & \overrightarrow{\text{CR}}_2 \end{pmatrix}:$$

$$\begin{pmatrix} \overrightarrow{\text{CR}}_2 \\ \coprod \\ \overrightarrow{\text{CR}}_1 \end{pmatrix} \circ\diamond \begin{pmatrix} \overleftarrow{\text{CR}}_2 \\ \coprod \\ \overleftarrow{\text{CR}}_1 \end{pmatrix} = \begin{pmatrix} \overrightarrow{\text{CR}}_2 & \circ & \overleftarrow{\text{CR}}_2 \\ \coprod & \diamond & \coprod \\ \overrightarrow{\text{CR}}_1 & \circ & \overleftarrow{\text{CR}}_1 \end{pmatrix}$$

Interchangeability, also called bifunctoriality in category theory albeit less complex, is a kind of a generalization of distributivity. Polycontextural interchangeability is a generalization of the mono-contextural categorical concept for polycontextural categories.

1.2.2. Diamond theoretic interchangeability

Hint to a diamond categorical formula for further modeling of interchangeability in the context of crossbar architectures.

$$m = 3,\ n = 2,\ env = 1$$

$$\left(\begin{array}{c|c} \mathcal{U}_1 & \\ \mathcal{U}_2 & \mathcal{U}_4 \\ \mathcal{U}_3 & \end{array}\right), \quad \begin{array}{cccc} g_1 & g_2 & g_3 & f_4 \\ \end{array} \left[\begin{array}{ccc|c} f_1 & f_2 & f_3 & g_4 \\ \mathrm{sys} & \mathrm{sys} & \mathrm{acc} & \mathrm{rej} \end{array}\right]:$$

$$\left(\begin{array}{c} \left(\begin{array}{c} f_1 \\ \amalg_{1.2\times.0\times.0} \\ f_2 \end{array}\right) \\ \amalg_{0.2\times.3\times.4} \\ f_3 \mid g_4 \end{array}\right) \left(\begin{array}{c} \circ_{1.2\times.3\times.0} \\ \Diamond_{1.2\times.0\times.0} \\ \shortparallel_{0.0\times.0\times.4} \end{array}\right) \left(\begin{array}{c} \left(\begin{array}{c} g_1 \\ \amalg_{1.2\times.0\times.0} \\ g_2 \end{array}\right) \\ \amalg_{0.2\times.3\times.4} \\ g_3 \mid f_4 \end{array}\right) =$$

$$\left(\begin{array}{c} \left(\begin{array}{ccc} (f_1 & \circ_{1.0\times.0\times.0} & g_1) \\ \Diamond_{1.2\times.0\times.0} & \amalg_{1.2\times.0} & \Diamond_{1.2\times.0\times.0} \\ (f_2 & \circ_{0.2\times.0\times.0} & g_2) \end{array}\right) \\ \amalg_{0.2\times.3\times.4} \\ \left(f_3 \circ_{0.0\times.3\times.0} g_3\right) \mid \left(g_4 \shortparallel_{0.0\times.0\times.4} f_4\right) \end{array}\right)$$

$\amalg$: mediation between contextures

$\circ$: composition of morphisms

$\Diamond$: cross – interchange between levels

$=$: equivalence

$\shortparallel$: saltisition

Interchangeability with $\blacksquare \equiv \left(\amalg \quad \diamond \right)$

$m = 3,\; n = 2,\; env = 1$

$$\left(\begin{array}{c|c} \mathcal{U}_1 & \\ \mathcal{U}_2 & \mathcal{U}_4 \\ \mathcal{U}_3 & \end{array} \right), \quad \begin{array}{cccc} g_1 & g_2 & g_3 & f_4 \end{array} \left[\begin{array}{cccc} f_1 & f_2 & f_3 & g_4 \end{array} \right] : \quad \begin{array}{ccc|c} sys & sys & acc & rej \end{array}$$

$$\left(\begin{array}{c} \left(\begin{array}{c} f_1 \\ \blacksquare_{1.2\times.0\times.0} \\ f_2 \end{array} \right) \\ \amalg_{0.2\times.3\times.4} \\ f_3 \;\Big|\; g_4 \end{array} \right) \begin{pmatrix} \circ_{1.2\times.3\times.0} \\ \shortparallel_{0.0\times.0\times.4} \end{pmatrix} \left(\begin{array}{c} \left(\begin{array}{c} g_1 \\ \blacksquare_{1.2\times.0\times.0} \\ g_2 \end{array} \right) \\ \amalg_{0.2\times.3\times.4} \\ g_3 \;\Big|\; f_4 \end{array} \right) =$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \left(f_1 \quad \circ_{1.0\times.0\times.0} \quad g_1 \right) \\ \blacksquare_{1.2\times.0} \\ \left(f_2 \quad \circ_{0.2\times.0\times.0} \quad g_2 \right) \end{array} \right) \\ \amalg_{0.2\times.3\times.4} \\ \left(f_3 \;\circ_{0.0\times.3\times.0}\; g_3 \right) \;\Big|\; \left(g_4 \;\shortparallel_{0.0\times.0\times.4}\; f_4 \right) \end{array} \right)$$

This is, as usual for *diamond-theoretic* approaches, a radicalization of the *category-theoretic* and the *poly-contextural* constructions designed before. This approach, like the *kenomic/morphogrammatic* approach, which is based on kenogrammatics and morphograms, needs a special introduction, and will therefore be introduced at another place. The diamond-categorical approach enables a direct implementation of the features of *dromic* and simultaneously *antidromic* movements of crossbar activities into the formal approach itself as the difference of *acceptive,* categorical and *rejective,* saltatorical properties of a diamond category.

An important strategy of the diamond category approach is the mechanism of *'in-sourcing'* of the matching conditions of compositions and saltisitions. This might be seen, again, as a hint, how to overcome the strict difference of inscription and matter (silicon) on a category-theoretical level, which is different from the morphogrammatic thematization.

1.2.3. Morphogrammatic approach to interchangeability

A morphogrammatic modeling of the *dromic/antidromic* behavior of memristive systems, implemented as multi-layer crossbar systems, is based on morphogrammatic de/composition principles.
Independent of the importance of the density of chips, what is much more provoking, is the paradigm shift involved with memristive systems. Chip design today is still following the paradigm of *writing* on/in stones. That is, chips are build by inscription onto silicon. But again, silicon is nothing more than a passive carrier of inscriptions. In contrast, the memristive approaches tries to go beyond such 'matter/mind'-barriers by directly modeling its matter. As a consequence, a new paradigm of 'writing' has to emerge.
Rudolf Kaehr, The Abacus of Universal Logics
http://works.bepress.com/thinkartlab/17/

> *"All memory storage elements today use silicon transistors as the memory element," said Gartner's Reynolds. "The thing about the crossbar that I really like is that it moves the memory element out of the silicon and puts it on top." Because "relatively conventional silicon circuits" are used for control and management logic below the array, "the crossbar memory array can be much more dense than the underlying transistor technology," he added."*
> R. . Colin Johnson, Will memristors prove irresistible?
> http://www.eetimes.com

1.2.4. Metamorphic interchangeability

Things are getting slightly more complex if we consider the full wording of what happens with the role of B in the up/downwards double strategy. The double strategy means that B is in a double functionality: B as operator, and simultaneously, B as operand.

