DERRIDA'S MACHINES PART III

BYTES & PIECES

of

PolyLogics, m-Lambda Calculi, ConTeXtures

PolyLogics.

Towards A Formalization of Polycontextural Logics

© by Rudolf Kaehr

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"Interactivity is all there is to write about:
it is the paradox and
the horizon of realization."

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PolyLogics.

Towards A Formalization of Polycontextural Logics

A. Rhizomatics for PolyLogics

B. General Framework of PolyLogics

C. Tableaux rules for PolyLogics

D. Unification for PolyLogics

PolyLogics*

Towards A Formalization of Aspects of Polycontextural Logics

*Sponsored, partly, by the German Software ThinkTank **ALGoLL AG**, Munich, Germany.

Rhizomatics for PolyLogics

PolyLogics are a special thematization and formalization of the general idea of polycontextural logics. There is a nice little book *"Logic with Trees"* from Colin Howson. PolyLogics in addition, could be called "Logics with Forrests" or *"Logics in Rhizomes"* to connect to the tradition of tree-based methods.

General Strategies

Polycontextural complexions are composed by components, contextures. Complexions are compounds of components. Compounds can be de-composed into components. Compositions are *super-additive* in respect to their components.

In contrast to logocentric systems polycontexturality is involving complexity at the very beginning, not as a static decision but as a dynamic exchange between its fundamental architectonics and its metamorphic futures.

Metaphors and Motivations

PolyLogics are not based on propositions and their simple alphabet (atomicity, linearity), reducible to a one sign and a nil sign alphabet, based linguistically on speech but on written forms, hieroglyphic, like Chinese writing with its tabularity, sign-complexity, contextuality, interpretability and dynamic multitude from the very beginning.

PolyLogics, in this picture, are the logics of Chinese writing. Dialectics.

> **How many characters?**
> The Chinese writing system an open-ended one, meaning that there is no upper limit to the number of characters. The largest Chinese dictionaries include about 56,000 characters, but most of them are archaic, obscure or rare variant forms.
>
> **Strokes**
> Chinese characters are written with the following twelve basic strokes:
> Basic strokes which are combined to make up all Chinese characters.

The strokes themselves are not characters, thus, they don't have a meaning. They are the conditions of the possibility of meaning at all. In polycontextural terms they are not signs but kenograms inscribing morphograms which can be thematized, interpreted, transformed into logical meanings. First are the written characters, then the phonetic interpretations. But this alone wouldn't reflect the chiasm between writing and speech properly. The written characters can contain some phonetic elements; and the meaning of the written forms has to be negotiated.

> http://www.omniglot.com/writing/chinese.htm
> http://home.vicnet.net.au/~ozideas/writchin.htm
> http://cjvlang.com/Writing/writsys/writlinks.html

> **The Textual Dance: Allusion in the Oldest and Newest Poetry**
> A single Sumerian sign may have five, ten, twenty or more values. But the traditional and historical method of reading those signs ignores that multi-valent quality and instead of plasticizing and expanding meaning, traditional translation hardens meaning. Traditional readings do this by building a transliteration, a version of the text in which specific and singular meanings are assigned to each of the signs, establishing a single definitive or "Ur" text.
> http://resonantconcept.typepad.com/experimental/2003/11/the_textual_dan.html

PolyLogics are fundamentally non-fundamentalist. There is no Ur-origin, only multitudes of beginnings and ends.

1 Problems with logic and the logics of problem soving

1.1 A motivational scenario

Why not simply asking the experts from the MIT?

The Panalogy Principle: If you 'understand' something in only one way then you scarcely understand it at all—because when something goes wrong, you'll have no place to go. But if you represent something in several ways, then when one of them fails you can switch to another. That way, you can turn things around in your mind to see them from different points of view —until you find one that works well for you now. And that's one of the things that ‘thinking” means!

We cannot expect much resourcefulness from a program that uses one single technique—because if that program works in only one way, then it will get stuck when that method fails. However, a program with multiple ‘ways to think’ could behave more like a person does: whenever you get frustrated enough, then you can switch to a different approach—perhaps through a change in emotional state.

Marvin Minsky

Implementing Panalogy

I will use the term Panalogy to refer to a family of techniques for synchronizing and sharing information between different ways of thinking concerned with the same or similar problems. The term derives from ‘parallel analogy’. By maintaining panalogies between ways of thinking, we can rapidly switch from one way of thinking to another.

We can also make more partial changes like the representation language they are using, the types of assumptions they are making, the methods that are available to them for solution, and so forth. The key idea is to support representing multiple problem solving contexts simultaneously and the links between them. A graphical depiction of panalogy at work is shown below.

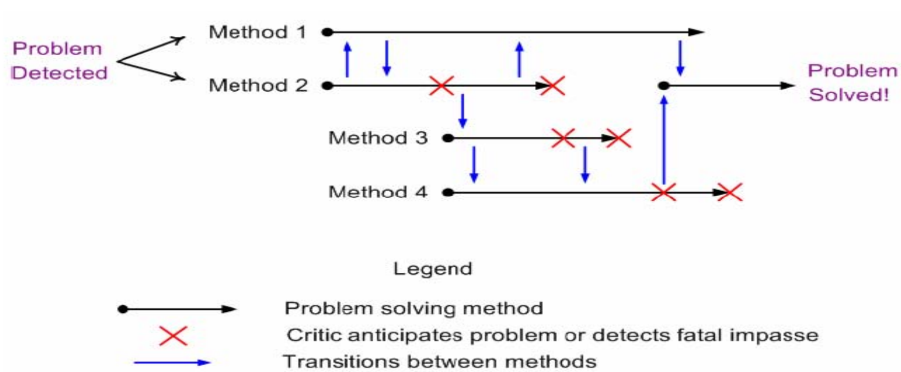


Figure 2. Switching between parallel methods or ways of thinking.

But is this exactly what I am looking for? Obviously not. To have the same wording and to have the same diagram doesn’t yet mean that we are thematizing the same situation in the same way of thinking and implementation.

The main difference between panalogy and PolyLogics is this. Panalogy is mono-contextural, always only one method is running, not several at once and there is no interactivity and reflectionality between successively different methods. They are applied only one after the other. If one method doesn’t work, take another.

PolyLogics with its proemiality is ruling the interplay of different methods running and cooperating together at once.

> Marvin Minsky offered the Six Level Model from his forthcoming book *The Emotion Machine* as an initial proposal for such an architecture. This architecture is being developed jointly by himself and Aaron Sloman, and is based on several key ideas:
>
> *1. Use several approaches, at once, to each problem.*
>
> *2. Have many ways to recognize and respond to internal and external problems.*
>
> *3. Support many different "ways of thinking".*
>
> Marvin Minsky, Push Singh, MIT, May 13, 2002

The wording *"Switching between parallel methods of thinking"* sounds quite promising, but it doesn't gives us a hint *how* the switch is working, what is the mechanism of the switch, and, how do we know that we are dealing with the same problem after the switch to another domain. How much is the problem itself transformed by the switch of context? And what is the notion of sameness involved in this switch? What do we mean by "parallel" in this context?

> *I will use the term Panalogy to refer to a family of techniques for synchronizing and sharing information between different ways of thinking concerned with the same or similar problems.*

The common term between the different domains of panalogy is obviously *information*. But how can we know that all the domains are ruled by the very same concept of information? Why is the term information not in itself panalogical?

> *By maintaining panalogies between ways of thinking, we can rapidly switch from one way of thinking to another.*

This sounds really good! But, again, *how* does it work and who is operating these deliberating switches?

Minsky´s question is "*What* could cause the change?" and not "*How* does it happen?" or "What is the *mechanism* of change?"

The key idea is to support representing multiple problem solving contexts simultaneously and the links between them.

Different thematization, different strategies

The complementary aspect of Minsky´s approach to the polycontextural approach is expressed by the statement

> *"We'll try to **design** (as opposed to **define**) machines that can do all those 'different things'."* Minsky

The question of definition is a logical one, the process of design belongs to the domains of modeling, simulation, implementation and not to formalization.

It seems not to be easy to escape the challenge of logics. All the tools and methods of design, programming languages, LISP obviously too, are based on logic. The same is the case for the machines.

Why should the process of design be restricted by the structure of its classical tools?

http://web.media.mit.edu/~push/Push.Phd.Proposal.pdf
http://www.thinkartlab.com/pkl/media/DERRIDA/Panalogy.html

Peter Wegner on logics

Because of my focus on foundational studies my realization of the category of explanation is worked out in a more philosophical sense, and the category of implementation too, is more foundational than empirical, that is, it is implementing new formalisms and programming languages with the help of today´s monolitical methods (existing programming languages) on monolitical machines.

> *"I first presented the idea that Turing machines cannot model interaction at the 1992 closing conference of the Japanese 5th generation computing project, showing that the project´s failure to reduce computation to logic was due not to lack of cleverness on the part of logic programming researchers, but to theoretical impossibility of such a reduction. The key argument is the inherent trade-off between logical completeness and commitment. Commitment choice to a course of action is inherently incomplete because commitment cuts off branches of the proof tree that might contain the solution, and commitment is therefore incompatible with complete exploration." Wegner, ECOOP ´99, p.1/2*
>
> *http://www.cs.brown.edu/people/pw/*

As mentioned above, Wegners strategy to surpass this limiting situation is not to deliberate the paradigm of formality which is defining the very concept of logic and all the concrete logical systems, but some form of regression to empiricism.

> *"Logic can in principle be extended to interaction by allowing nonlogical symbols to be interactively modified (reinterpreted) during the process of inference, for example by updating a database of facts during the execution of logic programs. However, interactive discovery of facts negates the monotonic property that true facts always remains true."* Wegner, p. 25

This strategy of extending logical systems by non-logical symbols for modeling interaction introduces into logic some non-logical elements of empiricism. For practical reasons this approach has its merits. Nevertheless, from a structural point of view of operativity and formality nothing has changed. Still the old logic is ruling the situation and dictating the possibilities of design.

http://www.cse.uconn.edu/~dqg/papers/#interaction

The polycontextural approach

Thus, we can distinguish 3 levels of reflection or even 3 different paradigm to computation and logic:

1. The *classic* paradigm of logic, arithmetics, computation, etc.

2. The *interactive* paradigm (Peter Wegner, Marvin Minsky, Rolf Pfeifer, et al). With themes of interaction, morphologic computation based on a new interpretation of computing reality in modeling and design, accepting the first paradigm of formality, but correcting it with new more realistic models based in empirical approaches. This paradigm is more heterogeneous and interactive than the first but doesn't have any formalism to mediate the interacting heterarchic parts of the complex system in an operativ way. This approach is still descriptive and not operative.

3. The *polycontextural* paradigm which accepts both previous paradigms in their context, but is trying to surpass the very limits of operativity and formality posed by the first paradigm. Polycontexturality tries to keep the level of operativity of the classic paradigm but concerns with needs of a operative theory of mediation of interacting and reflecting parts which has no place in the 2 previous paradigms.

Short: Simultaneity on a strict logico-mathematical level. Paradox approach to the idea of an extension of logics: there is no extension of logic, in principle, but a dissemination of the very same logic and its logical systems over a grid of contextural loci.

http://www.ifi.unizh.ch/ailab/people/iida/research/pfeifer_iida_JSM05.pdf
http://www.ifi.unizh.ch/ailab/people/llicht/morphcomp/slides/MorphologicalComputation.htm

2 From Propositions to Glyphs

"The idea of *naming* something is a process of *abstraction*."

O-LUDICS.pdf, Jean-Yves Girard, 2000

One last word : this book has been written during the year 2000, the year of commemorative frenzy. So let me review last century, from the viewpoint of logical foundations.

1900-1930, the time of illusions : Naive foundational programs, like Hilbert's, refuted by Gödel's theorem.

1930-1970, the time of codings : Consistency proofs, monstrous ordinal notations, *ad hoc* codings, a sort of voluntary bureaucratic self-punishment.

1970-2000, the time of categories : From the mid sixties the renewal of natural deduction, the Curry-Howard isomorphism, denotational semantics, system $\mathbb{F}$ ... promoted (with the decisive input of computer science) an approach in which the objects looked natural and reasonably free from foundational anguish.

Proof-theory started as a justification of the rules of logic, as they were given to us, classical logic. The rules became in turn an object of study, inducing their own logic, which is not the original (classical) one. Intuitionistic logic, and later linear logic, not to speak of ludics are part of this logic of rules... whence the subtitle :

From the rules of logic to the logic of rules.

Time is changing quickly, now, we are in 2005, and it seems that category theory has lost its leading function to *polymathematics* with its m-categories.

"I shall take heart from this dream and extend here a scheme I outlined in Chapter 10 of my book, an amalgamation of a scheme of Sir Michael Atiyah with one of Baez and Dolan, which derives in part from another giant of the twentieth century, Alexandre Grothendieck:

19th century
The study of functions of one (complex) variable
The codification of 0-category theory (set theory).

20th century
The study of functions of many variables
The codification of 1-category theory

21st century
Infinite-dimensional mathematics
The codification of n-category theory,
and infinite dimensional-category theory." David Corfield

http://www-users.york.ac.uk/~dc23/phorem.htm

To connect motivations and metaphors for an introduction of PolyLogics to the 21st century additional to grammatological speculations about Chinese writing, the event of a revolution in category theory could play a significant role. It will still be an analogy and its interpretation full of risks but easier to handle. The nice symmetry between logic, computation and 1-category theory is in a process of displacement by the new movement of n-category theory, challenges of interactivity in computing (Peter Wegner) and approaches in polycontexturality to transform logic; and more.

The Tale of n-Categories: http://math.ucr.edu/home/baez/week78.html#tale
n-Categories: Foundations and Applications: http://www.ima.umn.edu/categories/

Even if not well studied, we know well that 1-categories are based on triadic concepts. We know well dyadic concepts and their logics. But we still try to understand triadic concepts (semiotics, categories) in the framework of dyadics. Maybe of combined dyadics. But do we have an idea about n-categories? Are they iterations, even indefinite iterations ("infinite dimensionality") of triadic concepts of 1-category, still based on dyadic logics? Is the term "infinite" not understood as a dyadic and not as a genuine n-category theoretical term? What is the difference in the meaning of the notion "indefinite" in the 3 different conceptualisations; the dyadic, the triadic 1-categorial and the magic n-categorial?

Is the notion of the infinite chain of 1-categorial concepts constituting n-categories itself a 1-categorial concept?

PolyLogics are both: combinations of dyadic logics and genuinely m-categorial. Because mediation (combination) in PolyLogics is super-additive, decomposition into single dyadic systems is not working without reduction, that is, denying the interactional and reflectional parts of the whole. PolyLogics are based on morphograms. Morphogrammatics: The calculus of kenomic loci.

> *Thus, category theory is philosophically relevant in many ways and which will undoubtedly have to be taken into account in the years to come.*
> Introduction to CT: http://plato.stanford.edu/entries/category-theory/
> Manifesto for CT: http://www-cse.ucsd.edu/users/goguen/pps/manif.ps
> CT and Computer Science: http://www.cwru.edu/artsci/math/wells/pub/ctcs.html

Locus Solum

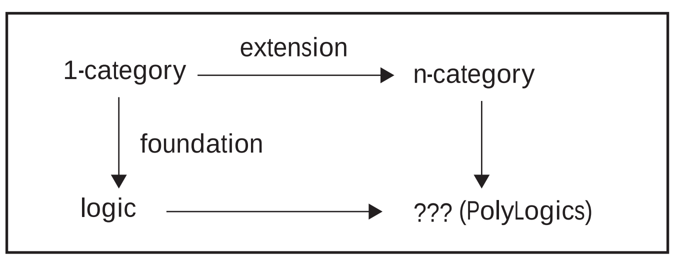
Only locations matters. Jean-Yves Girard

What's about the location of different n-categorial systems, where are they placed, do they occupy a locus? And what's the calculus of these loci? Locations in the sense of Girard are intra-contextural loci of a system, they are not thematizing the genuine locus of the system itself. Does n-category theory reflect any loci?

Logic and category theory

Classic category theory is well founded in classic logic. On the other hand, logical systems can well be modeled in category theory.

William S. Hatcher: http://www.rbjones.com/rbjpub/philos/bibliog/hatch82.htm



Now, n-category theory claims to be a kind of a revolution transforming the old concepts of 1-category to new concepts of n-categories. My question remains, what are the logics of n-category theory? The plural of logics means the different roles logic can play in the construction of n-categories. What is the use of logic in developing n-categories, what is the deduction system for n-categories and what is the foundational role of logic for the new category theory?

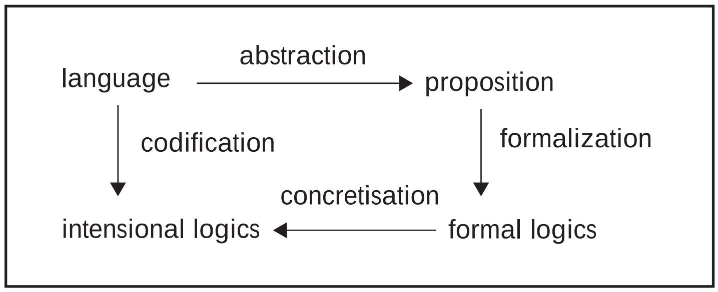
Ultimative presentation: Tom Leinster: http://www.maths.gla.ac.uk/~tl/#book

2.1 Abstracting propositions

"Abstraktion schwächt, Reflexion stärkt". Novalis

2.1.1 Abstracting sentences as propositions

Writing in the phono-logocentric conception of rationality is a secondary event. It is the inscription of thoughts and spoken language into a linear scripture based on atomic elements. The process of abstracting propositions is modeled along the concept of naming, giving something a name, producing the whole machinery of identity. Abstractionn as giving something a name is emphazing the act of classification in contrast to creation.



Abstraction

Out of the dynamic complexity of language *sentences* are abstracted which are decontextualized enough to play the role of *propositions*[1]. Logical propositions are propositions which can be true or false independent of all kind of modalities and parameters. That is, independent of who, where, how, when, why, etc. a proposition is stated. Propositions are formal statements which can be true or false.

Formalization

Formalization is connecting the abstract concept of propositions with operational methods, mainly mathematical, like geometric, algebraic, functional, categoric etc. to develop formal logics, which is also called symbolic or mathematical (propositional) logic. In other words, formalization is producing what we know today as logic and logical systems, universal logic and combined logics.

Codification

Intensional logics are based on propositional logic. But enriched, step by step, by what had to be excluded from language to develop abstract propositional logic. That is, all sorts of intensional operators recognized by linguists are transformed from their vagueness in ordinary language via mathematical linguistics to mathematization, that is concretization, in intensional logics.

Concretization

Concretizations happens by parametrization of aspects of formal logics and by adopting further formal features of natural language, like temporal, modal, spacial, etc. aspects leading to modal logics, temporal logics, multi-valued logics, etc.

Formal semantics, operational hermeneutics, common sense logics, etc. are other labels for intensional logics.

1. Usually I don't give references for obvious things. But there seems to be some demand to justify my thoughts by referencing it to the well known (?) academic literature. *Thus:* The procedure of "abstracting" logical propositions out of ordinary language is developed with high accuracy by: *Wilhelm Kamlah/Paul Lorenzen, Logische Propädeutik, Vorschule des Vernünftigen Redens, B.I. 227-227a, Mannheim, 1967, 242pp*.

Again, alphabetism and signatures

To speak about alphabetism in formal systems, with its atomicity, linearity, iterability, and ideality is not forgetting the conceptual move from alphabets as *sign repertoires* to the more abstract concept of *signatures* of institutions introduced by Goguen. This move is connected with the move from set to category theoretic conceptualizations.

> Institutions accomplish this formalization by passing from "vocabularies" to signatures, which are abstract objects, and from "translations among vocabularies" to abstract mappings between objects, called signature morphisms;
>
> then the parameterization of sentences by signatures is given by as assignment of a set Sen(S) of sentences to each signature S, and a translation Sen(f) from Sen(S) to Sen(S') for each signature morphism f: S -> S', while the parameterization of models by signatures is given by an assignment of a class Mod(S) of models for each signature S, and a translation Mod(S') -> Mod(S) for each f: S -> S' (please note the contravariance here).
>
> More technically, an institution consists of an abstract category Sign, the objects of which are signatures, a functor Sen: Sign -> Set, and a contravariant functor Mod: Sign -> Setop (more technically, we might uses classes instead of sets here).
>
> Satisfaction is then a parameterized relation $|=_S$ between Mod(S) and Sen(S), such that the following satisfaction condition holds, for any signature morphism f: S -> S', any S-model M, and any S'-sentence e:
>
> $M \; |=_S f(e) \quad \text{iff} \quad f(M) \; |=_{S'} e$
>
> **This condition expresses the invariance of truth under change of notation.**
>
> http://www.cs.ucsd.edu/users/goguen/projs/inst.html

Ideality: Abstractness of the change of notation

Signatures are even better realizing alphabetism than sign repertoires because they are empathizing the abstractness of alphabetical signs, that is, the ideality of signs, and sugn systems, in contrast to concrete occurrence of signs, independent of the content of the sign repertoire, i.e., the concrete notational material. That is, sign systems are not only characterized by atomicity, linearity, iterability, but also by *ideality*. Ideality is the medium of realization of signs. Sign systems are not concrete systems but ideal systems. Notational systems of sign systems are, to some degree, the concrete realizations, that is, the representations of abstract sign systems. And signatures as they are defined in the theory of institutions are the themes of thematizations.