The wording is not simply *"operators becomes operands and operands become operators"* but more explicitly: *"an operator **as** an operator becomes an operand, and at the same time the operator as an operator remains in its role **as** an operator"*.
Therefore, an operator as an operator changes roles to become an operand, and simultaneously, as an operator it remains an operator. And this holds for all parts of the construction.

Full wording of metamorphosis

Operators

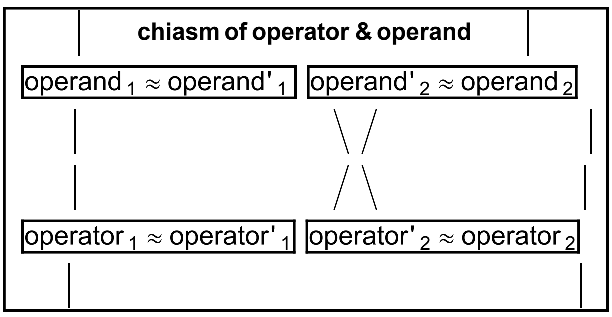
$[\approx, \diamond, \circ, \amalg] = [$as, transvers, composition, mediation$]$

Wording

1. f_1 as f_1, $f_1 \equiv f_1$, is connected with g_1 as g_1, $g_1 \equiv g_1$, by composition : $(f_1 \circ g_1)$
2. f_2 as f_2, $f_2 \equiv f_2$, is connected with g_2 as g_2, $g_1 \equiv g_1$, by composition : $(f_2 \circ g_2)$;
3. f_1 as f_1 is connected with f_2 as f_2 by mediation : $\begin{pmatrix} f_1 \\ \amalg \\ f_2 \end{pmatrix}$
4. g_1 as g_1 is connected with g_2 as g_2 by mediation : $\begin{pmatrix} g_1 \\ \amalg \\ g_2 \end{pmatrix}$;
5. f_1 as f'_1, $(f_1 \approx f'_1)$, is connected with g_2 as g'_2, $(g_2 \approx g'_2)$, by transversality : $\begin{pmatrix} f'_1 \\ \diamond \\ g'_2 \end{pmatrix}$
6. g_1 as g'_1, $(g_1 \approx g'_1)$, is connected with f_2 as f'_2, $(f_2 \approx f'_2)$, by transversality : $\begin{pmatrix} g'_1 \\ \diamond \\ f'_2 \end{pmatrix}$.

Hence, the term "f" as (f, f′) is *at once* in a *compositional* relation with "g"

and in a *transversal* relation with "g′", as well as in a *mediational* relation with the composition "$\circ$".

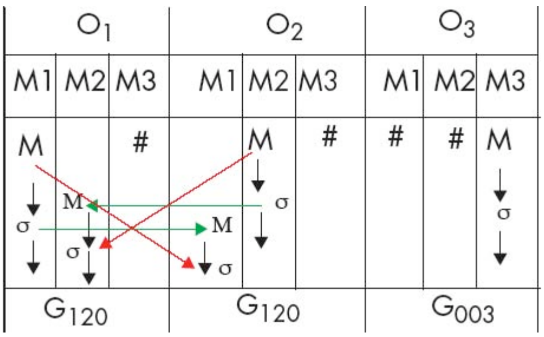


Crossbars in the kenomic matrix

Following the accessible literature, the preceding paragraphs considered just the simplest case of a mediation of crossbars. If we map the concept onto the kenomic matrix it is just covering the diagonal structure of the matrix.
In fact, the wording "operator *as* operand" and "operand *as* operator" suggests a more reflectional modeling.
Because an operator is not identical with an operand, the '*sameness*' of the terms, operator as operand, and, operand as operator, has to be *represented* at another place of the matrix.
This opens up possibilities for reflectional and interactional configurations.

Hence, the little chiasm of operator M and operand σ, gets the following wording:
1. the operator M at the place (O_1M_1) becomes an operand σ at the place (O_2M_1), and the operand σ at the place (O_1M_1) becomes an operator M at the place (O_2M_1) and
2. the operator M at the place (O_2M_2) becomes an operand σ at the place (O_1M_2), and the operand σ at the place (O_2M_2) becomes an operator M at the place (O_1M_2).
3. The systems (O_iM_i), i=1,2,3 *per se* remain unchanged.

This is depicted in the diagram below:



Without the arrows, this chiastic situation is caught by the positional matrix for the systems involved :

[reply, reply, id]	O_1	O_2	O_3
M_1	$S_{1.1}$	$S_{1.2}$	–
M_2	$S_{2.1}$	$S_{2.2}$	–
M_3	–	–	$S_{3.3}$

Both, the diagram and the matrix, shall be put together into the operative formulation for the interchangeability of operator and operand in the considered construction. In other words, the locus $O_1$is offering the system at locus O_2 a place M_2 (O_1M_2) to realize its 'identity' interchange. Mutually, the locus O_2 is offering the system O_1 a place M_1 (O_2M_1) to realize its 'identity' interchange.

General scheme for replication of system S_1 and S_2

$$\begin{pmatrix} O_1 \\ \amalg \\ O_2 \\ \amalg \\ O_3 \end{pmatrix}\begin{pmatrix} \circ\ -\ - \\ \amalg \\ -\ \circ\ - \\ \amalg \\ -\ -\ \circ \end{pmatrix}\begin{pmatrix} M_1 \square M_2 \\ \amalg \\ M_2 \square M_1 \\ \amalg \\ M_3 \end{pmatrix} = \begin{pmatrix} (O_1 \circ M_1) \square (O_1 \circ M_2) \\ \amalg \\ (O_2 \circ M_2) \square (O_2 \circ M_1) \\ \amalg \\ (O_3 \circ M_3) \end{pmatrix}$$

$\square$: replication

$\circ$: composition

$\amalg$: mediation

This general scheme becomes a formula for interchangeability by the variables *f* and *g* the different terms.

Interchangeability and replication for S_1, S_2

$$\begin{pmatrix} f_1 \square_{1.2} f_2 \\ \amalg_{1.2} \\ f_2 \square_{2.1} f_1 \\ \amalg_{2.3} \\ f_3 \end{pmatrix} \begin{bmatrix} \circ_{1.2} \\ \\ \circ_{2.1} \\ \\ \circ_{3.3} \end{bmatrix} \begin{pmatrix} g_1 \square_{1.2} g_2 \\ \amalg_{1.2} \\ g_2 \square_{2.1} g_1 \\ \amalg_{2.3} \\ g_3 \end{pmatrix} =$$

$$\begin{pmatrix} \left(\left(f_1 \circ_{1.1} g_1\right) \square_{1.2} \left(f_2 \circ_{1.2} g_2\right)\right) \\ \amalg_{1.2} \\ \left(f_2 \circ_{2.2} g_2\right) \square_{2.1} \left(f_1 \circ_{2.1} g_1\right)\right) \\ \amalg_{2.3} \\ \left(f_3 \circ_{3.3} g_3\right) \end{pmatrix}$$

After this tedious preparation, the interchangeability formula for crossbar functions is available for implementation.