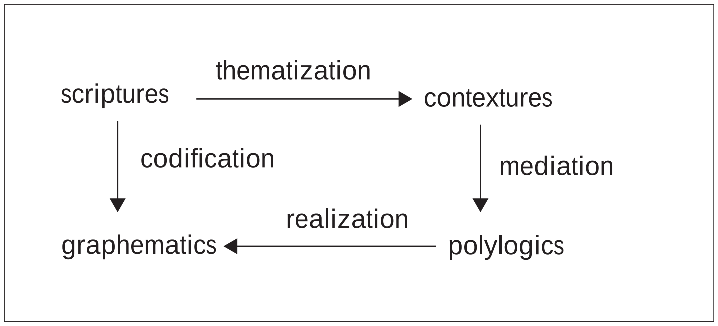
Goguen’s *"This condition expresses the invariance of truth under change of notation."*
and Makowski’s *„Computing does not deal with the creation of notational systems.“*
Makowsky, in: Herken, p. 457,
was quite motivational for my studies in *"Strukturationen der Interaktivität"* (SKIZZE).

More about semiotic, grammmatological and graphematic reflexions in/of formal languagues (systems, logics, computation) can be found in my German texts:

http://www.thinkartlab/.com/pkl/media/SKIZZE-0.9.5-Prop.pdf
http://www.thinkartlab.com/pkl/media/DISSEM-final.pdf

2.2 Generating contextures out of scriptures

Thematization is not only classification, like abstraction, but creation and evocation of new horizons of thinking.



Thematization

An abstraction is delivering an abstract proposition excluding its context of abstraction. Thematizations are "abstractions" of the difference "proposition/context" delivering propositions as propositions of a context, that is, a contexture. As a consequence of this as-abstraction there are always a plurality of contextures involved. Therefore, a proposition as belonging to one contexture is conceptually different to a proposition as belonging to another contexture.

Mediation

Contextures and their propositions are not isolated but mediated to complexions of contextures and propositions. The mechanisms of such textual mediations have to be analyzed and to be formalized towards a realization of polycontextural logics, especially to PolyLogics.

Codification

General patterns of writing have to be thematized. Not along the lines of phono-logical linguistics but specific to the practice of writing. Such grammatological and graphematic rules are delivering the basis for operational realizations.

Realization

A realization of graphematics is a further abstraction froom the rules of polycontextural logics connected with the insights into the rules of scriptural codifications. Polycontexturality and graphematics are the two new domains of scriptural research.

2.3 Sentence vs. textures

The name giving process is identifying its object and installing the laws of identity, thus these name givers are also called "identifiers". Like the lambda abstraction, which is defined or introduced intra-contexturally, the new samba abstraction is a trans-contextural operation. To distinguish it from the lambda abstraction, it should be called *samba thematization*.

Philosophically, to give a name is a special linguistic operation of the general mode of thematizing. Thematizing is more textual, to give a name is propositional or sententional. It is connected with the concept of a sentence as a statement. The philosophy of the lambda calculus is even stressing this further to the point, that the definition of a sentence is build by naming. To give a name is fundamental for the lambda calculus and radicalized, brought to the point by A++.

Hermeneutics and in radicalizing it, deconstructivism, tried to surpass this restriction and to focus more on texts, intertextuality, interpretability, iterability and ambiguity in contrast to well-formed single isolated sentences and propositions. In this sense poly-A++ can be considered as a further extension of the lambda calculus not from the inside but by distribution of the very idea and apparatus of the lambda calculus over different loci, empty places. Surely not in changing at all anything of the lambda calculus itself, but in disseminating it over the loci of the graphematic matrix.

Every programming language must somehow provide a `name giving' mechanism. Thus, every polycontextural programming language-system must somehow provide a general '*thematizing*' mechanism as a general feature allowing disseminated *'name giving' mechanisms* which each of them *allows to call procedures or functions and have the possibility to refer to variable*s inside the 'name giving' systems and between different 'name giving' systems.

A '*name giving*' procedure is also an *identifier*. To be able to identify something it has to be separated from its environment, but something can be separated from others only if it can be identified. We don't want to go into this paradoxical situation which is nevertheless the beginning of all formalism at all. But it should be mentioned that to identify something is including also a semiotic-ontological principle of identity: the named has not to be changed in the process of its naming. To name is to identify and not to change. But this is true only for the very special class of identical beings. It doesn't apply for living systems and even quantum physics is running in some troubles with this identity principle.

Said all that, it seems to be obvious, that the "references" of ConTeXtures are not symbols, variables and the data like for the intra-contextural ARS systems but the processes of interaction and reflection between ARS systems itself as it is realized in the texturality/textuality of texts, that is contextures. In this sense, ConTeXtures are abstracting from the process of abstraction as it is realized in the Lambda Calculus. The reference is the processuality of the abstraction and not its topics.

Abstraction, again:

"The idea of *naming* something is a process of *abstraction*.
When we calculate 2+1, 3+1, 5+1, 16+1 we detect a pattern and feel that it might be useful to calculate x+1 for any x – or at any rate for a numeral x. This concept is of course central to mathematics and to computing where it is of the essence that we should try to develop programs not just to do one job but to be as general as possible. The replacing of a whole class of objects by a name representative of an element of the class is roughly what we mean by abstraction and it allows us to approach functions naturally."
A J T Davie, An Introduction to Functional Programming Systems using HASKELL, pp. 79/80

To name a function by name, that is to name a name, as it happens in the definition for recursive function, can be done in two ways: intra-contextural, using the circularity of the Y-operator or distributed over different contextures, that is trans-contextural.

2.4 Signs vs. Patterns

ConTeXtures is not based on statements, but on intertextu(r)ality. It is not starting with signs but with complex graphemes. ConTextures, thus, are not primarily guided by philosophical concept of logos but by graphein. The logos, and all its secular derivations, like the lambda calculus statements, have to be listened. You have to listen the command sentences. ConTeXtures have to be read. Their elements are not semiotic atoms but scriptural patterns which have to be described and deciphered. Signs are based on atomic unities, patterns are complex, antagonistic, dynamic events. If there is a cultural change involved with ConTeXtures then it is the transition from the Greek alphabetism to the Chinese emblematics. ConTeXtures are not belonging to a new logical paradigm but to the grammatological subversion of Graphematics.

The polycontextural matrix can not be read like a sentence and to be listened to its meaning. The matrix has to be read, involving different viewpoints and memory. It can not be memorized internally as a subjective truth. There is no truth in ConTeXtures, but a multitude of truths space-ing multiple ways of practice. The play of differences has to be written in a mundane game of traces and marks. The polycontextural matrix is a first step / jump to Graphematics transforming idealistic logic to materialistic dialectics.

A further step is introduced with the "abstraction" (subversion) of morphizing contextures. This is realized by Morphogrammatics and Kenogrammatics, the grammar of kenos, the game of emptiness, studying the invariance of dynamic scriptural patterns.

2.5 Contextures vs. Names

The textuality of ConTeXtures is involving the game of the as-abstraction based on the proemial relation. Therefore there are no new stable beginnings of the calculus. ConTeXtures are not simply changing from a name dominated to a text dominated paradigm of disseminated contextures. In the same sense as names can become contextures, e.g. in a process of de-nominalization, contextures can be nominalized and can become names. We can call contextures by name. Contextures can be named and names can be contexturalized. This is not a return home to the name dominated logocentrism of the Lambda Calculus but part of the game of proemial metamorphosis.

There is no fixed hierarchy for the operators *samba* and *lambda.* It is allowed to use these operators in a "circular" and "parallel" way which also shows, that there is no ultimate beginning and origin of the game: (...(lambda(samba(lambda))...).This too, may give a hint to understand that ConTeXtures are combining algebraic and co-algebraic concepts and methods of formalizing.

ARS allows a very general use of the lambda abstraction. Everything named can be called everywhere and every time by name. It seems, that the conceptualization of ARS is more "self-referential" than other Lambda-based programming approaches. In general, ARS could even call itself by name. But this makes not much sense, because there is no conceptual space to use this way of naming and to put its named object (ARS) somewhere. In a polycontextural environment things are very different and it is not only possible but also necessary to call a whole system, like ARS, by name and to use it in other contextures.

3 More Metaphorics

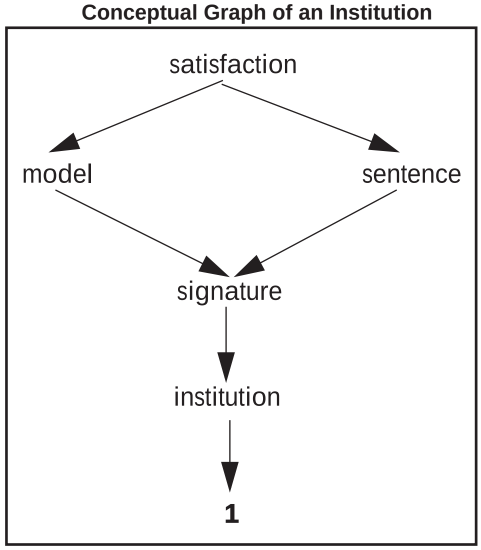
3.1 Tree farming of colored logics

To put all these more philosophical descriptions and ideas together in a more strict formal terminology and operative apparatus we connect these ideas of colored logics with the metaphor of the tree. In classical logic trees figure as metaphor and as mathematical concepts on a very basic level. We postulated that every colored logic is locally classical, e.g. each colored logic has the structure of a classic tree logic in its syntax, semantics and proof theory. It is helpful to represent the structure of the body or the tectonics of a logical system by its conceptual graph.

3.1.1 Notation of an institution

Diagramm 1

Conceptual Graph of an Institution



„The arrows in this diagram represents conceptual dependencies. The notation
model —> signature
for example, means that:
the concept of model varies as signature varies.
In particular, it means that the concept of model, the one that we have in mind, cannot be independent of the concept of signature and neither can a particular model be independent of its particular signature.
In a conceptual diagram, 1 represents the absolute. The notion
institution—>1
expresses that the institution notion is absolute, for it tells us that the institution notion varies as the absolute varies – which is not at all.“ p. 488

absolute

The absolute 1 expresses that there is only one logic as such. There are many different logical systems but from a more philosophical and logical and not only mathematical point of view all these logical systems are isomorphic to one and only one logic. This is a (not provable Hypo) thesis.

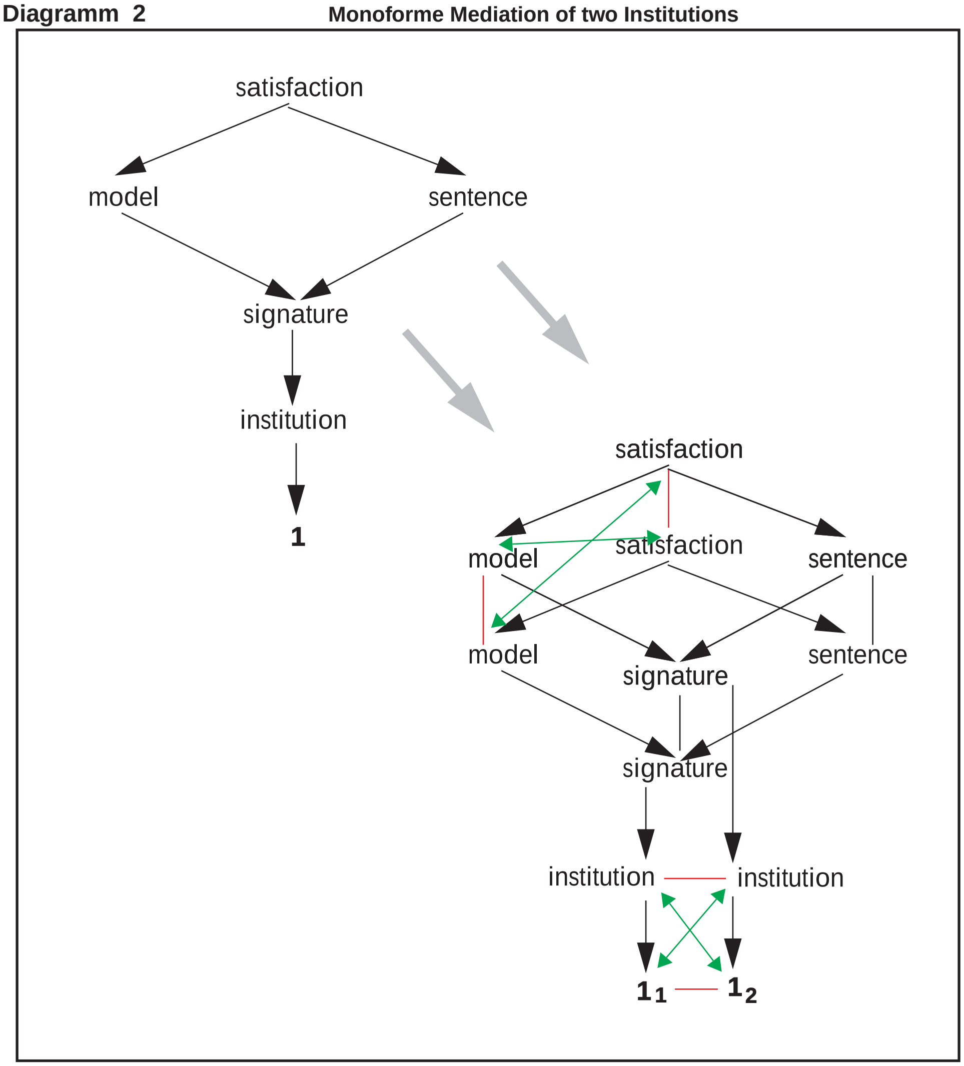
If we do not like this absolutism we should remember that the wording holds also in the more relativistic case. Considering a logical system working in it and with it means that we are working with this system and not at once or at the same time with another one. We can speak therefore of a relative absolute of the logic under consideration.

Even for mixed logics as in the project of *Combining Logics* there is a (relative) notion of the absolute of the system.

As we will see the situation will be totally different for polycontextural logics where a plurality of absolutes ordered in a heterarchical manner exists and the desire to have a mega-absolute for the whole complex system would turn out to be (simply) a new absolute within a plurality of other neighbored absolutes.

From the point of view of PCL the absolute means that the whole system is defined under the operation of identity ID. The system viewed as an object and as a morphism coincide.

Diagramm 2 **Monoforme Mediation of two Institutions**



3.2 Preliminary Comments

3.2.1 Syntax

For each tree of the colored logical systems the ancestral property of its formulae holds. In each tree there is a immediate predecessor relation which decompose the formula in its subformulas. All that is ruled by the principle of induction over the formulas for each logical system. The induction principle is distributed over all logical systems.

Additionally to the concept of a predecessor for each system we have to introduce in the trans-classical context the operation or relation of the immediate neighbor. These makes possible some kind of permutation of logical systems over the range of the complex of distributed systems.

Further on we have to introduce the very special concept of the immediate bifurcation of a formula of a logical system into subformulas distributed over other logical systems and one part of the formula remains in its original system

3.2.2 Unification

On a meta-logical level, total functions are supporting symmetrical classifications and categorisations of logical particles.

This nice property of a logical symmetry is lost in trans-classical logics because of their transjunctions which are composed of partial functions. But with the help of the concept of partial functions we can introduce a new idea of a slightly more complex symmetry composed by partiality. The classical case of symmetry is then introduced as a regular composition of partial functions. This idea of a complex symmetry composed of asymmetrical functions needs additionally to the classical operator of conjugation a new operator of composition of partial functions.

3.2.3 Semantics

Truth-values in classical systems are connotated with the formal logical explication of truth and false. Formal truth and formal false do not involve ontological questions about truth and falsehood of sentence. This belongs to the level of examples for formal logical sentences.

In PCL systems truth values, if we are choosing a truth value semantics, have to realize two jobs, the first is more or less the same as in classical systems, they represent the formal logical concept of true and false of propositions of their logic. The second job is very different, they have to mark in which logical system the difference of true/false holds. Therefore they have an index of the system they belong or origin: {Ti, Fi}. As the splitting function of transjunction shows these truth values can occur in different systems at once. As a result, the whole semantics of propositions, sentences, phrases and truth-values has to be deconstructed.

As explained metaphorically earlier these logics are not isolated from each other but combined to a complex logical, or ultra-logical, system. Otherwise they would behave totally in parallel and it would be at least at first only an application of one classical logic at different epistemological places without any interaction or mediation.

A first, quite natural and elementary, connection is given by a (special) linear ordering of the systems and their truth-values.

To not to confuse this kind of order with other ordering systems I call it a chiastic linear order of truth-values. A chiasm is defined as a tupel of order, exchange, coincidence and positioning relations.

Therefore the semantics of PCL is not defined over a set of truth-values but over an order, a chiastically ordered structure of truth-values.

The difference becomes obvious for the semantics of ternary and general n-ary logical functions or logical compositions. This difference between set based and order based semantics is hidden for the typical binary case.

As a natural consequence the notions of sentence, model and satisfaction have to be distributed over the indices of their semantics.

3.2.4 Consequence relations and proof theory

For each single logical system of the PCL complexion there exist a consequence relation and a proof theory for this logic. The consequence relation for the whole system of logic is composed of the distributed single consequence relations of each logic.

We will choose the analytical tableaux method as our proof procedure.

Proemiality of PolyLogics

4 Architectonics

4.1 Proemial relationship of mediated systems

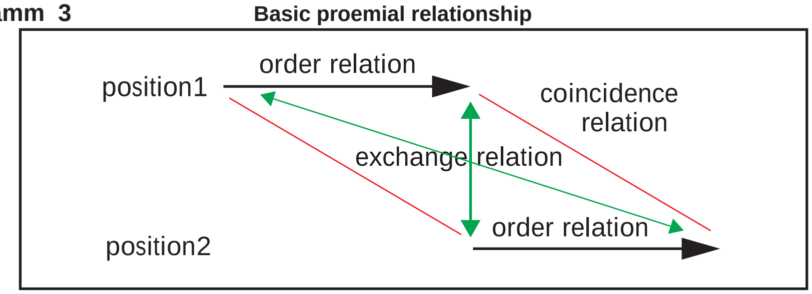
The architectonics of a polycontextural systems defines the kind and complexity of distribution and mediation of the contextures involved, short, the complexity of dissemination.

For introductory reasons the order of mediation of basic contextures will be restricted to linearity in excluding other combinatorial patterns like stars, etc. as architectonic basic structures. That doesn't mean, that polycontexturality is subsumed under the general logocentric principle of linearity. Simply because this logocentric principle of linearity is an intra-contextural and not a trans-contextural principle governing the polycontexturality of a multitude of contextures, all inhibited locally by a classical principle of linearity.

The modus of mediation will be realized by the *proemial relation*, its structure is often called chiasm. This too, is supporting tabularity and distribution of different principles of linearity, in contrast to classic linearity, even for the case of "linearily" mediated systems.

On this level the presentation and notation of architectonics and proemiality is restricted to conceptual graphs.