$$
\begin{pmatrix} \overleftarrow{CR}_1 & \overleftarrow{CR}_2 & \overleftarrow{CR}_3 \\ \overrightarrow{CR}_1 & \overrightarrow{CR}_2 & \overrightarrow{CR}_3 \end{pmatrix} :
$$

$$
\frac{
\begin{pmatrix} \overrightarrow{CR}_1 \, \square_{1.2} \, \overrightarrow{CR}_2 \\ \amalg_{1.2} \\ \overrightarrow{CR}_2 \, \square_{2.1} \, \overrightarrow{CR}_1 \\ \amalg_{2.3} \\ \overrightarrow{CR}_3 \end{pmatrix}
\begin{bmatrix} \circ_{1.2} \\ \circ_{2.1} \\ \circ_{3.3} \end{bmatrix}
\begin{pmatrix} \overleftarrow{CR}_1 \, \square_{1.2} \, \overleftarrow{CR}_2 \\ \amalg_{1.2} \\ \overleftarrow{CR}_2 \, \square_{2.1} \, \overleftarrow{CR}_1 \\ \amalg_{2.3} \\ \overleftarrow{CR}_3 \end{pmatrix}
}{
\begin{pmatrix} \left(\left(\overrightarrow{CR}_1 \circ_{1.1} \overleftarrow{CR}_1\right) \square_{1.2} \left(\overrightarrow{CR}_2 \circ_{1.2} \overleftarrow{CR}_2\right) \right. \\ \amalg_{1.2} \\ \left(\overrightarrow{CR}_2 \circ_{2.2} \overleftarrow{CR}_2\right) \square_{2.1} \left(\overrightarrow{CR}_1 \circ_{2.1} \overleftarrow{CR}_1\right) \\ \amalg_{2.3} \\ \left(\overrightarrow{CR}_3 \circ_{3.3} \overleftarrow{CR}_3\right) \end{pmatrix}
}
$$

1.4. Programming multi-layer crossbars

1.4.1. A conceptual sketch for a design of a multi-layered processor system

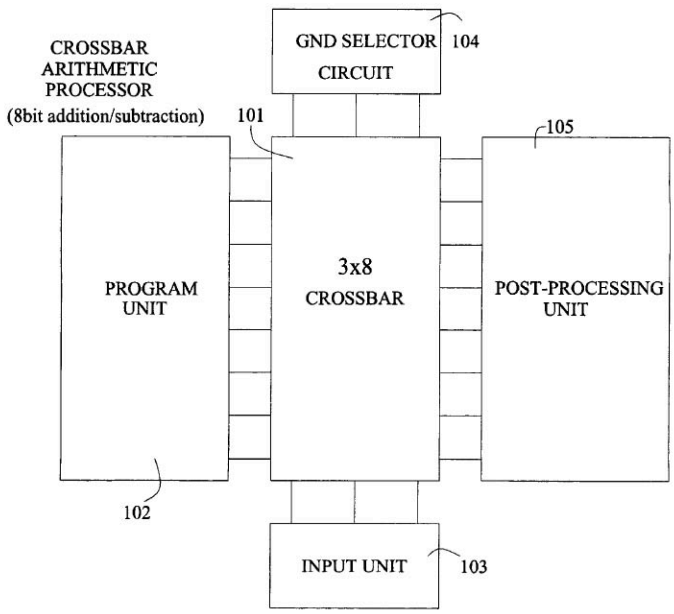
What appears now as a new challenge is the handling or programming of the dynamics of crossbars.
As much as the single crossbar has to be programmed, multi-layer systems has to be programmed too.

But in contrast to the programming of the classical multi-layer system in the framework of existing mono-contextural programming languages os a parallel processing unit, the new concept of a dynamic multi-layer systems has to be adequately programmed by programming languages and strategies based on polycontextural and morphogrammatic approaches, or similar. An interesting operator in polycontextural logic is introduced by "*transductions*". In contrast to logical junctions (conjunction, disjunction, implication), transjunctions are acting logically between different contextures, which might be modeled as different memristive crossbar layers.

> *"The results of modern brain research and the theory of poly-contexturality as a 'general theory of living systems' indicate the direction of future computer architecture which possibly may be designed on a molecular electronic basis. It can only be considered a first step that on a molecular level the old 'silicium structures' in their rigorous binarity are copied and miniaturized simply for purposes of optimizing quantities such as speed, size, etc. For a future computer science, founded on the basis of molecular electronics, the realization of a computer architecture has to be envisaged which models the dialectic and self-referential structure of matter as they appear, for example, in the brain."*
> R.Kaehr, E. von Goldammer, Again Computers and the Brain, Journal of Molecular Electronics Vol. 4 S31-S37 (1988)
> http://works.bepress.com/thinkartlab/28

A conceptual sketch for a design of a multi-layered processor system
An example might be given with a distribution of Blaise Laurent Mouttet's *Crossbar Arithmetic Processor*, 2006 .
www.freepatentsonline.com/y2007/0233761.html

An arithmetic processing system is taught to be formed by combining a crossbar array with programming circuitry, input circuitry, and post-processing circuitry. The programming circuitry is configured to set crosspoints of the crossbar array to either a relatively high conductivity or a relatively low conductivity state corresponding to a logic 1 or logic 0, thereby programming at least one programmed numerical value into the crossbar array. The input circuitry provides a binary input representative of an input numerical value to columns of the crossbar array. The post-processing circuitry converts an analog output vector produced from the rows of the crossbar array into a binary output representative of an output numerical value mathematically related to the at least one programmed numerical value and the input numerical value.



Simple mechanism for a distribution and mediation of crossbar arithmetical processors of a multi-layered system based on the interchangeability properties of multi-layered crossbar systems

.

Crossbar Arithmetical Processor = CAP
Program unit = PU,
Crossbar =CB
Input unit= IU

One of the basic relation in a crossbar arithmetic processor (CAP) seems to be the relation between programming unit (PU) and the crossbar (CR) itself. Hence, a very first conceptual idea to develop a genuine multi-layered CAP as a $CAP^{(m)}$ certainly would be achieved by a distribution of the PUs and CRs. Such an architecture obviously has at least to realized the conditions of interchangeability of its distributed and mediated functionalities. The distributed behaviors have to be mediated, otherwise it would produce a kind of a disjunctively separated parallelism.

Hence, at first, the interchangeability of the system (PU, CR) over the different layers available has to be implemented.
What has to be realized is the simple but necessary mechanism of bifunctoriality between CAP-functions and crossbar layers where the activity is localized and processed.

Thus, a composition ($\circ_{1.2}$) of 2 combinations ($\amalg_{1.2}$) , the mediative combination of PU_1 and PU_2 , and a same combination ($\amalg$) of CR_1 with CR_2, is realizing an equivalence (=) if a combination ($\amalg_{1.2}$) of the both compositions ($\circ_{1.2}$), PU_1 composed ($\circ_1$) with CR_1, and PU_2 composed ($\circ_2$) with CR_2, holds.

In a different category-theoretical style, this formula gets a commutative diagram, omitted here.