Diagramm 3 **Basic proemial relationship**

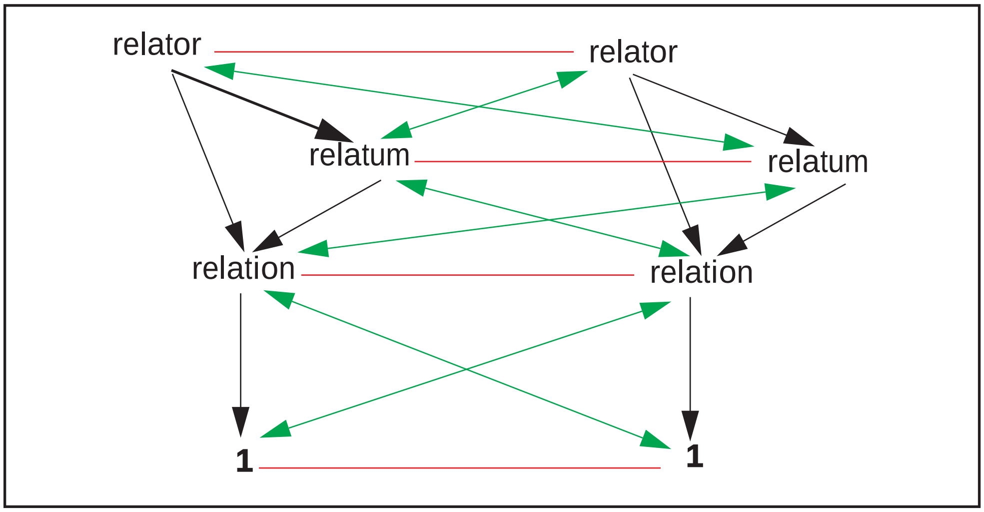


Terminology

The cascading order of the two order relation, mediated by the exchange and coincidence relations, an elementary case of "linearily" ordered mediation of subystems represented by order relations. Not included in the diagram is the additional sub-system at position3, produced super-additively by the difference of position1 and 2.

Diagramm 4 **Basic proemial relation table**

coinc (x y)	***exch (x y)***	***ord (x y)***
x1 coinc x2	*x1 exch y2*	*x1 ord y1*
y1 coinc y2	*y1 exch x2*	*x2 ord y2*



The mediator is defined by the proemial relationship of (coinc, exch, ord, pos).

The operator "*coinc*" is representing the binary relation of coincidence, in the sense of the sameness, categorial analogy, of two operands or two operators.

The operator "*ord*" is representing the binary relation of order, in the sense of the asymmetry of an ordered pair of operator, operand and uniqueness.

The operator "*exch*" is representing the binary relation of symmetrical exchange, in the sense of a symmetric difference between operator and operand.

The object **1** is the initial object defining the relative uniqueness of the relationship.

4.1.1 Objectionality of mediated proemial Objects

Atomic objects are not simple and identical in ConTeXtures but complexions of "parts", "aspects" belonging to different contextures. Contextures are mediated by the proemial relationship. Additional to this proemial or chiastic pattern we introduce a special kind of mediating systems, realizing the super-additivity of mediated or combined systems. I call their relationality towards the other systems "siml" for similarity.

samba (samba, 3, (ord, coinc, exch, siml, opp), (x, y))

Diagramm 5 **Diagram of mediated proemiality of objects**

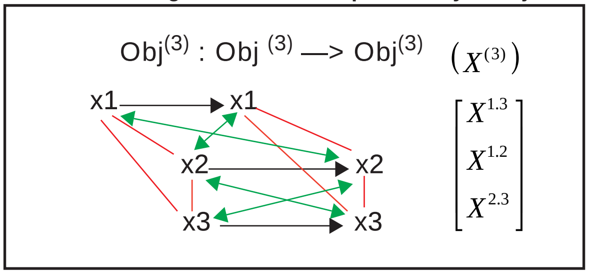


Diagramm 6 **Relational table of mediated proemial object**

coinc (x y)	*exch (x y)*	*siml (x y)*	*ord (x y)*	*opp (x y)*
x1 coinc x2	*x1 exch y2*	*x1 siml x3*	*x1 ord y1*	*x2 opp y3*
y1 coinc y2	*y1 exch x2*	*y2 siml y3*	*x2 ord y2*	*x3 opp y2*
y1 coinc y3	*x1 exch y3*		*x3 ord y3*	*x3 opp y1*
x2 coinc x3				

Diagramm 7 **Symmetric mediation table of mediated proemial object**

c-ob$^{(3)}$	x1	y1	x2	y2	x3	y3
x1	id	ord	coinc	exch	siml	exch
y1	ord	id	exch	coinc	opp	siml
x2	coinc	exch	id	ord	siml	opp
y2	exch	coinc	ord	id	opp	siml
x3	siml	opp	siml	opp	id	ord
y3	exch	coinc	opp	siml	ord	id

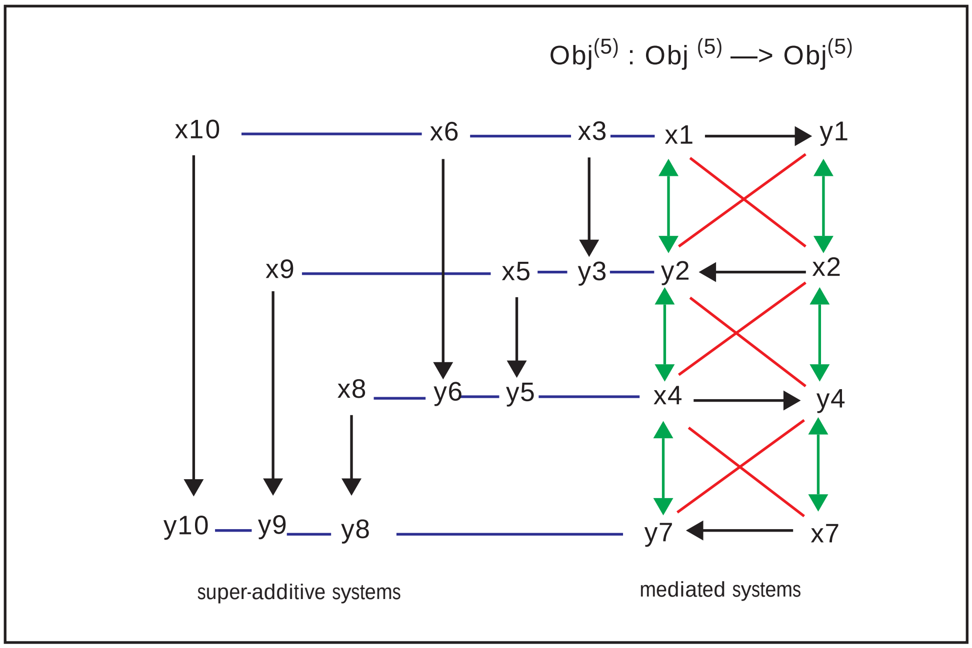
Proemial object

$x^{(3)}$ == (x^1 coinc x^2 coinc x^3 , x1 siml x3)
$y^{(3)}$ == (y^1 coinc y^2 coinc y^3 , y^1 siml y^3)
$x^{(3)}$ exch $y^{(3)}$;; $x^{(3)}$ ord $y^{(3)}$

An atomic 3-contextural object is therefore introduced as a mapping in itself which makes sense only if the object is not an atomic object *ob* (formal object, Curry) but a complex *c-ob*$^{(m)}$ (proemial object). Obviously, an atomic object *ob* is a reduction of a c-ob to its mono-contextural status.

In formal systems atomic terms are distinguished from composed terms. Composed terms are linear compositions of concatenations. Polycontextural c-obs are complex, not in the linear but in the tabular sense. Tabular complex terms are composed not by concatenation but by mediation.

4.1.2 Super-additivity of mediated systems



Super-additivity, as well as super-subtractivity, appears naturally on all levels of poly-contextural systems and is "based" in the super-additivity of the proemial relationship.

Proem (A) + Proem (B) < Proem (A + B)

4.1.3 Numbering of sub-systems

3.4 Enumeration

In the PCL literature the following enumeration of the $n = \binom{m}{2}$ subsystems $L_1, \ldots, L_n$ of an m-valued PCL is used:

For the values $1, 2, \ldots, m$ each unordered pair $\{i, j\}$ determines exactly one subsystem L_k; its "number" is defined by $k = \binom{j}{2} - i + 1$. So, we obtain $n = \binom{m}{2}$ subsystems $L_1, \ldots, L_n$ in an m-valued PCL system.

The truth values i, j of L_k are given by: $i = j(j-1)/2 - k + 1$.
and $j = [3/2 + \sqrt{2k - 7/4}]$ (the integer part), cf. [K2, p. 264].
Within L_k the two (classical) values are then determined by $T_k = \min(i, j)$, $F_k = \max(i, j)$.
Locally, this defines an order on these two values.
The global values $1, 2, \ldots, m$ then determine the corresponding identifications of local truth values: for example, if $T_k = i$ and $F_l = i$, then $T_k \equiv F_l$.

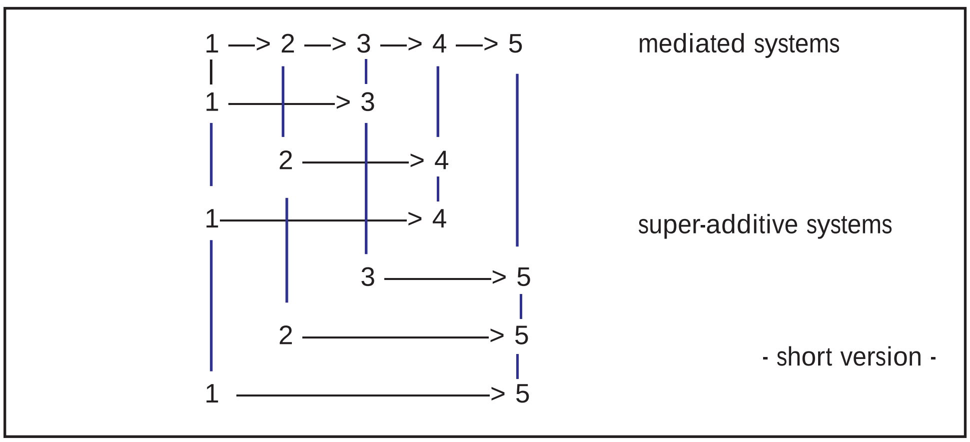


Diagramm 8 **Enumeration of sub-systems**

S1 S2 S4 S7	mediated
S3 S5 S8	super-
S6 S9	additive
S10	sub-systems

Directly mediated systems like S1, S2, S4, S7 are mediated by indirect mediated super-additive systems which becomes mediating systems relative to their positions.
((S1 med S2) add S3),
((S2 med S4) add S5),
((S4 med S7) add S8),
((S3 med S5) add S6),
((S5 med S8) add SS9), ((S6 med S9) add S10).

5 Notational problems with complexity

5.1 Linear vs. tabular notations

For traditional and historical reasons formulas have been written in linear form. But this comes to an end if we introduce more complex situations. The following examples are considering only the case of balanced matrices and OiMj, i=j, that is, the *diagonal* systems only.

$$X^{(4)} \vee \wedge \vee \wedge \vee \wedge Y^{(4)}$$
$$X^{(5)} \vee \wedge \vee \wedge \vee \wedge \vee \wedge \vee \wedge Y^{(5)}$$
$$X^{(6)} \vee \wedge \vee \wedge \vee \wedge \vee \wedge \vee \wedge \wedge \vee \wedge \vee \wedge Y^{(6)}$$
$$and\ so\ on\ binomial: \binom{m}{2}$$
$$1\ 3\ 6\ 10\ 15\ 21\ ...$$

To write down and to deal with, say a 47-contextural logic, would simply fall out of the scope of perception. Linearity, even for the representations of the operators of a formula, not mentioning composed formulas, has failed to work.

5.1.1 Operational patterns

A kind of a tabular numeration of the operators may help a step further in dealing with notational complexity.

On the half way to combinatory logic operational patterns are abstracting, as far as possible, from the variables of the formulas and inscribing only the operational patterns and their transformations.

Numeration

Diagramm 9

Enumeration of sub-systems, O-matrix

$$O_{num}: [1\ 2\ 3\ 4\ 5] \Rightarrow \begin{bmatrix} O_{10} \\ O_6O_9 \\ O_3O_5O_8 \\ O_1O_2O_4O_7 \end{bmatrix}$$

Binomial numbers onto operator patterns.

5.1.2 Examples: Dualizations

Diagramm 10 **Dualization**

$$D_4^{(5)}: \begin{bmatrix} \vee_{10} & & & \\ \wedge_6 & \vee_9 & & \\ \vee_3 & \wedge_5 & \vee_8 & \\ \vee_1 & \wedge_2 & \vee_4 & \wedge_7 \end{bmatrix} \Rightarrow \begin{bmatrix} \vee_{10} & & & \\ \wedge_3 & \vee_9 & & \\ \wedge_6 & \wedge_2 & \wedge_7 & \\ \vee_1 & \wedge_5 & \wedge_4 & \vee_8 \end{bmatrix}$$

Duality of sub-system 6 and transpositions of affected neighbor-systems

Green: not affected, red: dualized, black: transposed sub-systems.

Interpretation: sub-systems 1, 9, 10 are isolated in respect to the dualization of sub-system 4. Thus, sub-system 4 can be logically manipulated without disturbing the basic sub-system 1 and the mediating sub-systems 9 and 10.

The dualizations D_1 and D_4 are commutative: $D_1 (D_4) = D_4 (D_1)$.

5.1.3 Dualization cycles

Dualization cycles

$$D_1(D_3) = D_3(D_1)$$
$$D_1(D_2(D_1)) = D_2(D_1(D_2))$$
$$D_2(D_3(D_2)) = D_3(D_2(D_3))$$

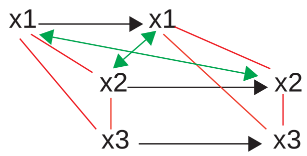
$$DC1: D_{1.2.1.2.1.2}$$
$$DC2: D_{1.3.1.3}$$
$$DC3: D_{1.2.1.2.1.3.1.2.1.2.3.1.2.3.2.1.2.1.2.3.2.1.3}$$

5.1.4 N, K and D calculus

$$K^1_i := (N_i \ X \ \odot\odot\odot \ Y)$$
$$K^2_i := (X \ \odot\odot\odot \ N_i \ Y)$$
$$D_i := N_i(K^1_i(K^2_i)), \ i = 1, 2$$

5.1.5 Proemiality in ConTeXtures

$Obj^{(3)} : Obj^{(3)} \longrightarrow Obj^{(3)}$ $\quad (X^{(3)})$



$$\begin{bmatrix} X^{1.3} \\ X^{1.2} \\ X^{2.3} \end{bmatrix}$$

Despite the intricate complexity of proemial objects ConTeXtures will be based mainly on the basic relations of proemiality, *order*, *exchange* and for the mediated systems *siml*. Without producing confusion *siml* will also be called the *coincidence* relation.

5.1.6 Comparability of proemial objects

5.1.6.1 Unary objects

$x^{(3)} = (x^1, x^2, x^3)$, more explicit because it is not a tuple but a mediation and again visualized by the diagram below.

$x^{(3)} = (x^1 \S\ x^2 \S\ x^3)$

Signatures of $x^{(3)}$

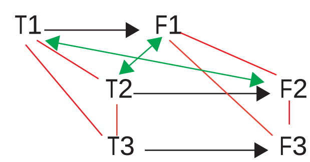
$T_{1,3}$ x

§

$F_{1,2}$ x (or F_1, T_2 x)

§

$F_{2,3}$ x (or $F_{2,3}$ x)

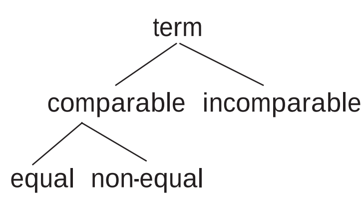


$$(X^{(3)}) \quad \begin{bmatrix} X^{1.3} \\ X^{1.2} \\ X^{2.3} \end{bmatrix} \qquad \begin{bmatrix} T_1, & \varnothing, & T_3 \\ F_1, & T_2, & \varnothing \\ \varnothing, & F_2, & F_3 \end{bmatrix} \qquad \begin{bmatrix} X^{1.1} \\ X^{1.1} \\ X^{1.1} \end{bmatrix}$$

There are 5 unary morphogrammatic objects for a 3x1-matrix with max 3 different occupations, giving base for all possible interpretations, mapping of obs onto it.

5.1.6.2 Binary Objects

There are 3281 morphograms for a 3x3-matrix with max 3 different occupations (kenograms).



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General Framework of PolyLogics

PolyLogics

- $sketch-logic-horizon^{(m)}$
  - $build-architectures$
    - $\{interactional, reflectional, deductional\}$
    - $thematize-scenarios$
      - $choose-logic-types$
        - $\{classic, intuitionistic, dialogical, multiple-valued, contextual\}$
        - $decide-proof-style$
          - $\{Hilbert, Smullyan, Fitch, Gentzen\}$
          - $select-logic-topics$
            - $\{Boolean, class, relational, reflectional, actional\}$
            - $identify-frameworks$
              - $define-operations$
                - $abstract-function$
                  - $\{propose-statements\}$

- $sketch-horizon^{(m)}$
  - $build-architectures$
    - $thematize-scenarios$
      - $identify-frameworks$
        - $define-operations$
          - $abstract-function$
            - $\{propose-statements\}$

Tableaux rules for PolyLogics

1 Tableaux rules for junctions

1.1 Pattern [id, id, id]

$$\frac{t_1\ X \wedge\wedge\wedge Y}{\begin{array}{c} t_1\ X \\ t_1\ Y \end{array}} \quad \frac{f_1\ X \wedge\wedge\wedge Y}{f_1\ X \mid f_1\ Y} \quad \frac{t_1\ X \vee\vee\vee Y}{t_1\ X \mid t_1\ Y} \quad \frac{f_1\ X \vee\vee\vee Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}}$$

$$\frac{t_2\ X \wedge\wedge\wedge Y}{\begin{array}{c} t_2\ X \\ t_2\ Y \end{array}} \quad \frac{f_2\ X \wedge\wedge\wedge Y}{f_2\ X \mid f_2\ Y} \quad \frac{t_2\ X \vee\vee\vee Y}{t_2\ X \mid t_2\ Y} \quad \frac{f_2\ X \vee\vee\vee Y}{\begin{array}{c} f_2\ X \\ f_2\ Y \end{array}}$$

$$\frac{t_3\ X \wedge\wedge\wedge Y}{\begin{array}{c} t_3\ X \\ t_3\ Y \end{array}} \quad \frac{f_3\ X \wedge\wedge\wedge Y}{f_3\ X \mid f_3\ Y} \quad \frac{t_3\ X \vee\vee\vee Y}{t_3\ X \mid t_3\ Y} \quad \frac{f_3\ X \vee\vee\vee Y}{\begin{array}{c} f_3\ X \\ f_3\ Y \end{array}}$$

PM	$O1$	$O2$	$O3$
$M1$	$log1$	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$log2$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$log3$

PM	$O1$	$O2$	$O3$
$M1$	and	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	and	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	and

$$(JJJ) : L^{(3)} \ * \ L^{(3)} \longrightarrow L^{(3)} : [L_1,\ L_2,\ L_3]$$

$$\left[\begin{array}{l} Log_1 : L_1 \ * L_1 \xrightarrow[junct\ J]{} L_1 \\ Log_2 : L_2 \ * L_2 \xrightarrow[junct\ J]{} L_2 \\ Log_3 : L_3 \ * L_3 \xrightarrow[junct\ J]{} L_3 \end{array}\right.$$

$$\frac{t_1\ X \vee\wedge\vee Y}{t_1\ X \mid t_1\ Y} \qquad \frac{f_1\ X \vee\wedge\vee Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}} \qquad \frac{t_1\ X \wedge\vee\wedge Y}{\begin{array}{c} t_1\ X \\ t_1\ Y \end{array}} \qquad \frac{f_1\ X \wedge\vee\wedge Y}{f_1\ X \mid f_1\ Y}$$

$$\frac{t_2\ X \vee\wedge\vee Y}{\begin{array}{c} t_2\ X \\ t_2\ Y \end{array}} \qquad \frac{f_2\ X \vee\wedge\vee Y}{f_2\ X \mid f_2\ Y} \qquad \frac{t_2\ X \wedge\vee\wedge Y}{t_2\ X \mid t_2\ Y} \qquad \frac{f_2\ X \wedge\vee\wedge Y}{\begin{array}{c} f_2\ X \\ f_2\ Y \end{array}}$$

$$\frac{t_3\ X \vee\wedge\vee Y}{t_3\ X \mid t_3\ Y} \qquad \frac{f_3\ X \vee\wedge\vee Y}{\begin{array}{c} f_3\ X \\ f_3\ Y \end{array}} \qquad \frac{t_3\ X \wedge\vee\wedge Y}{\begin{array}{c} t_3\ X \\ t_3\ Y \end{array}} \qquad \frac{f_3\ X \wedge\vee\wedge Y}{f_3\ X \mid f_3\ Y}$$

1.2 Pattern [id, id, red]

$$\frac{t_1\ X \vee\rightarrow\vee Y}{f_1\ X \mid f_1\ Y \parallel f_3\ X \mid f_3\ Y} \qquad \frac{f_1\ X \vee\rightarrow\vee Y}{\begin{array}{c\|c} f_1\ X & f_3\ X \\ f_1\ Y & f_3\ Y \end{array}}$$

$$\frac{t_2\ X \vee\rightarrow\vee Y}{f_2\ X \mid t_2\ Y} \qquad \frac{f_2\ X \vee\rightarrow\vee Y}{\begin{array}{c} t_2\ X \\ f_2\ Y \end{array}}$$

PM	$O1$	$O2$	$O3$	PM	$O1$	$O2$	$O3$
$M1$	$log1$	$\varnothing$	$\varnothing$	$M1$	or	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$log2$	$\varnothing$	$M2$	$\varnothing$	$impl$	$\varnothing$
$M3$	$log3$	$\varnothing$	$\varnothing$	$M3$	or	$\varnothing$	$\varnothing$

$$(\vee \rightarrow \vee) : L^{(3)} * L^{(3)} \longrightarrow L^{(3)} : \left[(L_1 \mid L_3),\ L_2,\ \varnothing \right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[disjunction\ \vee]{} L_1 \mid L_3 \\ Log_2 : L_2 * L_2 \xrightarrow[imlication\ \rightarrow]{} L_2 \\ Log_3 : L_3 * L_3 \xrightarrow[disjunction\ \vee]{} L_1 \end{array} \right.$$

1.3 Pattern [id, id, red], part II

$$\frac{t_1\ X \rightarrow\rightarrow\rightarrow Y}{f_1\ X \mid t_1\ Y \parallel f_3\ X \mid t_3\ Y} \qquad \frac{f_1\ X \rightarrow\rightarrow\rightarrow Y}{\begin{array}{c|c} t_1\ X & t_3\ X \\ f_1\ Y & f_3\ Y \end{array}}$$

$$\frac{t_3\ X \rightarrow\rightarrow\rightarrow Y}{f_2\ X \mid t_2\ Y} \qquad \frac{f_3\ X \rightarrow\rightarrow\rightarrow Y}{\begin{array}{c} t_2\ X \\ f_2\ Y \end{array}}$$

$$\frac{t_1\ X \rightarrow\Rightarrow\rightarrow Y}{f_1\ X \mid t_1\ Y \parallel f_2\ X \mid t_2\ Y \parallel f_3\ X \mid t_3\ Y} \qquad \frac{f_1\ X \rightarrow\Rightarrow\rightarrow Y}{\begin{array}{c|c|c} t_1\ X & t_2\ X & t_3\ X \\ f_1\ Y & f_2\ Y & f_3\ Y \end{array}}$$

$$\frac{t_i\ X \rightarrow\Rightarrow\rightarrow Y}{f_i\ X \mid t_i\ Y} \qquad \frac{f_i\ X \rightarrow\Rightarrow\rightarrow Y}{\begin{array}{c} t_i\ X \\ f_i\ Y \end{array}}\ , i = 1, 2, 3$$

$$\frac{t_3\ X \leftarrow\leftarrow\leftarrow Y}{t_2\ X \mid f_2\ Y \parallel t_3\ X \mid f_3\ Y} \qquad \frac{f_3\ X \leftarrow\leftarrow\leftarrow Y}{\begin{array}{c|c} f_2\ X & f_3\ X \\ t_2\ Y & t_3\ Y \end{array}}$$

$$\frac{t_i\ X \leftarrow\Leftarrow\leftarrow Y}{t_i\ X \mid f_i\ Y} \qquad \frac{f_i\ X \leftarrow\Leftarrow\leftarrow Y}{\begin{array}{c} f_i\ X \\ t_i\ Y \end{array}}\ , i = 1, 2, 3$$

$$\frac{t_1\ X \rightarrow\leftarrow\rightarrow Y}{f_1\ X \mid t_1\ Y \parallel f_3\ X \mid t_3\ Y} \qquad \frac{f_1\ X \rightarrow\leftarrow\rightarrow Y}{\begin{array}{c|c} t_1\ X & t_3\ X \\ f_1\ Y & f_3\ Y \end{array}}$$

$$\frac{t_3\ X \rightarrow\leftarrow\rightarrow Y}{t_2\ X \mid f_2\ Y} \qquad \frac{f_3\ X \rightarrow\leftarrow\rightarrow Y}{\begin{array}{c} f_2\ X \\ t_2\ Y \end{array}}$$

1.4 Pattern [id, red, red]

$$\frac{t_1\ X \rightarrow\Leftarrow\rightarrow Y}{f_1\ X \mid t_1\ Y \parallel t_2\ X \mid f_2\ Y \parallel f_3\ X \mid t_3\ Y} \qquad \frac{f_1\ X \rightarrow\Leftarrow\rightarrow Y}{\begin{matrix} t_1\ X \\ f_1\ Y \end{matrix} \Bigg\Vert \begin{matrix} f_2\ X \\ t_2\ Y \end{matrix} \Bigg\Vert \begin{matrix} t_3\ X \\ f_3\ Y \end{matrix}}$$

PM	$O1$	$O2$	$O3$
$M1$	$log1$	$\varnothing$	$\varnothing$
$M2$	$log2$	$\varnothing$	$\varnothing$
$M3$	$log3$	$\varnothing$	$\varnothing$

PM	$O1$	$O2$	$O3$
$M1$	$impl$	$\varnothing$	$\varnothing$
$M2$	$repl$	$\varnothing$	$\varnothing$
$M3$	$impl$	$\varnothing$	$\varnothing$

$$(\rightarrow\Leftarrow\rightarrow) : L^{(3)} * L^{(3)} \longrightarrow L^{(3)}$$
$$: [L_1, L_1, L_1]$$
$$\begin{bmatrix} Log_1 : L_1 \xrightarrow[impl1]{} L_1 \\ Log_2 : L_2 \xrightarrow[repl1]{} L_1 \\ Log_3 : L_3 \xrightarrow[impl1]{} L_1 \end{bmatrix}$$

2 Tableaux rules for negations Neg$^{(3)}$

2.1 Permutational patterns

$$\frac{t_1\left(\neg_1\ X^{(3)}\right)}{f_1\ X^{(3)}} \qquad \frac{t_2\left(\neg_1\ X^{(3)}\right)}{t_3\ X^{(3)}} \qquad \frac{t_3\left(\neg_1\ X^{(3)}\right)}{t_2\ X^{(3)}}$$

$$\frac{f_1\left(\neg_1\ X^{(3)}\right)}{t_1\ X^{(3)}} \qquad \frac{f_2\left(\neg_1\ X^{(3)}\right)}{f_3\ X^{(3)}} \qquad \frac{f_3\left(\neg_1\ X^{(3)}\right)}{f_2\ X^{(3)}}$$

$$\frac{t_1\left(\neg_2\ X^{(3)}\right)}{t_3\ X^{(3)}} \qquad \frac{t_2\left(\neg_2\ X^{(3)}\right)}{f_2\ X^{(3)}} \qquad \frac{t_3\left(\neg_2\ X^{(3)}\right)}{t_1\ X^{(3)}}$$

$$\frac{f_1\left(\neg_2\ X^{(3)}\right)}{f_3\ X^{(3)}} \qquad \frac{f_2\left(\neg_2\ X^{(3)}\right)}{t_2\ X^{(3)}} \qquad \frac{f_3\left(\neg_2\ X^{(3)}\right)}{f_1\ X^{(3)}}$$

$$\frac{t_1t_2t_3\left(\neg_1\ X^{(3)}\right)}{f_1t_3t_2\ X^{(3)}} \qquad \frac{f_1f_2f_3\left(\neg_1\ X^{(3)}\right)}{t_1f_3f_2\ X^{(3)}}$$

$$\frac{t_1t_2t_3\left(\neg_2\ X^{(3)}\right)}{t_3f_2t_1\ X^{(3)}} \qquad \frac{f_1f_2f_3\left(\neg_2\ X^{(3)}\right)}{f_3t_2f_1\ X^{(3)}}$$

$$(non_1): L^{(3)} \to L^{(3)} : \left[L_1, L_3, L_2\right] \qquad (non_2): L^{(3)} \to L^{(3)} : \left[L_3, L_2, L_1\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 \xrightarrow[neg1]{} L_1 \\ Log_2 : L_2 \xrightarrow[perm1]{} L_3 \\ Log_3 : L_3 \xrightarrow[perm1]{} L_2 \end{array}\right. \qquad \left[\begin{array}{l} Log_1 : L_1 \xrightarrow[perm2]{} L_3 \\ Log_2 : L_2 \xrightarrow[neg2]{} L_2 \\ Log_3 : L_3 \xrightarrow[perm2]{} L_1 \end{array}\right.$$

2.2 Bifurcational patterns: Tableaux rules for transjunctions

Transjunctions are the primary interactional operations in polylogical systems.

Diagramm 11 **Tableaux rules for (X trans and and Y)**

$$\frac{t_1 \; X \; <>\wedge\wedge \; Y}{\begin{array}{c} t_1 \; X \\ t_1 \; Y \end{array}} \qquad \frac{f_1 \; X \; <>\wedge\wedge \; Y}{\begin{array}{c} f_1 \; X \\ f_1 \; Y \end{array}}$$

$$\frac{t_2 \; X \; <>\wedge\wedge \; Y}{\begin{array}{c|c} t_2 \; X & f_1 \; X \\ t_2 \; Y & f_1 \; Y \end{array}} \qquad \frac{f_2 \; X \; <>\wedge\wedge \; Y}{f_2 \; X \mid f_2 \; Y \; \Bigg\| \begin{array}{c|c} f_1 \; X & t_1 \; X \\ t_1 \; Y & f_1 \; Y \end{array}}$$

$$\frac{t_3 \; X \; <>\wedge\wedge \; Y}{\begin{array}{c} t_3 \; X \\ t_3 \; Y \end{array} \Bigg\| \begin{array}{c} t_1 \; X \\ t_1 \; Y \end{array}} \qquad \frac{f_3 \; X \; <>\wedge\wedge \; Y}{f_3 \; X \mid f_3 \; Y \; \Bigg\| \begin{array}{c|c} f_1 \; X & t_1 \; X \\ t_1 \; Y & f_1 \; Y \end{array}}$$

PM	$O1$	$O2$	$O3$	PM	$O1$	$O2$	$O3$
$M1$	$log1$	$log1$	$log1$	$M1$	$trans$	$trans$	$trans$
$M2$	$\varnothing$	$log2$	$\varnothing$	$M2$	$\varnothing$	and	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$log3$	$M3$	$\varnothing$	$\varnothing$	and

$$\left(<>\wedge\wedge\right) : L^{(3)} \; * \; L^{(3)} \longrightarrow L^{(3)} : \left[L_1, (L_2 \,||\, L_1), \left(L_3 \,||\, L_1\right)\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 \; * L_1 \xrightarrow[transjunct \; <>]{} L_1 : \begin{cases} f_1 \; * \; t_1 \to f_2, \, f_3 \\ t_1 \; * \; f_1 \to t_1, \, t_3 \end{cases} \\ Log_2 : L_2 \; * L_2 \xrightarrow[conjunction]{} L_2 \; || \; L_1 \\ Log_3 : L_3 \; * L_3 \xrightarrow[conjunction]{} L_3 \,||\, L_1 \end{array}\right.$$

Matrix representation of (oto)			*of (taa)*		
$T_{1,3}$	T_1	T_3	$T_{1,3}$	$F_{2,3}$	F_3
T_1	$f_{1,2}$	$T_{1,3}$	$F_{2,3}$	$f_{1,2}$	F_2
T_3	$T_{1,3}$	$F_{2,3}$	F_3	F_2	$F_{2,3}$

```
T1,3 : truth value true for systems 1 an3

f1: value false for system 1 (= f1)
f2: value true for system 2 (= t2)

F2,3: values false for systems 2, 3

t: transjunction
o: disjunction
a: conjunction
(this terminology (o, a, t, i, j ) holds
for the ML implementation)
```

Diagramm 12 **Tableaux rules for (or trans or)**

$$\frac{t_1\ X\ \vee <> \vee\ Y}{t_1\ X \;\Big|\; t_1\ Y \;\Big\|\; \begin{array}{c} f_2\ X \\ t_2\ Y \end{array} \;\Big|\; \begin{array}{c} t_2\ X \\ f_2\ Y \end{array}} \qquad \frac{f_1\ X\ \vee <> \vee\ Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array} \;\Big\|\; \begin{array}{c} t_2\ X \\ t_2\ Y \end{array}}$$

$$\frac{t_2\ X\ \vee <> \vee\ Y}{\begin{array}{c} t_2\ X \\ t_2\ Y \end{array}} \qquad \frac{f_2\ X\ \vee <> \vee\ Y}{\begin{array}{c} f_2\ X \\ f_2\ Y \end{array}}$$

$$\frac{t_3\ X\ \vee <> \vee\ Y}{t_3\ X \;\Big|\; t_3\ Y \;\Big\|\; \begin{array}{c} f_2\ X \\ t_2\ Y \end{array} \;\Big|\; \begin{array}{c} t_2\ X \\ f_2\ Y \end{array}} \qquad \frac{f_3\ X\ \vee <> \vee\ Y}{\begin{array}{c} f_3\ X \\ f_3\ Y \end{array} \;\Big\|\; \begin{array}{c} f_2\ X \\ f_2\ Y \end{array}}$$

$$(\vee <> \vee) : L^{(3)} * L^{(3)} \longrightarrow L^{(3)} : \left[(L_1 \,||\, L_2), L_2, \left(L_3 \,||\, L_2\right)\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[disjunction]{} L_1 \,||\, L_2 \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct\ <>]{} L_2 : \left\{\begin{array}{l} f_2 * t_2 \to t_1, t_3 \\ t_2 * f_2 \to t_1, t_3 \end{array}\right. \\ Log_3 : L_3 * L_3 \xrightarrow[disjunction]{} L_3 \,||\, L_2 \end{array}\right.$$

$$\frac{t_1\ X\ J <> J\ Y}{t_1\ J \left\| \begin{array}{c} f_2\ X \\ t_2\ Y \end{array} \right| \begin{array}{c} t_2\ X \\ f_2\ Y \end{array}} \qquad \frac{f_1\ X\ J <> J\ Y}{f_1\ J \left\| \begin{array}{c} t_2\ X \\ t_2\ Y \end{array} \right.} \qquad \frac{t_1\ X\ <> JJ\ Y}{\begin{array}{c} t_1\ X \\ t_1\ Y \end{array}} \qquad \frac{f_1\ X\ <> JJ\ Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}}$$

$$\frac{t_2\ X\ J <> J\ Y}{\begin{array}{c} t_2\ X \\ t_2\ Y \end{array}} \qquad \frac{f_2\ X\ J <> J\ Y}{\begin{array}{c} f_2\ X \\ f_2\ Y \end{array}} \qquad \frac{t_2\ X\ <> JJ\ Y}{J_2 \left| \begin{array}{c} f_1\ X \\ f_1\ Y \end{array} \right.} \qquad \frac{f_2\ X\ <> JJ\ Y}{J_2 \left\| \begin{array}{c} f_1\ X \\ t_1\ Y \end{array} \right| \begin{array}{c} t_1\ X \\ f_1\ Y \end{array}}$$

$$\frac{t_3\ X\ J <> J\ Y}{t_3\ J \left\| \begin{array}{c} f_2\ X \\ t_2\ Y \end{array} \right| \begin{array}{c} t_2\ X \\ f_2\ Y \end{array}} \qquad \frac{f_3\ X\ J <> J\ Y}{f_3\ J \left\| \begin{array}{c} f_2\ X \\ f_2\ Y \end{array} \right.} \qquad \frac{t_3\ X\ <> JJ\ Y}{J_3 \left\| \begin{array}{c} t_1\ X \\ t_1\ Y \end{array} \right.} \qquad \frac{f_3\ X\ <> JJ\ Y}{J_3 \left\| \begin{array}{c} f_1\ X \\ t_1\ Y \end{array} \right| \begin{array}{c} t_1\ X \\ f_1\ Y \end{array}}$$

For total transjunctions and junctions.

J is representing junctional operators. The compposition of (J<>J) has, as for all other cases too, to fuliil the conditions of mediation (CM).

$$(J <> J) : L^{(3)} * L^{(3)} \longrightarrow L^{(3)} : \left[(L_1 \,||\, L_2), L_2, \left(L_3 \,||\, L_2 \right) \right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[junct\ J]{} L_1 \,||\, L_2 \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct\ <>]{} L_2 : \begin{cases} f_2 * t_2 \rightarrow t_1, t_3 \\ t_2 * f_2 \rightarrow t_1, t_3 \end{cases} \\ Log_3 : L_3 * L_3 \xrightarrow[junct\ J]{} L_3 \,||\, L_2 \end{array} \right.$$

$$(<> JJ) : L^{(3)} * L^{(3)} \longrightarrow L^{(3)} : \left[L_1, \left(L_2 \,||\, L_1 \right), \left(L_3 \,||\, L_1 \right) \right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[transjunct\ <>]{} L_1 : \begin{cases} f_1 * t_1 \rightarrow f_2, f_3 \\ t_1 * f_1 \rightarrow f_2, f_3 \end{cases} \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct]{} L_2 \,||\, L_1 \\ Log_3 : L_3 * L_3 \xrightarrow[junct\ J]{} L_3 \,||\, L_1 \end{array} \right.$$

2.2.1 A Classification of transjunctions

We can distinguish between *total* and *partial* transjunctions.

Total transjunctions can be *full* total transjunctions (<>) or *reduced* (<>', '<>) total transjunctions.

Full total transjunctions can be separated into equivalential or into kontra-valential full transjunctions.

Partial transjunction, also called conditional transjunctions, can be classified as *replicative* (<) and *implicative* (>) partial transjunctions.

Implicative and replicative partial transjunctions can be devided into *conjunctive* and *disjunctive* parts.

For systems with more than 3 contexture a new type of transjunctions and its differentiations occurs: the *global* transjunction. In contrast to the above transjunctions, no value is repeated in global transjunctions. Each place of the morphogram is occupied by a different value from different systems.

This terminology is modeled, obviously, on the names of the classical junctions, other classifications are in use, too.