Hence, a 'double' crossbar arithmetic processor 2-CAP or $CAP^{(2)}$ we are dealing with a double program unit $PU^{(2)}$ and a double crossbar $CR^{(2)}$. The structure of the general interplay between $PU^{(2)}$ and $CR^{(2)}$ is shown by the formula below.
Obviously, there are 4 parts involved: 2 PUs and 2 CRs. Hence enough to construct a reasonable interchangeability between the parts constituting the whole system.

$$\begin{pmatrix} Layer_2 \\ Layer_1 \end{pmatrix}, \begin{pmatrix} CR_1 & CR_2 \\ PU_1 & PU_2 \end{pmatrix}:$$

$$\begin{pmatrix} PU_2 \\ \amalg_{1.2} \\ PU_1 \end{pmatrix} \circ_{1.2} \begin{pmatrix} CR_2 \\ \amalg_{1.2} \\ CR_1 \end{pmatrix} = \begin{pmatrix} PU_2 & \circ_2 & CR_2 \\ & \amalg_{1.2} & \\ PU_1 & \circ_1 & CR_1 \end{pmatrix}$$

$\amalg$: mediation between contextures
$\circ$: composition of morphisms
$=$: equivalence

$\forall f_i, g_i \in Layer\,\mathcal{U}_i,\ i \in \mathcal{N},\ i = 1, 2$ et
$\forall i \neq j:\ Layer\,\mathcal{U}_i \bigcap Layer\,\mathcal{U}_j = \emptyset$

If we want to emphasizes the internal interchangeability of the two-layered crossbar processor CR $^{(2)}$as part of the processor system, we simply have to ‘zoom in’ into CR $^{(2)}$.

Hence, $CR_1 = \overrightarrow{CR}_1, \overleftarrow{CR_1}$ and
$CR_2 = \overrightarrow{CR}_2, \overleftarrow{CR_2}$

$$\begin{pmatrix} Layer_2 \\ Layer_1 \end{pmatrix}, \begin{pmatrix} \overleftarrow{CR_1} & \overleftarrow{CR_2} \\ \overrightarrow{CR_1} & \overrightarrow{CR_2} \end{pmatrix}:$$

$$\begin{pmatrix} \overrightarrow{CR_2} \\ \amalg \\ \overrightarrow{CR}\ _1 \end{pmatrix} \circ\Diamond \begin{pmatrix} \overleftarrow{CR_2} \\ \amalg \\ \overleftarrow{CR_1} \end{pmatrix} = \begin{pmatrix} \overrightarrow{CR_2} & \circ & \overleftarrow{CR_2} \\ \amalg & \Diamond & \amalg \\ \overrightarrow{CR_1} & \circ & \overleftarrow{CR_1} \end{pmatrix}$$

$$\begin{pmatrix} \text{Layer}_2 \\ \text{Layer}_1 \end{pmatrix}, \left[\underset{PU_1}{\begin{pmatrix} \overleftarrow{CR}_1 \\ \overrightarrow{CR}_1 \end{pmatrix}} \; \underset{PU_2}{\begin{pmatrix} \overleftarrow{CR}_2 \\ \overrightarrow{CR}_2 \end{pmatrix}} \right]:$$

$$\begin{pmatrix} PU_2 \\ \amalg_{1.2} \\ PU_1 \end{pmatrix} \circ_{1.2} \begin{pmatrix} \overrightarrow{CR}_2 \\ \amalg \\ \overrightarrow{CR}_1 \end{pmatrix} \circ_{1.2} \diamond_{1.2} \begin{pmatrix} \overleftarrow{CR}_2 \\ \amalg \\ \overleftarrow{CR}_1 \end{pmatrix} =$$

$$\begin{pmatrix} PU_2 & \circ_2 & \left(\overrightarrow{CR}_2 \circ_2 \overleftarrow{CR}_2 \right)_2 \\ & \amalg_{1.2} & \amalg \quad \diamond_{1.2} \quad \amalg \\ PU_1 & \circ_1 & \left(\overrightarrow{CR}_1 \circ_1 \overleftarrow{CR}_1 \right)_1 \end{pmatrix}$$

The program unit (UP) which is programming the arithmetical processor needs a pre-programming unit (prPU), that is responsible for the management of the different arithmetics of the different input units (IU). It is supposed that their inputs are disjunct but mediated too. This pre-programming is one of the new serious costs. Similar to the costs for the management of parallel programming.

The interchangeability scheme is easily extended for complexity and complication for all parts included in a crossbar arithmetic processor system.

Scheme for 3 – layers and 3 – units with $\left[\amalg, \circ, \diamond\right]$

m = n = 3

$$\begin{bmatrix} q_1 & q_2 & q_3 \\ p_1 & p_2 & p_3 \\ h_1 & h_2 & h_3 \end{bmatrix} :$$

$$\begin{pmatrix} h_3 \\ \amalg_{0.2\times.3} \\ h_2 \\ \amalg_{1.2\times.0} \\ h_1 \end{pmatrix} \circ_{1.2\times.3} \begin{pmatrix} p_3 \\ \amalg_{0.2\times.3} \\ p_2 \\ \amalg_{1.2\times.0} \\ p_1 \end{pmatrix} \circ_{1.2\times.3} \begin{pmatrix} q_3 \\ \amalg_{0.2\times.3} \\ q_2 \\ \amalg_{1.2\times.0} \\ q_1 \end{pmatrix}$$

$$\begin{pmatrix} (h_3 & \circ_{0.0\times.3} & p_3 & \circ_{0.0\times.3} & q_3) \\ \amalg_{0.2\times.3} & \diamond_{0.2\times.3} & \amalg_{0.2\times.3} & \diamond_{0.2\times.3} & \amalg_{0.2\times.3} \\ (h_2 & \circ_{0.2\times.0} & p_2 & \circ_{0.2\times.0} & q_2) \\ \amalg_{1.2\times.0} & \diamond_{1.2\times.0} & \amalg_{1.2\times.0} & \diamond_{1.2\times.0} & \amalg_{1.2\times.0} \\ (h_1 & \circ_{1.0\times.0} & p_1 & \circ_{1.0\times.0} & q_1) \end{pmatrix}$$

The same conditions holds for the output unit, i.e. the post-processing unit (poPU), it needs a post-poPU to be able to manage the multi-layered, probably interacting, outputs delivered as a result of the programming units ($PU^{(m)}$), the input units ($IU^{(m)}$) and the multi-layered crossbar processor ($CR^{(m)}$).

This is not *concept art* as we know it, it is *paradigm design,* disseminated by the ThinkArt Lab.

1.5. Problems of realizations

It seems to be straight forward to understand the possibility to realize mono-contextural operators physically. As Bo Coecke emphasized with much clearness, operators like compositions and yuxtapositions are easily be found in physical systems. The operator “composition” obviously is composing sequential physical processes, while “yuxtaposition” is at work with parallel processes. Both, sequential and parallel processes, are quite ubiquitous.

Other operators, which had been introduced in polycontextural category theory, like replication, permutations, bifurcations are accessible too, assumed the mechanisms of mediation are known and realized. But this is just the main obstacle to understand how to realize polycontextural systems as distributed and mediated systems.

Many example to understand and realize mediation had been proposed, and are working in the defined domain of their introduction.

A new chance to realize beyond simulation and emulation features of mediation are opened up by the concepts and realizations of memristive systems.

1.5.1. Theoretical conditions of realizations

Matching conditions for composition and yuxtaposition (Coecke)

Primitive data :

processes / **operations** : f, g, h, ...

which are **typed** as $A \longrightarrow B,\ B \longrightarrow C,\ A \longrightarrow A,\ \ldots$

where A, B, C, ... are kinds / **names** of systems.