2.2.2 Conditional transjunctivity

$$\frac{t_1\ X\ J \triangleright J\ Y}{t_1\ J \left\| \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}\right.} \qquad \frac{f_1\ X\ J \triangleright J\ Y}{f_1\ J \left\| \begin{matrix} t_2\ X \\ t_2\ Y \end{matrix}\right.} \qquad \frac{t_1\ X\ J > J\ Y}{t_1\ J \left\| \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}\right.} \qquad \frac{f_1\ X\ J > J\ Y}{f_1\ J \left\| \begin{matrix} t_2\ X \\ t_2\ Y \end{matrix} \right| \begin{matrix} f_2\ X \\ t_2\ X \end{matrix}}$$

$$\frac{t_2\ X\ J \triangleright J\ Y}{\begin{matrix} t_2\ X \\ t_2 Y \end{matrix}} \qquad \frac{f_2\ X\ J \triangleright J\ Y}{f_2\ X} \qquad \frac{t_2\ X\ J > J\ Y}{\begin{matrix} t_2\ X \\ t_2 Y \end{matrix} \left| \begin{matrix} f_2\ X \\ t_2\ Y \end{matrix}\right.} \qquad \frac{f_2\ X\ J > J\ Y}{\begin{matrix} f_2\ X \\ f_2\ Y \end{matrix}}$$

$$\frac{t_3\ X\ J \triangleright J\ Y}{t_3\ J \left\| \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}\right.} \qquad \frac{f_3\ X\ J \triangleright J\ Y}{f_3\ J \,\|\, f_2\ X} \qquad \frac{t_3\ X\ J > J\ Y}{t_3\ J \left\| \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}\right.} \qquad \frac{f_3\ X\ J > J\ Y}{f_3\ J \left\| \begin{matrix} f_2\ X \\ f_2\ Y \end{matrix}\right.}$$

Conditional transjunction, also called implicative (>) and replicative (<) transjunctions, only one part of the two alternatives is chosen.

$$(J > J) : L^{(3)} * L^{(3)} \rightarrow L^{(3)} : \left[(L_1 \,||\, L_3), L_2, \left(L_3 \,||\, L_2\right)\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[junct\ J]{} L_1 \,||\, L_2 \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct\ >]{} L_2 : \begin{cases} f_2 * t_2 \rightarrow t_1, t_3 \\ t_2 * f_2 \rightarrow f_1, t_2 \end{cases} \\ Log_3 : L_3 * L_3 \xrightarrow[junct\ J]{} L_3 \,||\, L_2 \end{array}\right.$$

$$(J < J) : L^{(3)} * L^{(3)} \rightarrow L^{(3)} : \left[(L_1 \,||\, L_3), L_2, \left(L_3 \,||\, L_2\right)\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[junct\ J]{} L_1 \,||\, L_2 \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct\ <]{} L_2 : \begin{cases} f_2 * t_2 \rightarrow f_1, t_2 \\ t_2 * f_2 \rightarrow t_1, f_3 \end{cases} \\ Log_3 : L_3 * L_3 \xrightarrow[junct\ J]{} L_3 \,||\, L_1 \end{array}\right.$$

$$\frac{t_1\ X\ J < J\ Y}{t_1\ J \left\| \begin{matrix} f_2\ X \\ t_2\ Y \end{matrix}\right.} \qquad \frac{f_1\ X\ J < J\ Y}{f_1\ J \left\| \begin{matrix} t_2\ X \\ t_2\ Y \end{matrix} \right| \begin{matrix} t_2\ X \\ f_2 X \end{matrix}}$$

$$\frac{t_2\ X\ J < J\ Y}{\begin{matrix} t_2\ X \\ t_2 Y \end{matrix} \left| \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix}\right.} \qquad \frac{f_2\ X\ J < J\ Y}{\begin{matrix} f_2 X \\ f_2 Y \end{matrix}}$$

$$\frac{t_3\ X\ J < J\ Y}{t_3\ J \left\| \begin{matrix} f_2\ X \\ t_2\ Y \end{matrix}\right.} \qquad \frac{f_3\ X\ J < J\ Y}{f_3\ J \left\| \begin{matrix} f_2\ X \\ f_2\ Y \end{matrix}\right.}$$

$$\left[\begin{matrix} f_2\ X \\ f_2\ Y \end{matrix} \left| \begin{matrix} t_2\ X \\ f_2\ Y \end{matrix} \right. \right] = f_2\ X$$

2.2.3 implicative and replicative transjunctivity

```
both wrong !!!
```

$$\frac{t_1\ X\ J \triangleleft J\ Y}{t_1\ J\ \left\| \begin{array}{c|c} t_1\ X & f_2\ X \\ t_1\ Y & t_2\ Y \end{array}\right.} \qquad \frac{f_1\ X\ J \triangleleft J\ Y}{f_1\ J\ \left\| \begin{array}{c|c} f_1\ X & t_2\ X \\ f_1\ Y & f_2\ Y \end{array}\right.} \qquad \frac{t_1\ X\ J \triangleright J\ Y}{t_1\ J\ \left\| \begin{array}{c} t_2\ X \\ f_2\ Y \end{array}\right.} \qquad \frac{f_1\ X\ J \triangleright J\ Y}{f_1\ J\ \left\| \begin{array}{c} t_2\ X \\ t_2\ Y \end{array}\right.}$$

$$\frac{t_2\ X\ J \triangleleft J\ Y}{\begin{array}{c} t_2\ X \\ t_2 Y \end{array}} \qquad \frac{f_2\ X\ J \triangleleft J\ Y}{\begin{array}{c} f_2\ X \\ f_2\ Y \end{array}} \qquad \frac{t_2\ X\ J \triangleright J\ Y}{\begin{array}{c} t_2\ X \\ t_2 Y \end{array}} \qquad \frac{f_2\ X\ J \triangleright J\ Y}{\begin{array}{c|c} f_2\ X & f_2\ X \\ f_2\ Y & f_2\ Y \end{array}}$$

$$\frac{t_3\ X\ J \triangleleft J\ Y}{t_3\ J\ \left\| \begin{array}{c} f_2\ X \\ t_2\ Y \end{array}\right.} \qquad \frac{f_3\ X\ J \triangleleft J\ Y}{f_3\ J\ \left\| \begin{array}{c} f_2\ X \\ f_2\ Y \end{array}\right.} \qquad \frac{t_3\ X\ J \triangleright J\ Y}{t_3\ J\ \left\| \begin{array}{c} t_2\ X \\ f_2\ Y \end{array}\right.} \qquad \frac{f_3\ X\ J \triangleright J\ Y}{f_3\ J\ \left\| \begin{array}{c|c} f_2\ X & f_2\ X \\ f_2\ Y & f_2\ Y \end{array}\right.}$$

Systematics and semantics

$$(J \triangleright J) : L^{(3)} * L^{(3)} \rightarrow L^{(3)} : \left[(L_1 \| L_3), L_2, (L_3 \| L_2)\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[junct\ J]{} L_1 \| L_2 \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct\ \triangleright]{} L_2 : \begin{cases} f_2 * t_2 \rightarrow f_2,\ f_3 \\ t_2 * f_2 \rightarrow t_1,\ t_3 \end{cases} \\ Log_3 : L_3 * L_3 \xrightarrow[junct\ J]{} L_3 \| L_2 \end{array}\right.$$

$$(J \triangleleft J) : L^{(3)} * L^{(3)} \rightarrow L^{(3)} : \left[(L_1 \| L_3), L_2, (L_3 \| L_2)\right]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[junct\ J]{} L_1 \| L_2 \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct\ \triangleleft]{} L_2 : \begin{cases} f_2 * t_2 \rightarrow t_1,\ t_3 \\ t_2 * f_2 \rightarrow t_2,\ f_1 \end{cases} \\ Log_3 : L_3 * L_3 \xrightarrow[junct\ J]{} L_3 \| L_1 \end{array}\right.$$

2.2.4 Reductional transjunctive constellations

$$\frac{t_1\;X\;J > J\;Y}{t_1\;J\;\left\|\begin{array}{c|c} t_2\;X & f_2\;X \\ f_2\;Y & t_2\;Y \end{array}\right.} \qquad \frac{f_1\;X\;J > J\;Y}{f_1\;J\;\left\|\begin{array}{c} t_2\;X \\ t_2\;Y \end{array}\right.} \qquad \frac{t_1\;X\;J < J\;Y}{t_1\;J} \qquad \frac{f_1\;X\;J < J\;Y}{f_1\;J}$$

$$\frac{t_2\;X\;J > J\;Y}{\begin{array}{c} t_2\;X \\ t_2\;Y \end{array}} \qquad \frac{f_2\;X\;J > J\;Y}{\begin{array}{c} f_2\;X \\ f_2\;Y \end{array}} \qquad \frac{t_2\;X\;J < J\;Y}{\begin{array}{c} t_2\;X \\ t_2\;Y \end{array}} \qquad \frac{f_2\;X\;J < J\;Y}{\begin{array}{c} f_2\;X \\ f_2\;Y \end{array}}$$

$$\frac{t_3\;X\;J > J\;Y}{t_3\;J} \qquad \frac{f_3\;X\;J > J\;Y}{f_3\;J} \qquad \frac{t_3\;X\;J < J\;Y}{t_3\;J\;\left\|\begin{array}{c|c} t_2\;X & f_2\;X \\ f_2\;Y & t_2\;Y \end{array}\right.} \qquad \frac{f_3\;X\;J < J\;Y}{f_3\;J\;\left\|\begin{array}{c} f_2\;X \\ f_2\;Y \end{array}\right.}$$

Total transjunctions, but reduced to only one sub-system interaction.

Reduced transjunctions occurs for all types of transjunctions. It has to be applied generally for all 8 transjunctional types.
>’

2.2.5 Composed transjunctional constellations

Logical operators with more than one transjunction can be composed out of the general tableaux rules for single transjunctions. Like for all mediated compositions they have to fulfil the conditions of mediation (CM).

$$\begin{array}{l} Composition\ of\ transjunctions \\ (trans, J, J) \oplus (J, trans, J) = (trans, trans, J) \\ Example: \\ (<>, J, J) \oplus (J, <>, J) = (<>, <>, J) \\ (<>, J, J) \in CM\ and\ (J, <>, J) \in CM \Rightarrow (<>, <>, J) \in CM \end{array}$$

Systematic semantics

$$(<><>J): L^{(3)} * L^{(3)} \xrightarrow[transtransjunct]{} L^{(3)} : [(L_1 \,||\, L_2), (L_2 \,||\, L_1), (L_3 \,||\, L_2 \,||\, L_1)]$$

$$\left[\begin{array}{l} Log_1 : L_1 * L_1 \xrightarrow[transjunct<>]{} L_1 \,||\, L_2 : \begin{cases} f_1 * t_1 \to f_2, f_3 \\ t_1 * f_1 \to f_2, f_3 \end{cases} \\ Log_2 : L_2 * L_2 \xrightarrow[transjunct\ <>]{} L_2 \,||\, L_1 : \begin{cases} f_2 * t_2 \to t_1, t_3 \\ t_2 * f_2 \to t_1, t_3 \end{cases} \\ Log_3 : L_3 * L_3 \xrightarrow[junct\ J]{} L_3 \,||\, L_2 \,||\, L_1 \end{array}\right.$$

Archtectonics

PM	$O1$	$O2$	$O3$
$M1$	$log1$	$log1$	$log1$
$M2$	$\varnothing$	$log2$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$log3$

PM	$O1$	$O2$	$O3$
$M1$	$log1$	$\varnothing$	$\varnothing$
$M2$	$log2$	$log2$	$log2$
$M3$	$\varnothing$	$\varnothing$	$log3$

PM	$O1$	$O2$	$O3$
$M1$	$log1$	$log1$	$log1$
$M2$	$log2$	$log2$	$log2$
$M3$	$\varnothing$	$\varnothing$	$log3$

$Op < trans, J, J >$:

PM	$O1$	$O2$	$O3$
$M1$	$tran$	$trans$	$trans$
$M2$	$\varnothing$	J	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	J

$Op < J, trans, J >$:

PM	$O1$	$O2$	$O3$
$M1$	J	$\varnothing$	$\varnothing$
$M2$	$trans$	$tran$	$trans$
$M3$	$\varnothing$	$\varnothing$	J

$Op < trans, trans, J >$:

PM	$O1$	$O2$	$O3$
$M1$	$tran$	$trans$	$trans$
$M2$	$trans$	$tran$	$trans$
$M3$	$\varnothing$	$\varnothing$	J

Diagramm 13 **Composition of a double transjunction tableaux**

$$Op < trans, J, J > \oplus\ Op < J, trans, J > = Op < trans, trans, J >:$$

$$\frac{t_1\ X\ <>JJ\ Y}{\begin{array}{c} t_1\ X \\ t_1\ Y \end{array}} \qquad \frac{t_1\ X\ J<>J\ Y}{t_1\ J \left\| \begin{array}{c|c} f_2\ X & t_2\ X \\ t_2\ Y & f_2\ Y \end{array}\right.} \qquad \frac{t_1\ X\ <><>J\ Y}{\begin{array}{c||c|c} t_1\ X & f_2\ X & t_2\ X \\ t_1\ Y & t_2\ Y & f_2\ Y \end{array}}$$

$$\frac{f_1\ X\ <>JJ\ Y}{\begin{array}{c} f_1\ X \\ f_1\ Y \end{array}} \qquad \frac{f_1\ X\ J<>J\ Y}{f_1\ J \left\| \begin{array}{c} t_2\ X \\ t_2\ Y \end{array}\right.} \qquad \frac{f_1\ X\ <><>J\ Y}{\begin{array}{c||c} f_1\ X & t_2\ X \\ f_1\ Y & t_2\ Y \end{array}}$$

$$\frac{t_2\ X\ <>JJ\ Y}{t_2\ J \left\| \begin{array}{c} f_1\ X \\ f_1\ Y \end{array}\right.} \qquad \frac{t_2\ X\ J<>J\ Y}{\begin{array}{c} t_2\ X \\ t_2\ Y \end{array}} \qquad \frac{t_2\ X\ <><>J\ Y}{\begin{array}{c|c} t_2\ X & f_1\ X \\ t_2\ Y & f_1\ Y \end{array}}$$

$$\frac{f_2\ X\ J<>J\ Y}{\begin{array}{c} f_2\ X \\ f_2\ Y \end{array}} \qquad \frac{f_2\ X\ <>JJ\ Y}{f_2\ J \left\| \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}\right.} \qquad \frac{f_2\ X\ <><>J\ Y}{\begin{array}{c||c|c} f_2\ X & f_1\ X & t_1\ X \\ f_2\ Y & t_1\ Y & f_1\ Y \end{array}}$$

$$\frac{t_3\ X\ <>JJ\ Y}{t_3\ J \left\| \begin{array}{c} t_1\ X \\ t_1\ Y \end{array}\right.} \qquad \frac{t_3\ X\ J<>J\ Y}{t_3\ J \left\| \begin{array}{c|c} f_2\ X & t_2\ X \\ t_2\ Y & f_2\ Y \end{array}\right.} \qquad \frac{t_3\ X\ <><>J\ Y}{t_3\ J \left\| \begin{array}{c||c|c} t_1\ X & f_2\ X & t_2\ X \\ t_1\ Y & t_2\ Y & f_2\ Y \end{array}\right.}$$

$$\frac{f_3\ X\ <>JJ\ Y}{f_3\ J \left\| \begin{array}{c|c} f_1\ X & t_1\ X \\ t_1\ Y & f_1\ Y \end{array}\right.} \qquad \frac{f_3\ X\ J<>J\ Y}{f_3\ J \left\| \begin{array}{c} f_2\ X \\ f_2\ Y \end{array}\right.} \qquad \frac{f_3\ X\ <><>J\ Y}{f_3\ J \left\| \begin{array}{c||c|c} f_2\ X & f_1\ X & t_1\ X \\ f_2\ Y & t_1\ Y & f_1\ Y \end{array}\right.}$$

Composition as definition

The composition operation Op+ is not simply a conjunction between two formulas because it overwrites the J-part.

But what does a conjunction of the parts mean?

$$
\begin{array}{l}
H = (X\ J <> J\ Y) \wedge\wedge\wedge (X\ <> \wedge\wedge\ Y\ \ ) \\
t_1\ (X\ J <> J\ Y) \wedge\wedge\wedge (X\ <> \wedge\wedge\ Y\ \ ) \\
t_1\ (X\ J <> J\ Y) \\
t_1\ (X\ <> \wedge\wedge\ Y\ \ ) \\
t_1\ X \\
t_1\ Y \\
t_1\ J \quad \left\| \begin{array}{c|c} f_2\ X & t_2\ X \\ t_2\ Y & f_2\ Y \end{array} \right.
\end{array}
$$

Part 1: Tabl1

$$
\begin{array}{l}
f_1\ ((X\ J <> J\ Y) \wedge\wedge\wedge (X\ <> \wedge\wedge\ Y\ \ )) \to\to\to X\ <><> J\ Y \\
t_1\ (X\ J <> J\ Y) \wedge\wedge\wedge (X\ <> \wedge\wedge\ Y\ \ ) \\
f_1\ X\ <><> J\ Y \\
t_1\ (X\ J <> J\ Y) \\
t_1\ (X\ <> \wedge\wedge\ Y\ \ ) \\
t_1\ X \\
t_1\ Y \\
t_1\ J \quad \left\| \begin{array}{c|c} f_2\ X & t_2\ X \\ t_2\ Y & f_2\ Y \end{array} \right. \\
\begin{array}{c|c} f_1\ X & t_2\ X \\ f_1\ Y & t_2\ Y \\ xx & xx \end{array}
\end{array}
$$

Part 2: Tabl2

$$
\begin{array}{l}
f_1\ \left(\left(X\ \ J <> J\ \ Y\right) \wedge\wedge\wedge \left(X\ \ <> \wedge\wedge\ Y\ \ \right)\right) \rightarrow\rightarrow\rightarrow\ X\ \ <><> J\ \ Y \\
t_3\ \left(X\ \ J <> J\ \ Y\right) \wedge\wedge\wedge \left(X\ \ <> \wedge\wedge\ Y\ \ \right) \\
f_3\ \ X\ \ <><> J\ \ Y \\
t_3\ \left(X\ \ J <> J\ \ Y\right) \\
t_3\ \left(X\ \ <> JJ\ \ Y\ \ \right) \\
\quad t_3\ \ J \left\| \begin{array}{l} t_1\ \ X \\ t_1\ \ Y \end{array} \right. \\
\quad t_3\ \ J \left\| \begin{array}{l} f_1\ \ X \\ t_1\ \ Y \end{array} \right| \begin{array}{l} t_1\ \ X \\ f_1\ \ Y \end{array} \\
\ ? \qquad\qquad xx \qquad\quad xx \\
\quad f_3\ \ J \left\| \begin{array}{l} f_1\ \ X \\ t_1\ \ Y \end{array} \right| \begin{array}{l} t_1\ \ X \\ f_1\ \ Y \end{array} \left\| \begin{array}{l} f_2\ \ X \\ f_2\ \ Y \end{array} \right. \\
\quad xx \qquad\ xx \qquad\ xx \qquad\ ?? \Rightarrow closed\ \ with\ \ Tabl_1
\end{array}
$$

2.3 Rules and Definitions in PCL$^{(3)}$

2.3.1 Unary and binary constellations

$$\neg_i \left(\neg_i \ X^{(3)}\right) \Leftrightarrow\Leftrightarrow\Leftrightarrow \ X^{(3)}, \ i = 1, 2$$
$$\neg_1 \left(\neg_2 \left(\neg_1 \ X^{(3)}\right)\right) \Leftrightarrow\Leftrightarrow\Leftrightarrow \ \neg_2 \left(\neg_1 \left(\neg_2 \ X^{(3)}\right)\right)$$

$$\begin{aligned} \neg_3 \ X^{(3)} &:= \neg_1 \left(\neg_2 X^{(3)}\right) \\ \neg_4 \ X^{(3)} &:= \neg_2 \left(\neg_1 X^{(3)} \right) \\ \neg_5 \ X^{(3)} &:= \neg_1 \left(\neg_2 \left(\neg_1 \ X^{(3)}\right)\right) \\ &:= \neg_2 \left(\neg_1 \left(\neg_2 \ X^{(3)}\right)\right) \end{aligned}$$

$$\neg_1 \ X^{(3)} \ \vee^{(3)} \ Y^{(3)} = X^{(3)} \rightarrow \oplus\oplus \ Y^{(3)}$$
$$\neg_2 \ X^{(3)} \ \vee^{(3)} \ Y^{(3)} = X^{(3)} \ \vee \rightarrow \vee \ Y^{(3)}$$

2.3.2 Dualities for junctions

$$\neg_5 \left(\neg_5 \ X^{(3)} \wedge \vee \wedge \ \neg_5 \ Y^{(3)}\right) \Leftrightarrow\Leftrightarrow\Leftrightarrow \ X^{(3)} \vee \wedge \vee \ Y^{(3)}$$
$$\neg_5 \left(\neg_5 \ X^{(3)} \wedge \vee \wedge \ \neg_5 \ Y^{(3)}\right) \Leftrightarrow\Leftrightarrow\Leftrightarrow \ X^{(3)} \vee \wedge \vee \ Y^{(3)}$$

$$\neg_5 \left(\neg_5 \ X^{(3)} \ \oplus\oplus\oplus \ \neg_5 \ Y^{(3)}\right) \Leftrightarrow\Leftrightarrow\Leftrightarrow \ X^{(3)} \ \otimes\otimes\otimes \ Y^{(3)}$$
$$\oplus, \otimes = \{\wedge, \vee\}$$

2.3.3 Dualities for transjunctions

Full total transjunction defined as a conjunction of 2 reductional transjunctions. (???)

$$\left(X\ \ J <> J\ \ Y\right) :=== \left(X\ \ J > J\ \ Y\right) \wedge\wedge\wedge \left(X\ \ J < J\ \ Y\right)$$

Ternary and n-ary constellations

2.4 Tableaux proofs in PolyLogics

2.4.1 Junctional and negational constellations

$$
\begin{array}{l}
polylog^{(3)}(\ id,\ id,\ red\ ) \\
\left[\begin{array}{l}
thematize\ (\ Boolean,\ definition\ ) \\
identify\ \ frames^{(3)} \\
\left[\begin{array}{l}
lambda\ (\ X^{(3)}\ \ Y^{(3)}) \\
lambda\ (\ neg_2\ ) \\
lambda\ (\ or^{(3)}\ ) \\
\left[\begin{array}{l}
identify\ \ frame^{1.1} \\
\left[\begin{array}{l}
define\ or^{1} \\
\left[\begin{array}{l}
lambda\ (\ X\ Y\ ) \\
\quad (\ perm_2\ \ X\ or^{1}\ Y)
\end{array}\right]
\end{array}\right]
\end{array}\right]
\left[\begin{array}{l}
identify\ \ frame^{2.2} \\
\left[\begin{array}{l}
define\ impl^{2} \\
\left[\begin{array}{l}
lambda\ (\ X\ Y\ ) \\
\quad (\ neg_2\ \ X\ or^{2}\ Y)
\end{array}\right]
\end{array}\right]
\end{array}\right] \\
\left[\begin{array}{l}
identify\ \ frame^{3.1} \\
\left[\begin{array}{l}
define\ or^{3} \\
\left[\begin{array}{l}
lambda\ (\ X\ Y\ ) \\
\quad (\ perm_2\ \ X\ or^{3}\ Y)
\end{array}\right]
\end{array}\right]
\end{array}\right]
\end{array}\right]
\end{array}\right]
\end{array}
$$

$$\neg_2\ X^{(3)}\ \vee^{(3)}\ Y^{(3)}\ =\ X^{(3)}\ \vee \rightarrow \vee\ Y^{(3)}$$

PM	$O1$	$O2$	$O3$	PM	$O1$	$O2$	$O3$
$M1$	$log1$	$\varnothing$	$\varnothing$	$M1$	or	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$log2$	$\varnothing$	$M2$	$\varnothing$	$impl$	$\varnothing$
$M3$	$log3$	$\varnothing$	$\varnothing$	$M3$	or	$\varnothing$	$\varnothing$

At a first glance it seems that our job can easily be realized by combining logics and to combine logics to combined logics, PolyLogics, seems to be an equally easy job. As usual, the devil is in the detail, and because PolyLogics are introduced on a very basic level, we are confronted, step by step, by such concrete problems.