Primitive connectives

Sequential composition is a primitive connective on processes

$$f \circ g : A \longrightarrow C \text{ for } f : A \longrightarrow \underline{B} \ \& \ g : \underline{B} \longrightarrow C$$

Parallel composition is a primitive connective both on systems and processes

$$f \bigotimes g : A \otimes C \longrightarrow B \otimes D \text{ for } f : A \longrightarrow B \ \& \ g : C \longrightarrow D$$

Conditions of mediation

$$\amalg\left(f_1, g_1, f_2, g_2\right) \in$$

$$\text{Mediation of } \mathcal{U}_1 \text{ and } \mathcal{U}_2,\ \left[\mathcal{U}_1, \mathcal{U}_2\right],$$

with $f_{1,}\, g_1 \in \mathcal{U}_1$ and $f_2, g_2 \in \mathcal{U}_2$

iff

1. Composition $\left(\circ_{1.2}\right)$ in $\mathcal{U}_1$ and $\mathcal{U}_2$, i.e.

$$\left(f_1 \circ_1 g_1\right) \in \mathcal{U}_1,\ \left(f_2 \circ_2 g_2\right) \in \mathcal{U}_2 \ \left(\text{OrdRel}\right)$$

$$\Longrightarrow \left(\left(f_1 \circ_1 g_1\right), \left(f_2 \circ_2 g_2\right)\right) \in \left[\mathcal{U}_1, \mathcal{U}_2\right]$$

2. Cross – interchange $(\diamond)$ between $\mathcal{U}_1$ and $\mathcal{U}_2$

$$\left(\left(f_1 \diamond g_2\right), \left(f_2 \diamond g_1\right)\right) \in \left[\mathcal{U}_1, \mathcal{U}_2\right] \quad \left(\text{ExchRel}\right)$$

3. $\left(\left(f_1 \cong f_2\right), \left(g_1 \cong g_2\right)\right) \in \left[\mathcal{U}_1, \mathcal{U}_2\right] \quad \left(\text{CoincRel}\right).$

Hence,

$$\begin{pmatrix} f_2 \\ \amalg \\ f_1 \end{pmatrix} \circ\diamond \begin{pmatrix} g_2 \\ \amalg \\ g_1 \end{pmatrix} = \begin{pmatrix} f_2 & \circ & g_2 \\ \amalg & \diamond & \amalg \\ f_1 & \circ & g_1 \end{pmatrix}.$$

With $\blacksquare \equiv \left(\amalg \quad \diamond\right)$ simplified to :

$$\blacksquare\left(f_1, g_1, f_2, g_2\right) = \begin{pmatrix} f_2 \\ \blacksquare \\ f_1 \end{pmatrix} \circ \begin{pmatrix} g_2 \\ \blacksquare \\ g_1 \end{pmatrix} = \begin{pmatrix} f_2 & \circ & g_2 \\ & \blacksquare & \\ f_1 & \circ & g_1 \end{pmatrix}.$$

Hence, instead of the yuxtapositional composition
we get a mediational composition for objects :

yuxtaposition : $f \otimes g : A \otimes C \longrightarrow B \otimes D$ for
$f : A \longrightarrow B$ & $g : C \longrightarrow D$, with objects in $\mathcal{U}_0$.

mediation : $f \blacksquare g : A_1 \blacksquare C_2 \longrightarrow B_1 \blacksquare D_2$ for
$f_1 : \left(A \longrightarrow B\right)_1$ med $g_2 : \left(C \longrightarrow D\right)_2$,
with objects in $\left[\mathcal{U}_1, \mathcal{U}_2\right]$.

Diagrams

For yuxtaposition

objects	functions	pattern
$\begin{array}{ccc} C & \longrightarrow & D \\ & \otimes & \\ A & \longrightarrow & B \end{array}$	$\begin{array}{ccc} f_2 & \circ & g_2 \\ & \otimes & \\ g_1 & \circ & f_1 \end{array}$	$\begin{array}{ccc} f_2 & \text{seq} & g_2 \\ \text{par} & & \text{par} \\ g_1 & \text{seq} & f_1 \end{array}$

For mediation :

objects	functions	pattern
$\begin{array}{ccc} D_2 & \longleftarrow & C_2 \\ \diamond & \cong & \diamond \\ A_1 & \longrightarrow & B_1 \end{array}$	$\begin{array}{ccc} f_2 & \circ & g_2 \\ \diamond & \cong & \diamond \\ g_1 & \circ & f_1 \end{array}$	$\begin{array}{ccc} f_2 & \text{OrdRel} & g_2 \\ \text{ExchRel} & \text{CoincRel} & \text{ExchRel} \\ g_1 & \text{OrdRel} & f_1 \end{array}$

Formula

$$\begin{pmatrix} \mathcal{U}_2 \\ \mathcal{U}_1 \end{pmatrix}, \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix}:$$

$$\begin{pmatrix} f_2 \\ \blacksquare \\ f_1 \end{pmatrix} \circ \begin{pmatrix} g_2 \\ \blacksquare \\ g_1 \end{pmatrix} = \begin{pmatrix} f_2 & \circ & g_2 \\ & \blacksquare & \\ f_1 & \circ & g_1 \end{pmatrix}$$

$\amalg$: mediation between contextures
$\circ$: composition of morphisms
$\diamond$: cross – interchange between levels
$\blacksquare \equiv (\amalg \quad \diamond)$
$=$: equivalence

$\forall f_i, g_i \in \text{Universe } \mathcal{U}_i, i \in \mathcal{N}$ et
$\forall i \neq j$: $\text{Universe } \mathcal{U}_i \bigcap \text{Universe } \mathcal{U}_j = \emptyset$

$\forall f_i \in \mathcal{U}_i, g_j \in \mathcal{U}_j, \; i \neq j = 1, 2$
$cod(f_2) \simeq dom(g_1)$
$cod(f_1) \simeq dom(g_2)$

$\forall f_i, g_i \in \mathcal{U}_i, \; i = 1, 2$
$cod(f_1) \cong dom(g_1)$
$cod(f_2) \cong dom(g_2)$

1.5.2. Physical realizations

Composition and yuxtaposition

Physical realizations of composition and yuxtaposition, and similar operations, are obvious, because their data are belonging to the same set of objects of a unique universe $\mathcal{U}$. Because data as signals appears concretely as highly different in quality (sound, images, text, ect.), the abstraction of *information* is unifying different ontological and semantical domains into the general domain or universe of informatic objects (information).

Mediation

Mediation is a form of interactivity between discontextural domains

(universes).
Interactivity is prior to information processing.
Hence, the mechanism of a “fusion” or “combination” of *logic* and *memory* has to be based on, at least, four distinctions to be able to function.
1. logic as logic,
2. memory as memory,
3. logic as memory,
4. memory as logic.
Therefore, a fusion of differences is conceptually perceived from an *external* point of view, where all four functionalities together seem to be one. Zooming into this structure uncovers the “*internal*” observation a chiastic dynamics of four functionalities.
Only if such a chiastic interplay is realized, something like data-processing is enabled.

1.6. Cloning HP’s hobby-horse “IMP"

1.6.1. Multi-layered systems and the destiny of implication

With the proposed hints and constructions it seems to be easy and even natural to distribute the logical connective *“material implication”* over different places in the kenomic matrix.

There is an application of implication between different layers of memristors. Is it necessary and reasonable to think that it is the same implication on all levels?
That is, does it make sense to think that there is one and only one logic and only one implication but many applications?
Is this not contradicting the attempt to *realize* a system in contrast to model it?