2.4.2 Junctional constellations without negations

$$0.\ \left(X \wedge^{(3)} Y\right) \rightarrow^{(113)} Z \equiv X \rightarrow^{(113)} \left(Y \rightarrow^{(113)} Z\right)$$

$$1.\ f_1 \left(\left(X \wedge^{(3)} Y\right) \rightarrow^{(113)} Z\ .\rightarrow^{(113)}.\ X \rightarrow^{(113)} \left(Y \rightarrow^{(113)} Z\right)\right) \quad (0)$$

$$\begin{array}{ll}
2.\ t_1 \left(X \wedge^{(3)} Y\right) \rightarrow^{(3)} Z & (1) \\
3.\ f_1\ X \rightarrow^{(3)} \left(Y \rightarrow^{(3)} Z\right) & (1) \\
4.\ t_1 \left(X \wedge^{(3)} Y\right) & (2) \\
5.\ f_1\ Z & (2) \\
6.\ t_1\ X & (4) \\
7.\ t_1\ Y & (4)
\end{array}$$

$$8.\ f_1\ X \ \left|\ \begin{array}{ll} t_1 \left(Y \rightarrow^{(3)} Z\right) & (3) \\ f_1\ Y \mid t_1\ Z & (8) \end{array}\right.$$

$$\overline{\ xx \qquad xx \qquad xx\ }$$

2.4.3 Junctional constellations with negations

Diagramm 14 **Junctional formula**

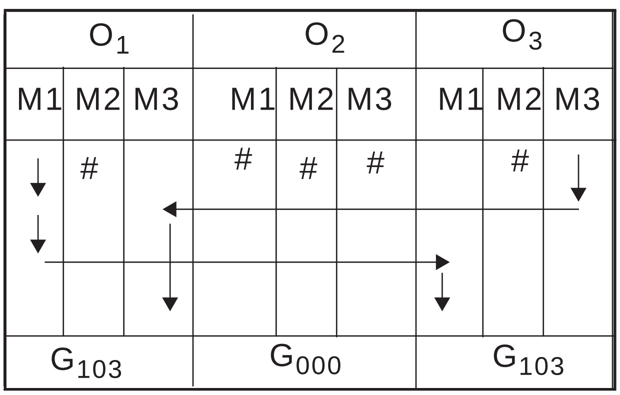
$$f_1\left(\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ Y^{(3)}\right) \rightarrow\rightarrow\rightarrow \left(X^{(3)}\ \vee\rightarrow\vee\ Y^{(3)}\right)\right)$$

$t_1\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ Y^{(3)}\right)$		$t_3\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ Y^{(3)}\right)$	
$f_1\left(X^{(3)}\ \vee\rightarrow\vee\ Y^{(3)}\right)$		$f_3\left(X^{(3)}\ \vee\rightarrow\vee\ Y^{(3)}\right)$	
$f_1\ X^{(3)}$		$f_3\ X^{(3)}$	
$f_1\ Y^{(3)}$		$f_3\ Y^{(3)}$	
$t_1\ \neg_2\ X^{(3)}$	$t_1\ Y^{(3)}$	$t_3\ \neg_2\ X^{(3)}$	$t_3\ Y^{(3)}$
$t_3\ X^{(3)}$	xx	$t_1\ X^{(3)}$	xx
??		??	

Incomparability occurs for the signatures f1 and t3, also for f3 and t1.

The tableau is decomposing the mediated formula into its sub-systems. Therefore the indices of the signatures have to be taken into account. There is no global signature fro them, say t and f, without indices.

In the tree example, the notational frame of different sub-systems OiMi is not used because the indices are doing the job.



Otherwise it would be even more clear that the signatures and terms are not comparable as the diagram shows.

That is, the branches to compare are localized in different sub-systems OiMj.

Our tableau development of the formula shows us that we are running into incompatibilities disproving the formula. But, interestingly, the definition of the local implication impl2 can be verified.

$$f_3\left(\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ Y^{(3)}\right)\rightarrow\rightarrow\rightarrow\left(X^{(3)}\ \vee\rightarrow\vee\ Y^{(3)}\right)\right)$$
$$t_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ Y^{(3)}\right)$$
$$f_2\left(X^{(3)}\ \vee\rightarrow\vee\ Y^{(3)}\right)$$
$$t_2\ X^{(3)}$$
$$f_2\ Y^{(3)}$$
$$\begin{array}{c|c} t_2\ \neg_2\ X^{(3)} & t_2\ Y^{(3)} \\ f_2\ X^{(3)} & xx \\ xx & \end{array}$$

The environment of impl2 is not to satisfy. Asymmetric permutations in binary logical function, (neg X opn Y), are often destroying the harmony of mediation of the whole function.

From a local point of view, the definition of impl via neg plus disj is working.

Obviously, the formula has to be set into a more complex environment to fulfill the conditions of mediation for the whole complexion. Another approach is to rethink the definitions of the involved implications.

Other methods of formalizing PCL may use additionally global values to tackle the problem. But global values don't tell us much about the local conditions. A mechanism of mappings could be introduced which is negotiating between local and global viewpoints and tries to resolve the local problem of disturbed harmony and incompatibility. But this would involve additional operators not yet accessible in the presented frameworks.

Comparability: Contradiction and Distance

Contradiction in PolyLogics is, until now, defined strictly local. The global aspect comes into play only with the collection of local situations. Thus, a formula is globally true iff it is true for all local sub-logics. There is nothing wrong with that.

Until now, strictly global considerations had been reductional, denying the local subs-system attributes. Thus reducing truth values to natural numbers as a set of truth values. Like in usual multi-valued logics, but surely with different use for functions.

A new approach can be introduced with the concept of *distance* between locally incomparable truth values. If a term is $true_i$ for $system_i$ and there is a term in $system_j$ with value $false_j$ and both occur in a common development, a new $system_k$ can be introduced reflecting in $system_k$ on the constellation ($true_i$, $false_j$) as the constellation ($true_k$, $false_k$) and observing a contradiction in $system_k$ with a specific distance between $system_i$ and $system_j$.

Thus, agent of $system_i$ insists on true, agent of $system_j$ insists on false.

And for an agent of a $system_k$, which is on a higher or lower level than both, it can be comparable because both are arguing still in the framework of semantics of truth and false. From the 3rd position it turns out to be a contradiction. But this new interpretation is not denying the incomparability between the local $system_i$ and $system_j$.

It could be called a *negotional mediation*. And is a resolution of a global conflictive situation. This negotiation as a re-interpretation happens between different local systems and is therefore not itself local but global. But the result of the re-interpretation is realized and localized in an own local system, thus, again local.

Thus, ($true_i$, $false_j$)=incomparabel, becomes ($true_k$, $false_k$)=contradiction.

Obviously, the old constellation has to be enlarged and re-interpreted by this modeling procedure. For this reason too, it isn't a reduction to global numeric truth values.

2.4.4 Balanced junctional formulas

Diagramm 15 **A DeMorgan formula**

$$f_1\left(\neg_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right)\longrightarrow\!\!\!\to\!\!\!\to\left(X^{(3)}\ \vee\wedge\vee\ Y^{(3)}\right)\right)$$

$$\begin{array}{c||c}
t_1\ \neg_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right) & \\
f_1\left(X^{(3)}\ \vee\wedge\vee\ Y^{(3)}\right) & t_3\ \neg_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right) \\
f_1\ X^{(3)} & f_3\left(X^{(3)}\ \vee\wedge\vee\ Y^{(3)}\right) \\
f_1\ Y^{(3)} & f_3\ X^{(3)} \\
t_1\ \neg_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right) & f_3\ Y^{(3)} \\
t_3\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right) & t_1\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right) \\
\begin{array}{c|c} t_3\ \neg_2\ X^{(3)} & t_3\ \neg_2\ Y^{(3)} \\ t_1\ X^{(3)} & t_1\ Y^{(3)} \\ xx & xx \end{array} &
\begin{array}{c|c} t_1\ \neg_2\ X^{(3)} & t_1\ \neg_2\ Y^{(3)} \\ t_3\ X^{(3)} & t_3\ Y^{(3)} \\ xx & xx \end{array}
\end{array}$$

$$f_3\left(\neg_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right)\longrightarrow\!\!\!\to\!\!\!\to\left(X^{(3)}\vee\wedge\vee\ Y^{(3)}\right)\right)$$

$$\begin{array}{c}
t_2\ \neg_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right) \\
f_2\left(\neg_2\ X^{(3)}\ \vee^{(3)}\ \neg_2\ Y^{(3)}\right) \\
f_2\left(X^{(3)}\ \vee\wedge\vee\ Y^{(3)}\right) \\
f_2\ \neg_2\ X^{(3)} \\
f_2\ \neg_2\ Y^{(3)} \\
t_2\ X^{(3)} \\
t_2\ Y^{(3)} \\
\begin{array}{c|c} f_2\ X^{(3)} & f_2\ Y^{(3)} \\ xx & xx \end{array}
\end{array}$$

Notational convention

If we know in which polylogical constellation we are working, all sorts of indices, indicating complexity, type of logic, etc., can be omitted.

If a formula is properly decomposable in its sub-systems we obviously also can omit its neighbor operators and variables and deal with the separated sub-systems only.

$$
\begin{array}{l}
f_3 \left(\neg_2 \left(\neg_2 \ X \ \vee\vee\vee \ \neg_2 \ Y \right) \rightarrow\rightarrow\rightarrow \left(X \vee\wedge\vee Y \right) \right) \\
\qquad t_2 \ \neg_2 \left(\neg_2 \ X \ \vee \ \neg_2 \ Y \right) \\
\qquad f_2 \left(\neg_2 \ X \ \vee \ \neg_2 \ Y \right) \\
\qquad f_2 \ \left(X \ \wedge Y \right) \\
\qquad f_2 \ \neg_2 \ X \\
\qquad f_2 \ \neg_2 \ Y \\
\qquad t_2 \ X \\
\qquad t_2 \ Y
\end{array}
$$

$$
\begin{array}{c|c}
f_2 \ X & f_2 \ Y \\
xx & xx
\end{array}
$$

2.4.5 Tautology property

The above formula H is closing for both signatures F1 and F3. The formula H is a tautology iff its tableaux Tabl close in all branches for the signatures F1 and F3.

A branch is closed iff it contains the signatures Ti, Fi for the same terms.

A tableaux is a connected graph with a tree structure. That is, with an origin and a succession of branches connected to the origin.

A polylogical complexion, a rhizome, is a ordered list of trees with an ordered set of origins. The order of the ordered set of origins is ruled by the proemial relation.

To use more familiar terminology we can say, a polylogical complexion is represented by a *list of trees*. More exactly it is a polycontextural list of trees. Polycontextural, because the listed trees are not only listed but mediated.

$$Pattern\ \{id, red\} : (S_{123} \to S_{113}\ ; S_{133}\ ; S_{131}) :$$
$$H^{(3)} \in\ taut\ \ iff\ \not\geq\!\!E\, TF^{1} H^{(3)}\ \ simul\ \not\geq\!\!E\, TF^{3} H^{(3)}$$

Also the tautology test or other formula development can be done successively, especially in the manual case of formula developments, it is ruled by definition that these different developments have to be realized simultaneously, say in parallel. It is the same formula which is tested for the different sub-system related semantic attributes. This has consequences for the implementation not considered in the present presentation. Formally the consequences are related to the fact of polycontexturally distributed syntactical induction principle. The decomposition steps are ruled by induction and therefore each sub-system has its own induction.

The definition of the tautology attribute thus can be condensed to the following formulation:

$$H^{(3)} \in\ taut^{13}\ \ iff^{13}\ \not\geq\!\!E^{(3)} T^{(3)} F^{1} F^{3} H^{(3)}$$

2.4.6 Transjunctional constellations without negations

2.5 Reductional constellations with negations

Incompatible situations turned into reductions.

incompatibility

$$p^{(3)} \wedge\wedge\wedge \neg^{1} p^{(3)} \equiv [\bot, \varnothing, \varnothing]$$
$$p^{(3)} \wedge\wedge\wedge \neg^{2} p^{(3)} \equiv [\varnothing, \bot, \varnothing]$$
$$p^{(3)} \wedge\wedge\wedge \neg^{3} p^{(3)} \equiv [\varnothing, \varnothing, \bot]$$
$$p^{(3)} \vee\wedge\wedge \neg^{1} p^{(3)} \equiv [T, \varnothing, \varnothing]$$
$$p^{(3)} \wedge\vee\wedge \neg^{2} p^{(3)} \equiv [\varnothing, T, \varnothing]$$
$$p^{(3)} \wedge\wedge\vee \neg^{3} p^{(3)} \equiv [\varnothing, \varnothing, T]$$

reduction rules

$$p^{(3)} \wedge\wedge\wedge \neg^{1} p^{(3)} \equiv [\bot, p^{2}, p^{2}]$$
$$p^{(3)} \vee\wedge\wedge \neg^{1} p^{(3)} \equiv [T, p^{3}, p^{3}]$$
$$p^{(3)} \wedge\vee\wedge \neg^{2} p^{(3)} \equiv [p^{1}, \bot, p^{1}]$$
$$p^{(3)} \wedge\vee\wedge \neg^{2} p^{(3)} \equiv [p^{3}, T, p^{3}]$$
$$p^{(3)} \wedge\wedge\vee \neg^{3} p^{(3)} \equiv [p^{1}, \neg^{1} p^{1}, \bot]$$
$$p^{(3)} \wedge\wedge\vee \neg^{3} p^{(3)} \equiv [\neg^{2} p^{2}, p^{2}, T]$$

PM	$O1$	$O2$	$O3$
$M1$	$\bot$	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	p	$\varnothing$
$M3$	$\varnothing$	p	$\varnothing$

PM	$O1$	$O2$	$O3$
$M1$	T	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	$\varnothing$	p
$M3$	$\varnothing$	$\varnothing$	p

PM	$O1$	$O2$	$O3$
$M1$	p	$\varnothing$	p
$M2$	$\varnothing$	$\bot$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$\varnothing$

PM	$O1$	$O2$	$O3$
$M1$	$\varnothing$	$\varnothing$	$\varnothing$
$M2$	$\varnothing$	T	$\varnothing$
$M3$	p	$\varnothing$	p

PM	$O1$	$O2$	$O3$
$M1$	p	$\neg p$	$\varnothing$
$M2$	$\varnothing$	$\varnothing$	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	$\bot$

PM	$O1$	$O2$	$O3$
$M1$	$\varnothing$	$\varnothing$	$\varnothing$
$M2$	$\neg p$	p	$\varnothing$
$M3$	$\varnothing$	$\varnothing$	T

2.6 Transjunctional constellations and tableaux proofs

Step-wise concretization of the presentation. First, simple formulas can be written without considering the OM-structures, using only the rules of the sub-system-indices of the formulas. Second, especially for transjunctional formulas, the sub-system structure, O (=S), is involved but omitting the M-structure. Third, for full interactional and reflectional formulas, the whole OM-structure has to be used. The following diagrams show the development of a simple negational and transjunctional formula, using only the sub-system structure. All those notational forms are for manual use only and to guide necessary further implementations. A further step follows by the application of meta-rules of the term calculus. Finally, a semi-automated proof by the LOLA-implementation is presented. A machine oriented presentation can be set into a different scheme.

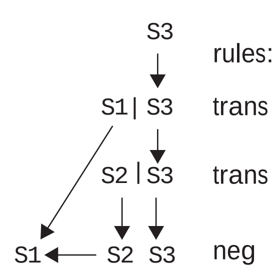
Diagramm 16 **Tableau presentation**

(0) f3 H1 = f3 ((X tr.et.et Y) .iij. N5 (N5 X vel.tr.vel N5 Y))

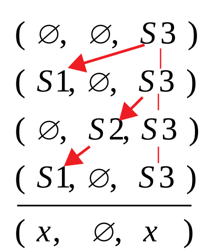
Nr	S1	S2	S3	Nr S3
1			t3 X tr.et.et Y	(0)
2			f3 N5 (N5 X vel.tr.vel N5 Y)	(0)
3	(S3)		t3 N5 X vel.tr.vel N5 Y	(2)
4	t1 X (1)		t3 X	(1)
5	t1 Y (1)	(S3)	t3 Y	(1)
6		t2 N5 X | f2 N5 X (3)	t3 N5 X | t3 N5 Y	(3)
7	(S2)	f2 N5 Y | t2 N5 Y (3)	f3 X | f3 Y	(6)
8	f1 X | t1 X (6)		x | x	
9	t1 Y | f1 Y (7)			
10	x | x			

Unification

$$\frac{\alpha^{(3)}}{(\alpha^3\ \ \gamma^1)\quad (\beta^3\ \ \delta^1)}$$

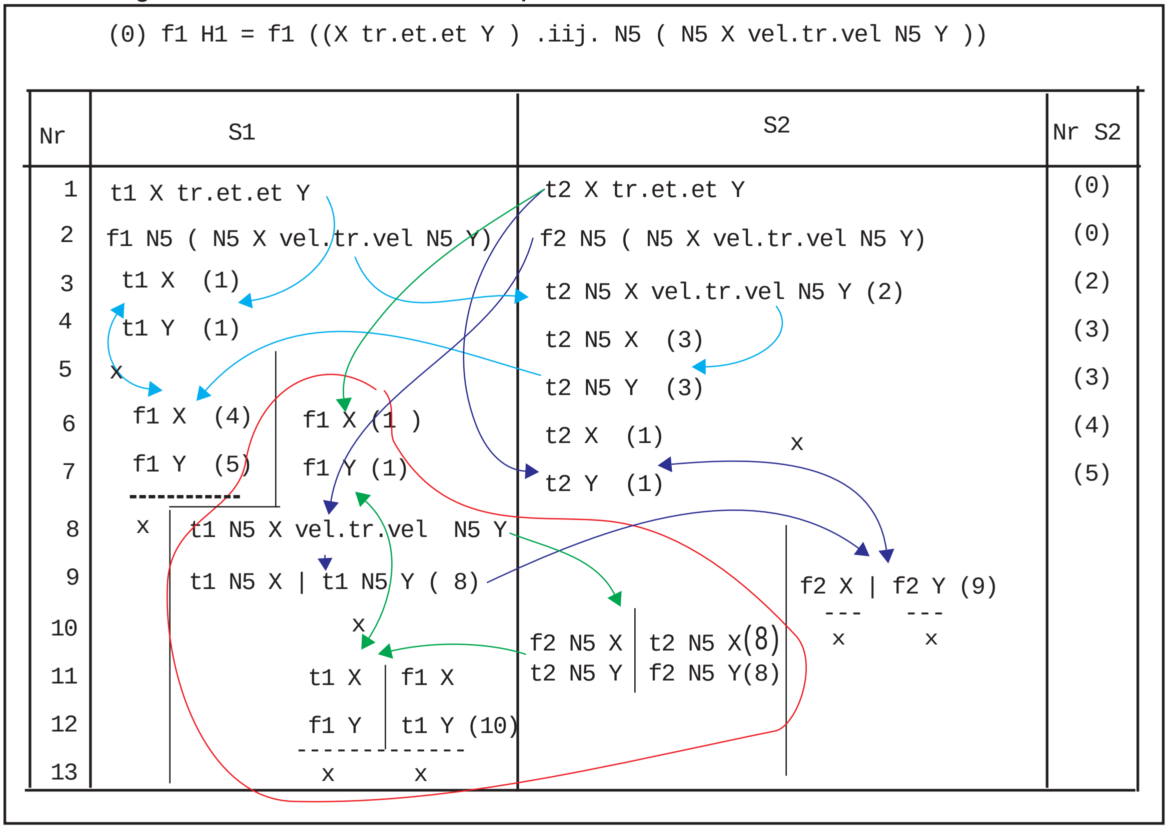


$$(\varnothing, \varnothing, S3M3) \downarrow (S1M3, \varnothing, S3M3) \downarrow (\varnothing, S2M3, S3M3) \downarrow (S1M3, \varnothing, S3M3)$$

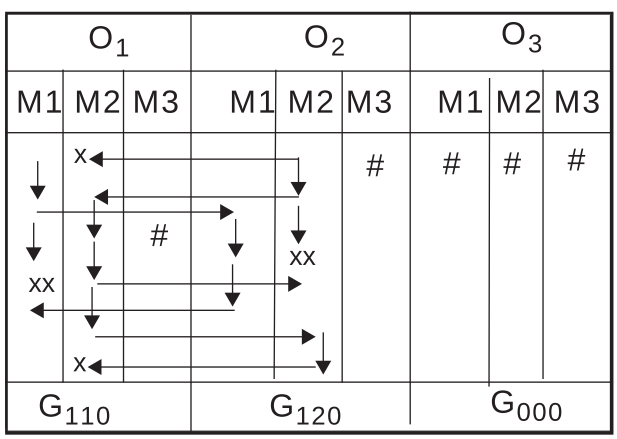


i,j: implication
tr: transjunction
et: conjunction
vel: disjunction

Diagramm 17 **Tableaux presentation**



Sub-systems S1 and S2 are closing directly, S1 at step 8 and S2 at step 9. This would be enough to close the tableaux for S1 and S2. But there is an additional part of the formula which is closing separately in S1, closing at step 13, encircled in red.