> *"The physical locations of the memristive devices are mapped to a four-dimensional logical address space such that unique access from the CMOS substrate is provided to every device in a stacked array of crossbars.”* (Williams, Strukov)

Implication plays at least two roles in memristive systems, one as a *realization* of memristor functionalities and one as an abstract, i.e. *conceptual* logical connective.
The difference of both is not considered in the literature. One reason is that there are no concepts to do it. The standard explication for such a difference is the difference of *theory and application* or *use/mention*. It is said, that the implications (and other logical connectives) are applied at different places, in

different situations, but they are always the very same concept of logical implication because there is no such thing as logically different implications. Material implication, e.g., is defined once and for ever by a set of rules or more traditionally by its truth table.

There are certainly different definitions for implications, two-valued, n-valued, fuzzy-valued, etc. But the above argument holds for each of them.

As long as we accept this kind of thinking there will never be a possibility to *realize* an implication at a specific place in a complex system. All such implications are simple applications and their epistemological status is that of being a *simulation*, in contrast to a realization.

Material implication got a revival by the memristor crossbar construction. This construction is extended to three-dimensional circuits.

Hence, two possible conceptualizations and therefore two different implementations or realizations are possible.

One is the decision for an abstract implementation where the fact that we are dealing with implications is depending on the interpretation of an *external* observer. Earlier on such a decision would have been called idealistic, in contrast to a materialist decision, where the realization is independent of an interpretation after it had been engineered and therefore implemented.

How could a realization of an implementation of a logical implication be observer-independent?
One simple answer is: we have to attribute the implication its place where in the system it is positioned, i.e. realized. Hence, in a tree-dimensional grid, a realization of an implication depends on its “place-value” defined by its *“place-designator”* in the three-dimensional grid.

Of the millions of implicative situations, the abstract solution is opting for *one and only one* concept of implication. The materialistic option is in principle opting for as many implications as necessary in a concrete realization. Hence, it is opting for the paradox conception of “one, but many” concept of a multitude of a single logical implication. Nobody is forced to build a formal logic with billions of material implications, even if there wouldn’t be a theoretical obstacle for that. It is probably enough to build by abstraction and classification an accessible set of *domains* (contextures), where each domain entails its own logical concept of material implication, and its resulting logic too.

The logic “behind” an application is not a concrete realization of the logic as such. Applications are hiding their background, realizations are inscribing and unmasking their background into the tissue.

With an option for polycontextural logic, both could be realized: A system of logic with multiple material implications only and a system of mixed logical connectives, conjunctions and others, too. One is delivering a dissemination of IMPL+Neg, $NIMPL^{(m, n)}$, the other a dissemination of realizations of $NIMPL^{(m, n)}$ and $NAND^{(m, n)}$ or $NOR^{(m, n)}$.

Again, at each place of a concrete realization of a logical connective, endless iterations of its *applications* are opened up - nothing is lost.

The whole narrative could have been compressed with introduction of the difference of logical *connective* and its *morphogram*.

This possibility of a mediation of different logical connectives in one complex computational logic grid might be of unforeseen importance for new chip designs. In such a case, multiple connectives are defined in parallel and are running simultaneously in a complexion. Additional to their own definition they have to fit into the logical architecture of the complexion, i.e. the compound logic and realizing conditions of mediation.

Hence, a dissemination of logical connectors, especially the material implication plus its negative value, offers two important informations: one is the place where it happens the other is the logic it is processing.

From the point of view of polycontextural systems theory, multi-layered systems have two options. One is to suppose a logical homogeneity of the compound system, hence applying one and only one logic for all layers. The second option is, to suppose a logical heterogeneity as a design for an implementation of different, but mediated logical systems for each layer or for different clusters of layers.

Hence, there are different options available to model “multi-layer crossbar arrays”.

The polycontextural strategy is to decompose the array into components with autonomous logics and then to mediated the components together to a polyconctextural compound systems with its specific compound logic and arithmetics.

1.6.2. Example of distributed implications

The highly abstract conceptual presentation of interchangeability of functions in categorical terms didn't yet contemplate on the hidden conditions of composition and mediation. Such conditions are necessary but they are also restrictive. That is, not every thing is composed and mediated per se.

Is there a realization for a mediation of implication on the diagonal of a kenomic matrix?
If we use, for reasons of simplicity, the classical definition of implication in a two-valued logic, the answer is no.
Why? Simply because the conditions of mediation are not fulfilled.

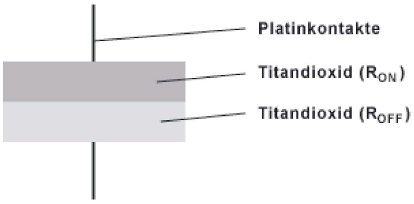
Polycontextural logic is not genuinely a multi-valued logic but much more a *place-valued logic*. But this is a historical constellation. Polycontextural logics, as generalizations and concretizations of place-valued logics, are distributing logical systems over the kenomic matrix. To function, conditions of mediation have to be applied. A simple approach to an understanding of the distribution mechanism is the given with the concept of fibering logical systems (Jochen Pfalzgraf). But this approach has additionally to take into account that the index set has to be at least to be tuples and not a simple set.

> *"The fiberings method is found to be very useful in modeling communication and interaction between cooperating agents, due to the possibility to switch between a local/global point of view which is inherent to this framework."*
> Pfalzgraf et al, Towards a General Approach for Modeling Actions and Change in Cooperating Agents Scenarios, 1996
> PFALZGRAF et al. Logic Jnl IGPL.1996; 4: 445-472 ,
> http://jigpal.oxfordjournals.org/cgi/content/abstract/4/3/445
> http://www.springerlink.com/index/7171543987226j53.pdf

The other idea is based on *morphogrammatics*. To accept, that two-valued-sequences (1211) and (1311) or also more suggestively, the sequences (3233) or (4144) are all representing a logical *implication* at different places, is based on the construct of the common morphogram for implication [IMP]. That is, the values are indicating a *place* in the matrix, and the structure or pattern of the sequence is defining the logical *functionality* of the logical connective, given by its morphogram.

Again, even on a logical level, what counts are not functions over identical elements, but *patterns* of behaviors in contexts and contextures.

Logical binarity of Ron and Roff, distributed over different loci of realization



A logical interpretation for the electronic polarity of R_{on} and R_{off} has to decide which situation is *prefered* befor the other. Hence, if R_{on} is designated as a positive state and R_{off} is designated as the negative state of the polarity, then the logical semantics is defined with $R_{on} \equiv$ true and $R_{of} \equiv$ false.

P M	O_1	O_2	O_3
M_1	Ron_1 $Roff_1$	–	–
M_2	□	Ron_2 $Roff_2$	□
M_3	–	□	Ron_3 – $Roff_3$

$$\begin{array}{ccccc} Ron_3 & \cong & Ron_1 & \longrightarrow & Roff_1 \\ \downarrow & & \updownarrow & x & \updownarrow \\ Roff_3 & \cong & Roff_2 & \longleftarrow & Ron_2 \end{array}$$

Distribution and mediation of Ron / Roff – binarity for 3 layers

$$\begin{bmatrix} Roff_1 & Roff_2 & Roff_3 \\ Ron_1 & Ron_2 & Ron_3 \end{bmatrix}:$$

$$\left(\begin{array}{c} \begin{pmatrix} Ron_1 \\ \amalg_{1.2\times.0} \\ Ron_2 \end{pmatrix} \\ \amalg_{1.2\times.3} \\ Ron_3 \end{array}\right) \begin{pmatrix} \circ_{1.2\times.3} \\ \diamond_{1.2\times.0} \end{pmatrix} \left(\begin{array}{c} \begin{pmatrix} Roff_1 \\ \amalg_{1.2\times.0} \\ Roff_2 \end{pmatrix} \\ \amalg_{1.2\times.3} \\ Roff_3 \end{array}\right) =$$

$$\left(\begin{array}{c} \begin{pmatrix} \left(Ron_1 \quad \circ_{1.0\times.0} \quad Roff_1\right) \\ \diamond_{1.2\times.0} \;\; \amalg_{1.2\times.0} \;\; \diamond_{1.2\times.0} \\ \left(Ron_2 \quad \circ_{0.2\times.0} \quad Roff_2\right) \end{pmatrix} \\ \amalg_{1.2\times.3} \\ \left(Ron_3 \quad \circ_{0.0\times.3} \quad Roff_3\right) \end{array}\right)$$

with $\blacksquare \equiv \left(\amalg \quad \diamond\right)$

One of the main problems in the understanding of a *technical* realization of the logical mechanism of *mediation*, here, as a simultaneity of interchanging $Roff_i$ and Ron_{i+1}, seems to be a step further to a solution within the possibilities of memristive systems. Multi-layered crossbar systems, “buffered” or not “buffered”, are enabling interactions on the nanosacle, which are not strictly determined and restricted by binary oppositions. Each layer might contain a ‘strict’ binarity but the compound system as a combination of different ‘binary’ systems in a multi-layered systems has not to be itself binary, neither multi-valued.

Rudolf Kaehr, PolyLogics. Towards a Formalization of Polycontextural Logics http://works.bepress.com/thinkartlab/25/

Diamond strategy

To fully realize the *complementary* and *antidromic* structure of memristive devices the concept of diamond strategies has to be involved. Hence, the antidromic system of diamonds has to be introduced even on the Ron/Roff-level of modeling. This is done by the system $S_4 : Ron_4 \longrightarrow Roff_4$. In words, the combination of two memristors is producing a combined behavior of

$S_1$and S_2 as the combination S_3. Such a combination is completed by its antidromic and complementary system S_4, realized as $Ron_4 \longrightarrow Roff_4$. S_4 functions as the complementary environment of S_3.
Hence,

the nucleus of a memristive crossbar architecture is composed of diamond –w structured memristive systems
here their internal structures are chiastic.

$$
\begin{array}{ccccccc}
Ron_3 & \cong & Ron_1 & \longrightarrow & Roff_1 & \cong & Roff_4 \\
\downarrow & & \updownarrow & x & \updownarrow & & \uparrow \\
Roff_3 & \cong & Roff_2 & \longleftarrow & Ron_2 & \cong & Ron_4
\end{array}
$$

Diamond of Ron / Roff – binarity for 3 layers, 1 environment

$$m = 3,\ n = 2,\ \text{env} = 1$$

$$\left(\begin{array}{c|c} \mathcal{U}_1 & \\ \mathcal{U}_2 & \mathcal{U}_4 \\ \mathcal{U}_3 & \end{array}\right),\ \left[\begin{array}{ccc|c} \text{Roff}_1 & \text{Roff}_2 & \text{Roff}_3 & \text{Roff}_4 \\ \text{Ron}_1 & \text{Ron}_2 & \text{Ron}_3 & \text{Ron}_4 \end{array}\right]:$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \text{Ron}_1 \\ \blacksquare_{1.2\times.0} \\ \text{Ron}_2 \end{array}\right) \\ \amalg_{1.2\times.3\times.4} \\ \text{Ron}_3 \,\Big|\, \text{Ron}_4 \end{array}\right) \left(\begin{array}{c} \circ_{1.2\times.3\times.0} \\ \diamond_{1.2\times.0\times.0} \\ \shortparallel_{0.0\times.0\times.4} \end{array}\right) \left(\begin{array}{c} \left(\begin{array}{c} \text{Roff}_1 \\ \blacksquare_{1.2\times.0} \\ \text{Roff}_2 \end{array}\right) \\ \amalg_{1.2\times.3\times.4} \\ \text{Roff}_3 \,\Big|\, \text{Roff}_4 \end{array}\right) =$$

$$\left(\begin{array}{c} \left(\begin{array}{c} \left(\text{Ron}_1 \ \circ^{1.0\times.0}\ \text{Roff}_1\right) \\ \blacksquare_{1.2\times.0} \\ \left(\text{Ron}_2 \ \circ^{0.2\times.0}\ \text{Roff}_2\right) \end{array}\right) \\ \amalg_{1.2\times.3\times.4} \\ \left(\text{Ron}_3 \circ_{0.0\times.3\times.0} \text{Roff}_3\right) \Big| \left(\text{Ron}_4 \shortparallel_{0.0\times.0\times.4} \text{Roff}_4\right) \end{array}\right)$$

$\shortparallel$: saltisition, $\amalg$: mediation, $\diamond$: cross – interchange, $\blacksquare \equiv (\amalg \ \diamond)$

It seems that memristics is offering better realizations of a new paradigm of computation than other approaches, like quantum, optical, molecular or DNA computing (Paun, Rozenberg, Salomaa, Deutsch) which are all embedded into the general paradigm of computability defined by Church-Rosser-Turing-Markov, etc.

Distribution of implications

PM	O_1	O_2	O_3
M_1	$IMP_{1.1}$	–	–
M_2	–	$IMP_{2.2}$	–
M_3	–	–	$IMP_{3.3}$

⇒

PM	O_1	O_2	O_3
M_1	12 11	–	–
M_2	–	23 22	–
M_3	–	–	13 11

$$\begin{array}{lcl} head_1 & = & head_3 \\ head_2 & \neq & tail_1 \\ tail_2 & = & tail_3 \end{array}$$

Diagonal values are violating the matching conditions.

[id, repl, repl]	O_1	O_2	O_3
M_1	$IMP_{1.1}$	–	–
M_2	$IMP_{1.2}$	–	–
M_3	$IMP_{1.3}$	–	–

⇒

[impl]	O_1	O_2	O_3
M_1	12 11	–	–
M_2	12 11	–	–
M_3	12 11	–	–

accepted for

$$\begin{array}{lcl} head_1 & = & head_3 \\ head_2 & = & tail_1 \\ tail_2 & = & tail_3 \end{array}$$

[id, repl, repl]	O_1	O_2	O_3
M_1	$IMP_{1.1}$	–	–
M_2	$IMP_{2.1}$	–	–
M_3	–	–	$MP_{3.3}$

⇒

[iimpl]	O_1	O_2	O_3
M_1	12 11	–	–
M_2	12 11	–	–
M_3	–	–	13 11

accepted for

$$\begin{array}{lcl} head_1 & = & head_3 \\ head_2 & = & tail_1 \\ tail_2 & = & tail_3 \end{array}$$

The conditions of mediation (matching conditions) are defined in terms of the head and tail of a function.