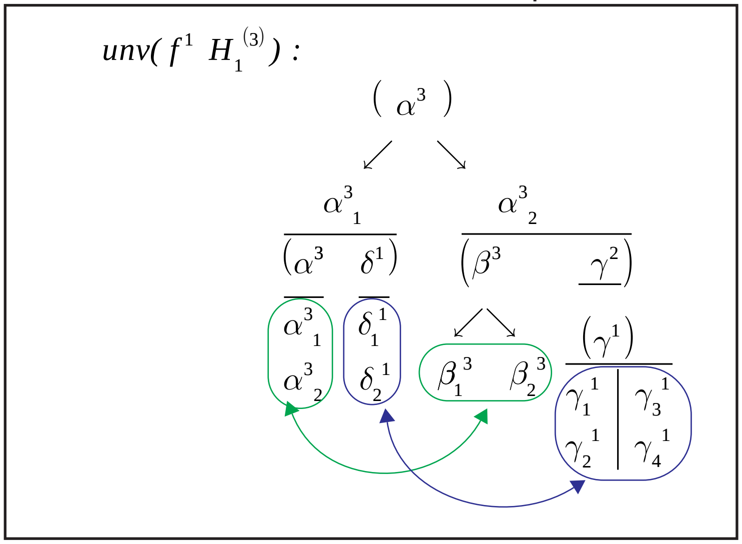


The different structural diagrams above should show clear enough how the formula development is working. The step-wise development of the formula guarantees the connectedness of the branches of the different trees, despite the jumps into other sub-systems, which are necessary to produce a semantic result on the base of the signatures.

2.6.1 Reduction of complexity by unification and meta-rules

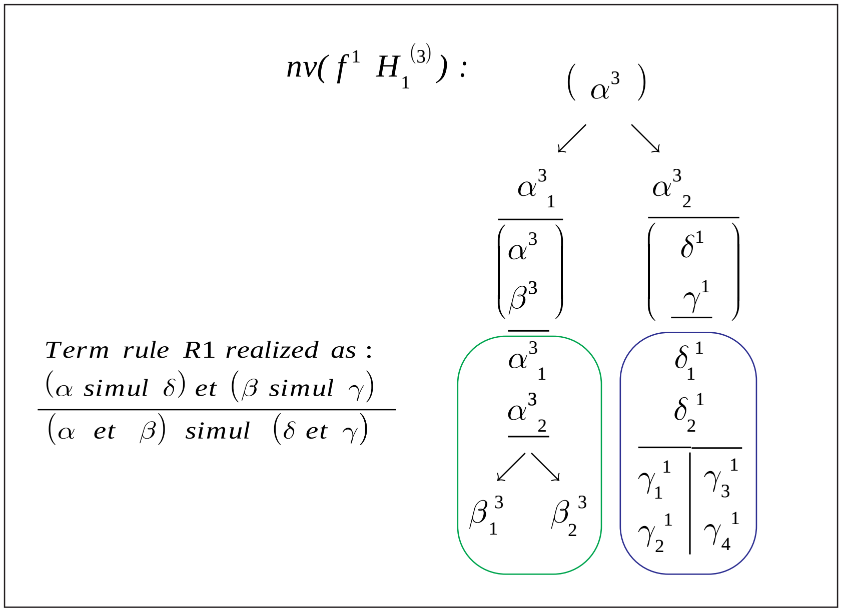
The concrete complexity of the tableaux tree of formula H1 can be reduced with the help of the unification method of Smullyan and, as a step further, with the application of some meta-rules over tableaux trees, that is, the term rules of polylogic. The diagram structure is represented by the indices of the sub-systems only.

Diagramm 18 **Unification tableaux tree development of f1H1**



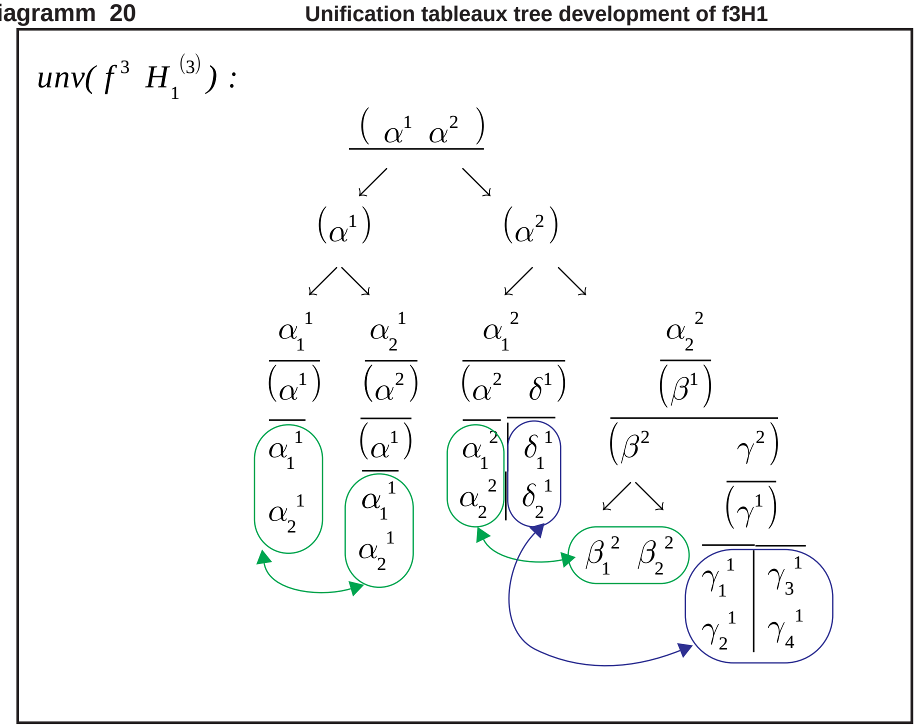
The example of unification preserves the tree structure induced by the formula. The next example, which applies additionally the term rule R1, is reducing and separating junctional and transjunctional parts, and therefore, offering a better economy of the sub-system parts which in the latter example are still distributed over the whole tree.

Diagramm 19 **Unification of f1H1 with meta-rules**



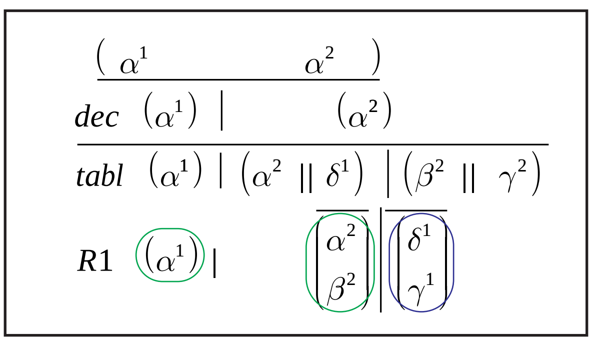
The tableau tree for f3H1 is even more confusing in its full concrete manual development, which first has to be studied before it can be reduced with the application of meta-rules to the following unification.

Diagramm 20 **Unification tableaux tree development of f3H1**



Again, the meta-rules are applied, producing a simple house holding of the branches and their sub-systems.

Diagramm 21 **Unification of f3H1 with meta-rules**



System changes, represented as a change in the index of the formula, say from gamma2 to gamma1, are produced by logical negators. Their rules are given with the tableaux rules for negations of signed formulas. Signed formulas are very convenient for complex logics but the polylogical rules are not depending on the method of signed formulas. Each method which is keeping the sub-system indices right is doing the job.

```
Comment:
dec:  decomposition of the complexion into alpha1 and alpha2 parts,
tabl: tableaux rules, producing intra-contextural sub-formulas,
R1:   term rule R1, collecting junctional and transjunctional
      parts separately.
```

2.7 Term representation of formula development H1

The formula development can be represented by a term calculus. The example includes rule R_0 and R_1 of the term list below. This, again, is a linearizations of the notational approach, but it is exactly what we need for machine readability of an implemented tableaux prover even if it has a tree representation to comfort the user.

Once, the rules of polylogical systems are clear, it is necessary to realize a (semi-)automated prove system. It makes not much sense to go one with manual work. Proof systems like LOLA are supporting the experimental exploration of polylogics, their results can be implemented to improve the theorem prover. Until now a strict mathematical approach hasn't produced much valuable insights beyond existing experimental knowledge.

Diagramm 22 **Term development for f_3 H1**

$f_3\ H:$

$$1.\ \left[\left[(t_3X\ et\ t_3Y)\ sim\ (t_1X\ et\ t_1Y)\right]\ et\ \left[(f_3X\ et\ f_3Y)\ sim\ ((f_1X\ et\ t_1Y)\ or\ (t_1X\ et\ f_1Y))\right]\right]$$

$$2.\ \left[\left[(t_3X\ et\ t_3Y)\ et\ (f_3X\ et\ f_3Y)\right]\ sim\ \left[(t_1X\ et\ t_1Y)\ et\ ((f_1X\ et\ t_1Y)\ or\ (t_1X\ et\ f_1Y))\right]\right] : R_1\ /1.$$

$$3.\ \left[\left[\varnothing_3\right]\ sim\ \left[((t_1X\ et\ t_1Y)\ et\ (f_1X\ et\ t_1Y))\ or\ ((t_1X\ et\ t_1Y)\ et\ (t_1X\ et\ f_1Y))\right]\right] \quad : R_0\ /2.$$

$$4.\ \left[\left[\varnothing_3\right]\ sim\ \left(\left[\varnothing_1\right]\ or\ \left[\varnothing_1\right]\right)\right]$$

$$5.\ \left[\left[\varnothing_3\right]\ sim\ \left[\varnothing_1\right]\right]$$

$$\frac{Term\ rule\ R1\ realized\ as:\quad (\alpha\ simul\ \delta)\ et\ (\beta\ simul\ \gamma)}{(\alpha\ et\ \beta)\ simul\ (\delta\ et\ \gamma)}$$

The structure of step-wise development of the terms can be shown by brackets only and using the LOLA notation. Term developments can be produced semi-automatically by LOLA.

$f_3\ H:$

$$1.\ \left[\left[(\ )\ |||\ (\ )\right]\ \&\&\ \left[(\ )\ |||\ ((\ )\ ||\ (\ ))\right]\right]$$

$$2.\ \left[(\ )\ \&\&\ (\ )\right]\ |||\ \left[(\ )\ \&\&\ ((\ )\ ||\ (\ ))\right]$$

$$3.\ \left[\left[\varnothing_3\right]\ |||\ \left[((\ )\ \&\&\ (\ ))\ ||\ ((\ )\ \&\&\ (\ ))\right]\right]$$

$$4.\ \left[\left[\varnothing_3\right]\ |||\ \left(\left[\varnothing_1\right]\ ||\ \left[\varnothing_1\right]\right)\right]$$

$$5.\ \left[\left[\varnothing_3\right]\ |||\ \left[\varnothing_1\right]\right]$$

Diagramm 23 **Semi-automated tree-proof of f3H in LOLA (1992)**

```
f3: ((X taa Y) iij n5 (n5 X oto n5 Y))


              |
      +------ && ----------+
      |                    |
t3: (X taa Y)    f3: n5 (n5 X oto n5 Y)


                                      |
                 +------------------ && ---------+
                 |                               |
                 |                       t3: (n5 X oto n5 Y)
      +-------  ///  -------+
      |                     |
      |              +--------------+
  +-- && --+         |      |       |
  |        |         |  +-- && --+  |
t3: X    t3: Y       |  |        |  |
                     |t1: X    t1: Y|
                     +--------------+


                                                                                                    |
                     +---------------------------------------------------------------------------  ///  -------+
                     |                                                                                         |
                     |                                                                                  +--------------+
         +---------- && ------------------------+                                                       |      |       |
         |                                      |                                                       |  +-- && --+  |
         |                                      |                                                       |  |        |  |
   +---- && ---+                 +----------  ///  ----------------------+                              |t1: X    t1: Y|
    |          |                 |                                       |                              +--------------+
+------+    +------+             |                 +--------------------------------------------+
|[t3:X]|    |[t3:Y]|       +---- || ---+           |                     |                      |
+------+    +------+       |           |           |         +---------- || ---------+          |
                        t3: n5 X    t3: n5 Y       |         |                       |          |
                                                   |         |                       |          |
                                                   |   +---- && ---+           +---- && ---+    |
                                                   |   |           |           |           |    |
                                                   |f2: n5 X    t2: n5 Y    t2: n5 X    f2: n5 Y|
                                                   +--------------------------------------------+


                                                                                        |
                                   +-------------------------------------------------  ///  -------+
                                   |                                                               |
                                   |                                                        +--------------+
               +---------------  ///  -----------------------+                              |      |       |
               |                                             |                              |  +-- && --+  |
               |                       +--------------------------------------------+       |  |        |  |
      +------- && ------+              |                     |                      |       |t1: X    t1: Y|
      |                 |              |         +---------- || ---------+          |       +--------------+
+------------+          |              |         |                       |          |
|[t3:X, t3:Y]|      +-- || --+         |         |                       |          |
+------------+      |        |         |   +---- && ---+           +---- && ---+    |
                  f3: X    f3: Y       |   |           |           |           |    |
                                       |f2: n5 X    t2: n5 Y    t2: n5 X    f2: n5 Y|
                                       +--------------------------------------------+


                                        |
               +---------------------  ///  --------------------------------+
               |                                                            |
               |                             +--------------------------------------------------------------+
      +------- && ---------+                 |               |                                              |
      |                    |                 |      +------- && ---------------------+                      |
+------------+             |                 |      |                                |                      |
|[t3:X, t3:Y]|       +---- || ---+           |      |                                |                      |
+------------+        |          |           |  +-- && --+               +---------- || ---------+          |
                  +------+    +------+       |  |        |               |                       |          |
                  |[f3:X]|    |[f3:Y]|       |t1: X    t1: Y             |                       |          |
```

```
                  +------+    +------+        |                    +---- && ---+            +---- && ---+   |
                                              |                    |           |            |           |   |
                                              |                 f2: n5 X    t2: n5 Y    t2: n5 X    f2: n5 Y|
                                              +-------------------------------------------------------------+


                             |
               +----------  ///  --------------------------------+
               |                                                 |
               |                 +---------------------------------------------------------------+
      +------- && ---+           |                |                                              |
      |              |           |      +------- && ---------------------+                       |
+------------+    +------+       |      |                                |                       |
|[t3:X, t3:Y]|    |[f3:Y]|       |      |                                |                       |
+------------+    |[f3:X]|       |  +-- && --+               +----------- || ---------+          |
                  +------+       |  |         |              |                        |          |
                                 |t1: X    t1: Y             |                        |          |
                                 |                     +---- && ---+            +---- && ---+    |
                                 |                     |           |            |           |    |
                                 |                  f2: n5 X    t2: n5 Y    t2: n5 X    f2: n5 Y|
                                 +---------------------------------------------------------------+


      |
 +-  ///  --------------------------------+
 |                                        |
+-+       +---------------------------------------------------------------+
|$|       |               |                                               |
+-+       |      +------- && ---------------------+                       |
          |      |                                |                       |
          |      |                                |                       |
          |  +-- && --+               +----------- || ---------+          |
          |  |         |              |                        |          |
          |t1: X    t1: Y             |                        |          |
          |                     +---- && ---+            +---- && ---+    |
          |                     |           |            |           |    |
          |                  f2: n5 X    t2: n5 Y    t2: n5 X    f2: n5 Y|
          +---------------------------------------------------------------+


      |
 +-  ///  -----------------------------+
 |                                     |
+-+       +---------------------------------------------------------+
|$|       |                     |                                   |
+-+       |         +---------- && ---------------+                 |
          |         |                             |                 |
          |         |                             |                 |
          |   +---- && ---+             +-------- || ------+        |
          |   |           |             |                  |        |
          |+------+    +------+         |                  |        |
          ||[t1:X]|    |[t1:Y]|      +-- && --+       +-- && --+    |
          |+------+    +------+      |        |       |        |    |
          |                        t1: X    f1: Y    f1: X    t1: Y|
          +---------------------------------------------------------+


      |
 +-  ///  --------------------------------+
 |                                        |
+-+       +---------------------------------------------------------------+
|$|       |               |                                               |
+-+       |      +------- && ---------------------+                       |
          |      |                                |                       |
          |+------------+                         |                       |
          ||[t1:X, t1:Y]|             +----------- || ---------+          |
          |+------------+             |                        |          |
          |                           |                        |          |
          |                     +---- && ---+            +---- && ---+    |
          |                     |           |            |           |    |
          |                  +------+    +------+    +------+    +------+|
          |                  |[t1:X]|    |[f1:Y]|    |[f1:X]|    |[t1:Y]||
          |                  +------+    +------+    +------+    +------+|
```

```
        +--------------------------------------------------------+



     |
 +-  ///  ----------------------+
 |                              |
+-+     +---------------------------------------------+
|$|     |             |                               |
+-+     |     +------- && ---------------+            |
        |     |                          |            |
        |+------------+                  |            |
        ||[t1:X, t1:Y]|         +------- || ------+   |
        |+------------+         |                 |   |
        |                +------------+    +------------+|
        |                |[t1:X, f1:Y]|    |[f1:X, t1:Y]||
        |                +------------+    +------------+|
        +---------------------------------------------+



     |
 +-  ///  ---------------+
 |                       |
+-+     +-----------------------------+
|$|     |             |               |
+-+     |     +------- && ------+     |
        |     |                 |     |
        |+------------+   +------------+|
        ||[t1:X, t1:Y]|   |[f1:X, t1:Y]||
        |+------------+   |[t1:X, f1:Y]||
        |                 +------------+|
        +-----------------------------+



     |
 +-  ///  --+
 |          |
+-+     +---+
|$|     |+-+|
+-+     ||$||
        |+-+|
        +---+
```

2.8 General Strategy

Distribution of the sub-systems over different places according the OM-matrix.

Using signed formulas to realize the sub-system management.

Developing the sub-formulas of the distributed sub-systems according to their tableaux rules.

Applying first alpha and delta rules to beta and gamma rules.

Applying the meta-rules of the term calculus to organize the distributed results collected in the trees

Searching for closed branches.

Alternatively, a strict term development which requires its own strategy can be applied.

2.8.1 Interpretations for formula H1

Junctions vs. transjunctions

The difference between junctional and transjunctional logical function has to be explained first.

Junctions are total functions accepting the values offered by their system.