1.6.3. Example of distributed negations

As it is well know, logical implication alone is not building a complete set for logical connective. Only in concert with negation, implication is able to replace NAND, i.e. conjunction and negation. Hence, it is not IMP, but NIMP which is replacing NAND or NOR.

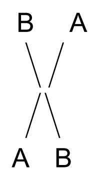
How are negations defined in distributed logical systems? As implications are taking place at a locus in a kenomic matrix, negations too are localized. Hence, a negation is not reduced to a function which is changing a value of a function only, but it is changing the places of subsystems of the compund system too.

Classical negation is connected to permutation. A nice example of permutation is given in the context of yuxtaposition as symmetry.

Symmetry

$\sigma_{A,B} : A \otimes B \longrightarrow B \otimes A$

$\sigma_{B,A} \circ \sigma_{A,B} = 1_{A,B}$



Symmetries in mediated compound systems are similarly constructed like permutations in monoidal categories. The main point to consider is that such formalisms are not equivalent to group-theoretical approaches, say of negations in many-valued logics.

Definitions of 3 – contextural symmetries

$$\sigma_{1\,(A,B,C)} : \begin{pmatrix} X_{1.1} \\ \sigma_1(\amalg_{1.2}) \\ X_{2.2} \\ \sigma_1(\amalg_{2.3}) \\ X_{3.3} \end{pmatrix} \longrightarrow \begin{pmatrix} \overline{X_{1.1}} \\ \amalg_{1.3} \\ X_{3.3} \\ \amalg_{3.2} \\ X_{2.2} \end{pmatrix}$$

$$\sigma_{2\,(A,B,C)} : \begin{pmatrix} X_{1.1} \\ \sigma_2(\amalg_{1.2}) \\ X_{2.2} \\ \sigma_2(\amalg_{2.3}) \\ X_{3.3} \end{pmatrix} \longrightarrow \begin{pmatrix} X_{3.3} \\ \amalg_{1.3} \\ \overline{X_{2.2}} \\ \amalg_{3.2} \\ X_{1.1} \end{pmatrix}$$

If the path or journeys in mediated compound systems comes into focus, theories of Hamilton cycles have to be considered.
http://www.thinkartlab.com/pkl/lola/Diamond Relations/Diamond Relations.pdf

Rules of 3 – contextural symmetries (permutations)

$$\sigma_{1\,(A,B,C)} : (X_{1.1}\ \amalg_{1.2}\ X_{2.2}\ \amalg_{2.3}\ X_{3.3}) \longrightarrow (\overline{X_{1.1}}\ \amalg_{2.3}\ X_{3.3}\ \amalg\ X_{2.2})$$

$$\sigma_{2\,(A,B,C)} : (X_{1.1}\ \amalg_{1.2}\ X_{2.2}\ \amalg_{2.3}\ X_{3.3}) \longrightarrow (X_{3.3}\ \amalg_{1.3}\ \overline{X_{2.2}}\ \amalg_{2.3}\ X_{1.1})$$

$$\sigma_{1\,(B,A,C)} \circ \sigma_{1\,(A,B,C)} = \mathbf{1}_{1\,(A,B,C)}$$

$$\sigma_{2\,(A,C,B)} \circ \sigma_{2\,(A,B,C)} = \mathbf{1}_{2\,(A,B,C)}$$

$$\sigma_{1\,(A,B,C)} \circ \sigma_{2\,(A,B,C)} \circ \sigma_{1\,(A,B,C)} = \sigma_{2\,(A,B,C)} \circ \sigma_{1\,(A,B,C)} \circ \sigma_{2\,(A,B,C)}$$

$$\sigma_{1\,(A,B,C)} \circ \sigma_{2\,(A,B,C)} \circ \sigma_{1\,(A,B,C)} \circ \sigma_{2\,(A,B,C)} \circ \sigma_{1\,(A,B,C)} \circ \sigma_{2\,(A,B,C)} = 1_{1.2 \times .3\,(A,B,C)}$$

1.6.4. Non-transitivity of implicational systems

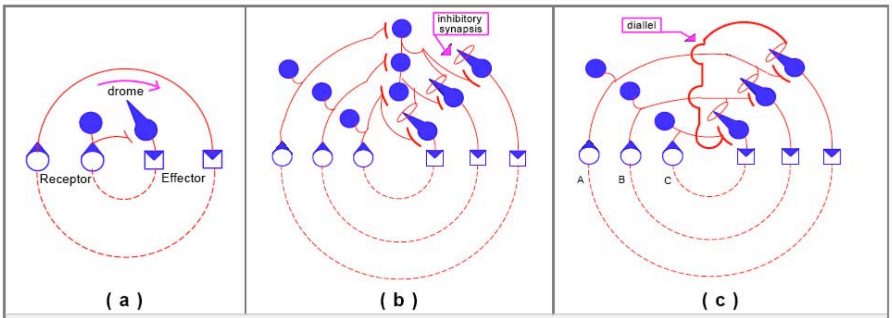


Figure 1: Neural nets, from Ref. 8
continuous line : nervous portion of path;
broken line : nervous somatic or environmental portion of path.

(a)	(b)	(c)
hierarchic and monocontextural	parallel and hierarchic $(A \to B) \wedge (B \to C) \to (A \to C)$	heterarchic (non-transitive) $(A \to B) \wedge (B \to C) \to (A \leftarrow C)$

Repeating some old insights about intransitivity of implications in neuromorph systems.

$$L^{(3)} : \begin{cases} L_1 : \left(\left(A \longrightarrow B\right) \wedge \left(B \longrightarrow C\right)\right) \Longrightarrow \left(A \longrightarrow C\right) \\ L_2 : \left(\left(A \longrightarrow B\right) \wedge \left(B \longrightarrow C\right)\right) \Longrightarrow \left(A \longrightarrow C\right) \\ L_3 : \left(\left(A \longleftarrow B\right) \wedge \left(B \longleftarrow C\right)\right) \Longrightarrow \left(A \longleftarrow C\right) \end{cases}$$

$$L^{(3)}_{(\mathrm{IMP,IMP,REP})} : \left(A \longrightarrow B\right)_1 \wedge^{(3)} \left(B \longrightarrow C\right)_2 \Longrightarrow^{(3)} \left(A \longleftarrow C\right)_3$$

$$: \left(A \longrightarrow B\right)_1 \wedge^{(3)} \left(B \longrightarrow C\right)_2 \wedge^{(3)} \left(\mathrm{neg}_3\, A \longrightarrow \mathrm{neg}_3\, C\right)_3.$$

> *"If, for example, the preference relation between the statements A, B and C about the nervous net in Fig.lc is given in three contextures as then the transitivity rule holds strictly in each of the three contextures. However, if the implication chain starts in the logical domain L_1(or L_2) and continues with a change of the contexture, say in L_3, then non-transitivity occurs trans-contexturally in the chain of implication without antinomy."* (Kaehr, von Goldammer, 1988)

Unfortunately, the new *"Neuromorphic Circuit Based on Memristor Synapses"*- movement is not aware about the importance of non-transitive, heterarchical and self-referential features of brain activities as they have been pointed out as early as 1945 with Warren McCulloch's fundamental papers. The empirical results of intransitivity still don't fit into the paradigm of computionalism.

Again, what is the meaning of *logic* and *memory* in the slogan of a *simultaneity of logic and memory?*