Transjunctions are composed of partial functions. They reject alternatives of their system in favor to values of other systems. Therefore they are composed of acception and rejection values distributed over different sub-systems. They are intrinsically intra-contextural. Transjunctions are obviously trans-contextural, involved in at least two different contextures (sub-systems).

Junctions, like conjunction and disjunction, but also implication and replication, are directly dual under DeMorgan operators.

Transjunctions

Between the term (tr.et.et) and the term (vel.tr.vel) a duality is established with help of the negator N5.

D5 (tr.et.et) = (vel.tr.vel)

This holds for junctions in general: D5 (tr. J.J) = (DJ, tr, DJ)

But this duality is, as all negational situations in polylogics, involved in permutations. The duality of the transjunction tr is due to its symmetric definition as a total transjunction.

In general, transjunction are classified in commutative and non-commutative, or symmetric and non-symmetric transjunctions. Only commutative transjunctions are directly dual. They are also called total transjunctions.

Duality-D5 for (tr. et.et): (S123, S2, S3) —> (S1, S213, S3)

Wordings

System S1 is modeling in itself situations of systems S2 and S3.

System S2 is modeling in itself situations of systems S1 and S3.

2.8.2 General interpretation of term rules: Separability of interactionality

It was emphasized before that the operators of interactionality in polylogical systems are realized by the logical operators of transjunctions while the intra-contextural logical situations are ruled by junctional operations.

The term rules for formulas with transjunctions are separating the junctional and the transjunctional parts of the complex formula. These rules correspond to the DNF of classical logics which are also included in polylogics.

Additional to their proof-technical meaning it is possible to understand these rules as the rules of separating the intra-systemic part from their trans-systemic relations which are exactly the interactional patterns. That is, the term rules for transjunctional formulas are the rules, or meta-rules, for interactionality in polylogical systems.

The set of term rules for transjunctional formulas are based on experiences, intuition and experiments with the implementation in ML of the theorem prover LOLA (1992) for polylogical systems. They are still waiting for a strict mathematical proof of correctnes!

The set above of meta-rules is not dealing with systems change produced by permutational operators, also not with reduction and other topics. But it seems that these topics are not producing any special obstacles.

These meta-rules are of great unificational importance because of their meta-logical status they help to reduce the enormous magnitude of concrete single tableaux rules to some understandable and accessible structures. The multitude of concrete logical operators shows the flexibility of polylogical systems to the challenge of real-world applications. Each concrete situation can be handled by its own concrete set of operators. This is not excluding the search for minimal sets of operators but these new kind of sets turn out to be flexible and adapted to concrete demands of logical modeling.

The combination of Smullyan unification and the meta-rules makes this polylogical complexity of concrete situations accessible to further studies.

Following the example of Gentzen and Moisil a calculus focussed mainly on the meta-rules can be developed. It would emphasize more clear the structure of interactionality between logical systems in addition to their intra-contextural deductibility.

From a polycontextural point of view, meta- and meta-meta-rules can be understood as a reflectional activity towards the object-system. Therefore it is possible to model meta-concepts of all degrees in the framework of reflectionality of polylogics. With this, meta-considerations are free to chose between hierarchical and also heterarchical strategies.

The term rules, as listed below, are emphasizing the separation, or separability, of transjunctional terms from the junctional. Therefore, they offer a method of separating from their actionality and of studying interactionality between systems as such.

3 General term rules

3.1 Term rules for junction and transjunctions

The term calculus for polycontextural logics was first introduced in the work "Tableaux Beweiser" by Bashford/Kaehr 1992 as a first attempt to deal with the question of meta-rules.

Term Rules

$$R_0 : \quad \frac{t_1 \; et \; (t_2 \; or \; t_3)}{(t_1 \; et \; t_2) \; or \; (t_1 \; et \; t_3)}$$

$$\frac{(t_1 \; or \; t_2) \; et \; t_3}{(t_1 \; et \; t_3) \; or \; (t_2 \; et \; t_3)}$$

$R1:$

$$\frac{(t \; simul \; ta) \odot (t' \; simul \; t'a)}{(t \odot t') \; simul \; (ta \odot t'a)}$$

$R2:$

$$\frac{t \; et \; (t' \; simul \; t'a)}{(t \; et \; t') \; simul \; ta}$$

$$\frac{(t \; simul \; ta) \; et \; t'}{(t \; et \; t') \; simul \; ta}$$

$R3:$

$$\frac{(\{t\} \; simul \; ta) \; or \; (\{t'\} \; simul \; ta')}{(t \; or \; t') \; simul \; (ta \; or \; t'a)}$$

$R4:$

$$\frac{\{t\} \; or \; (\{t'\} \; simul \; t'a)}{(t \; or \; t') \; simul \; t'a}$$

$$\frac{(\{t\} \; simul \; ta) \; or \; \{t'\}}{(t \; or \; t') \; simul \; ta}$$

$R5:$

$$\frac{(t \; simul \; ta) \; simul \; t'a}{t \; simul \; (ta \; et \; t'a)}$$

Unification for PolyLogics

1 Generalized Smullyan Unification for PolyLogics

Obviously, as easily visible, the full magnitude of combinatory possible functions is not simple to handle, therefore more classifications and unifications has to be introduced. The best guide is given by Raymond Smullyan's well known method of unification. We are studying the familiar case of linear-mediated polylogics in contrast to tabular architectonics.

1.1 General unification rules

Diagramm 24 **Unified junctions and total and partial transjunctions**

junctions and total transjunction scheme

$$\begin{array}{c} \alpha \\ \hline \alpha_1 \\ \alpha_2 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\beta} \\ \hline \beta_1 & \beta_2 \end{array} \qquad \begin{array}{c} \delta \\ \hline \delta_1 \\ \delta_2 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\gamma} \\ \hline \gamma_1 & \gamma_3 \\ \gamma_2 & \gamma_4 \end{array}$$

junctions and partial transjunction schemes

$$\begin{array}{c} \alpha \\ \hline \alpha_1 \\ \alpha_2 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\beta} \\ \hline \beta_1 & \beta_2 \end{array} \qquad \begin{array}{c||c} \multicolumn{2}{c}{\delta} \\ \hline \delta_1 & \gamma_3 \\ \delta_2 & \gamma_4 \end{array} \qquad \begin{array}{c} \gamma \\ \hline \gamma_1 \\ \gamma_2 \end{array}$$

$$\begin{array}{c} \alpha \\ \hline \alpha_1 \\ \alpha_2 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\beta} \\ \hline \beta_1 & \beta_2 \end{array} \qquad \begin{array}{c||c} \multicolumn{2}{c}{\delta} \\ \hline \delta_1 & \gamma_1 \\ \delta_2 & \gamma_2 \end{array} \qquad \begin{array}{c} \gamma \\ \hline \gamma_3 \\ \gamma_4 \end{array}$$

Diagramm 25 **Basic unification for total transjunctions**

$$\begin{array}{c} \alpha \\ \hline \alpha_1 \\ \alpha_2 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\beta} \\ \hline \beta_1 & \beta_2 \end{array} \qquad \begin{array}{c} \delta \\ \hline \delta_1 \\ \delta_2 \end{array} \qquad \begin{array}{c||c} \multicolumn{2}{c}{\gamma} \\ \hline \gamma_1 & \overline{\gamma_1} \\ \gamma_2 & \overline{\gamma_2} \end{array} \qquad \left[\frac{\overline{\gamma_1}}{\gamma_2}\right] = \begin{array}{c} \gamma_3 \\ \gamma_4 \end{array}$$

1.1.1 Signed formulas

$$\begin{array}{c} \pi\ \alpha \\ \hline \pi_1\ \alpha_1 \\ \pi_2\ \alpha_2 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\pi\ \beta} \\ \hline \pi_1\ \beta_1 & \pi_2\ \beta_2 \end{array} \qquad \begin{array}{c} \pi\ \delta \\ \hline \pi_1\ \delta_1 \\ \pi_2\ \delta_2 \end{array} \qquad \begin{array}{c|c} \multicolumn{2}{c}{\pi\ \gamma} \\ \hline \pi_1\ \gamma_1 & \pi_3\ \gamma_3 \\ \pi_2\ \gamma_2 & \pi_4\ \gamma_4 \end{array}$$

1.1.2 Junctional unification table

Diagramm 26

Unification for junctions

α^i	α_1^i	α_2^i
$t^i\ X \wedge Y$	$t^i\ X$	$t^i\ Y$
$f^i\ X \vee Y$	$f^i\ X$	$f^i\ Y$
$f^i\ X \rightarrow Y$	$t^i\ X$	$f^i\ Y$
$f^i\ X \leftarrow Y$	$f^i\ X$	$t^i\ Y$

β^i	β_1^i	β_2^i
$f^i\ X \wedge Y$	$f^i\ X$	$f^i\ Y$
$t^i\ X \vee Y$	$t^i\ X$	$t^i\ Y$
$t^i\ X \rightarrow Y$	$f^i\ X$	$t^i\ Y$
$t^i\ X \leftarrow Y$	$t^i\ X$	$f^i\ Y$

1.2 Classification of unificators

1.2.1 (f,c)- and (klor)-analysis

The rules of mediation are restricting the combinatory of distributed operators.

$$Junctional\ \ mediation:$$
$$\{\alpha,\ \beta\}_{/\,mediation} \in S^{(3)} \Leftrightarrow \{\alpha,\ \beta\} \in CF$$
$$CF = \{(ccc), (cff), (fcf), (ffc), (fff)\}$$
$$f = \{\wedge, \vee\}$$
$$c = \{\rightarrow, \leftarrow, \leftrightarrow\}$$

$$Transjunctional\ \ mediation:$$
$$\{\alpha,\ \beta,\ \delta,\ \gamma\}_{/\,mediation} \in S^{(3)} \Leftrightarrow \{\alpha,\ \beta\ \ \delta,\ \gamma\} \in CF$$
$$CF = \{(ccc), (cff), (fcf), (ffc), (fff)\}$$
$$f = \{\wedge, \vee, <, >, <>, \lhd, \rhd\}$$
$$c = \{\rightarrow, \leftarrow, \leftrightarrow, \Uparrow, \Downarrow\}$$

The sets f and c also contains the negated functions.

$$\frac{\alpha \in S^1,\ \alpha \in S^2,\ \alpha \in S^3}{(\alpha\alpha\alpha) \in S^{(3)}}$$

$$\frac{\{\alpha,\ \beta\} \in S^{(3)}}{\left\{\begin{matrix}(\alpha\alpha\alpha)\\(\alpha\alpha\beta)\\(\alpha\beta\alpha)\\(\alpha\beta\beta)\\(\beta\alpha\alpha)\\(\beta\alpha\beta)\\(\beta\beta\alpha)\\(\beta\beta\beta)\end{matrix}\right\} \in S^{(3)}}$$

1.2.2 Mediation and unification

1.2.2.1 Identity patterns

$$\begin{aligned}
&pattern\ [id, id, id]: S_{123} \xrightarrow[id]{} S_{123} \\
&med\left[\left(t^1\ X \wedge Y\right) \in \alpha^1, \left(t^2\ X \wedge Y\right) \in \alpha^2, \left(t^3\ X \wedge Y\right) \in \alpha^3\right] \in \left(fff\right) \Rightarrow \left(\alpha^1\alpha^2\alpha^3\right) \in S^{(3)} \\
&med\left[\left(f^1\ X \wedge Y\right) \in \alpha^1, \left(f^2\ X \wedge Y\right) \in \alpha^2, \left(f^3\ X \wedge Y\right) \in \alpha^3\right] \in \left(fff\right) \Rightarrow \left(\beta^1\beta^2\beta^3\right) \in S^{(3)} \\
&med\left[\left(t^1\ X \wedge Y\right) \in \alpha^1, \left(t^2\ X \vee Y\right) \in \alpha^2, \left(t^3\ X \wedge Y\right) \in \alpha^3\right] \in \left(fff\right) \Rightarrow \left(\alpha^1\beta^2\alpha^3\right) \in S^{(3)} \\
&med\left[\left(t^1\ X \wedge Y\right) \in \alpha^1, \left(t^2\ X \wedge Y\right) \in \alpha^2, \left(t^3\ X \vee Y\right) \in \alpha^3\right] \in \left(fff\right) \Rightarrow \left(\alpha^1\alpha^2\beta^3\right) \in S^{(3)} \\
&med\left[\left(t^1\ X \wedge Y\right) \in \alpha^1, \left(t^2\ X \vee Y\right) \in \alpha^2, \left(t^3\ X \vee Y\right) \in \alpha^3\right] \in \left(fff\right) \Rightarrow \left(\alpha^1\beta^2\beta^3\right) \in S^{(3)}
\end{aligned}$$

$$\begin{aligned}
&pattern\ [id, id, id]: S_{123} \xrightarrow[id]{} S_{123} \\
&\left(X \wedge\wedge\wedge\ Y\right) \in \left(fff\right) \Rightarrow \left(t^1\ t^2\ t^3\quad \alpha^1\alpha^2\alpha^3\right) \in S^{(3)} \\
&\left(X \wedge\wedge\wedge\ Y\right) \in \left(fff\right) \Rightarrow \left(f^1\ f^2\ f^3\ \beta^1\beta^2\beta^3\right) \in S^{(3)} \\
&\left(X \wedge\vee\wedge\ Y\right) \in \left(fff\right) \Rightarrow \left(t^1\ t^2\ t^3\quad \alpha^1\beta^2\alpha^3\right) \in S^{(3)} \\
&\left(X \wedge\wedge\vee\ Y\right) \in \left(fff\right) \Rightarrow \left(t^1\ t^2\ t^3\ \ \alpha^1\alpha^2\beta^3\right) \in S^{(3)} \\
&\left(X \wedge\vee\vee\ Y\right) \in \left(fff\right) \Rightarrow \left(t^1\ t^2\ t^3\ \ \alpha^1\beta^2\beta^3\right) \in S^{(3)}
\end{aligned}$$

1.2.2.2 Reductional patterns

$$pattern\ [id,\ red,\ red]:\ S_{123} \xrightarrow[idredred]{} S_{111}$$
$$med\left[\left(t^1\ X \wedge Y\right) \in \alpha^1,\ \left(t^2\ X \wedge Y\right) \in \alpha^2,\ \left(f^3\ X \rightarrow Y\right) \in \alpha^3\right] \in (ffc) \Rightarrow \left(\alpha^1\alpha^1\alpha^1\right) \in S^{(3)}$$

$$pattern\ [id,\ red,\ id]:\ S_{123} \xrightarrow[idredrid]{} S_{113}$$
$$med\left[\left(t^1\ X \wedge Y\right) \in \alpha^1,\ \left(t^2\ X \wedge Y\right) \in \alpha^2,\ \left(f^3\ X \rightarrow Y\right) \in \alpha^3\right] \in (ffc) \Rightarrow \left(\alpha^1\alpha^1\alpha^3\right) \in S^{(3)}$$

1.2.3 Transjunctional unification table

The general unificational schemes for transjunctions are not as simple as the junctional counterparts but there is a clear structure behind the transjunctional table (situation, configuration).

signature schemes :

partial transjunction $(<),\ i = 1,2,3\ mod 3$

$$\frac{t_i\ \gamma''}{\begin{matrix} f_{i+1}\gamma_3 \\ t_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_i\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{t_{i+1}\ (\delta\ \gamma')}{\begin{array}{c||c} t_{i+1}\ \delta_1 & t_{i+1}\ \gamma_1 \\ t_{i+1}\ \delta_2 & f_{i+1}\ \gamma_2 \end{array}} \quad \frac{f_{i+1}\ \delta}{\begin{matrix} f_{i+1}\ \delta_1 \\ f_{i+1}\ \delta_2 \end{matrix}} \quad \frac{t_{i+2}\ \gamma''}{\begin{matrix} f_{i+1}\ \gamma_3 \\ t_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_{i+2}\ \delta}{\begin{matrix} f_{i+1}\ \delta_1 \\ f_{i+1}\ \delta_2 \end{matrix}}$$

partial transjunction $(>),\ i = 1,2,3\ mod 3$

$$\frac{t_i\ \gamma''}{\begin{matrix} f_{i+1}\gamma_3 \\ t_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_i\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{t_{i+1}\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{f_{i+1}\ (\delta\ \gamma')}{\begin{array}{c||c} t_{i+1}\ \delta_1 & t_{i+1}\ \gamma_1 \\ t_{i+1}\ \delta_2 & f_{i+1}\ \gamma_2 \end{array}} \quad \frac{t_{i+2}\ \gamma''}{\begin{matrix} f_{i+1}\ \gamma_3 \\ t_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_{i+2}\ \delta}{\begin{matrix} f_{i+1}\ \delta_1 \\ f_{i+1}\ \delta_2 \end{matrix}}$$

partial transjunction $(\lhd),\ i = 1,2,3\ mod 3$

$$\frac{t_i\ \gamma''}{\begin{matrix} t_{i+1}\gamma_3 \\ f_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_i\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{t_{i+1}\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{f_{i+1}\ (\delta\ \gamma')}{\begin{array}{c||c} f_{i+1}\ \delta_1 & f_{i+1}\ \gamma_1 \\ f_{i+1}\ \delta_2 & t_{i+1}\ \gamma_2 \end{array}} \quad \frac{t_{i+2}\ \gamma''}{\begin{matrix} t_{i+1}\ \gamma_3 \\ f_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_{i+2}\ \delta}{\begin{matrix} f_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}}$$

partial transjunction $(\rhd),\ i = 1,2,3\ mod 3$

$$\frac{t_i\ \gamma''}{\begin{matrix} f_{i+1}\gamma_3 \\ t_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_i\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{t_{i+1}\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{f_{i+1}\ (\delta\ \gamma')}{\begin{array}{c||c} f_{i+1}\ \delta_1 & f_{i+1}\ \gamma_1 \\ f_{i+1}\ \delta_2 & t_{i+1}\ \gamma_2 \end{array}} \quad \frac{t_{i+2}\ \gamma''}{\begin{matrix} f_{i+1}\ \gamma_3 \\ t_{i+1}\ \gamma_4 \end{matrix}} \quad \frac{f_{i+2}\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ f_{i+1}\ \delta_2 \end{matrix}}$$

total transjunction $(<>),\ i = 1,2,3\ mod 3$

$$\frac{t_i\ \ \delta}{\begin{matrix} t_i\ \ \delta_1 \\ t_i\ \ \delta_2 \end{matrix}} \quad \frac{f_i\ \gamma}{\begin{array}{c|c} f_{i+1}\ \gamma_1 & t_{i+1}\ \gamma_3 \\ t_{i+1}\ \gamma_2 & f_{i+1}\ \gamma_4 \end{array}} \quad \frac{t_{i+1}\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ t_{i+1}\ \delta_2 \end{matrix}} \quad \frac{f_{i+1}\ \delta}{\begin{matrix} f_{i+1}\ \delta_1 \\ f_{i+1}\ \delta_2 \end{matrix}} \quad \frac{t_{i+2}\ \gamma}{\begin{array}{c|c} f_{i+1}\ \gamma_1 & t_{i+1}\ \gamma_3 \\ t_{i+1}\ \gamma_2 & f_{i+1}\ \gamma_4 \end{array}} \quad \frac{f_{i+2}\ \delta}{\begin{matrix} t_{i+1}\ \delta_1 \\ f_{i+1}\ \delta_2 \end{matrix}}$$

$$f_{i+1}\ (\delta\ \gamma'):$$

$$\begin{array}{c||c} f_{i+1}\ \delta_1 & f_{i+1}\ \gamma_1 \\ f_{i+1}\ \delta_2 & t_{i+1}\ \gamma_2 \end{array}\ \ is\ \ \begin{array}{c|c} f_{i+1}\ \delta_1 & f_{i+1}\ \gamma_1 \\ f_{i+1}\ \delta_2 & t_{i+1}\ \gamma_2 \end{array}\ \ is\ \ \begin{array}{c|c} f_{i+1}\ \delta_1 & f_{i+1}\ \delta_1 \\ f_{i+1}\ \delta_2 & t_{i+1}\ \delta_2 \end{array}\ \ is\ \ f_{i+1}\ \delta_1$$

1.2.4 Consistency properties

Diagramm 27 **Propositional consistency properties for PolyLogic(3)**

Propositional consistency property for PolyLogics

$\forall i \forall j \in s(m):$

$$\left.\begin{array}{l} 1.\ F_i \notin S^i\ ;\ \neg T_i \notin S^i \\ 2.\ \neg\neg Z \in S^i \Rightarrow S^i \cup \{Z\} \in C^i \\ 3.\ \alpha \in S^i \Rightarrow S^i \cup \{\alpha_1, \alpha_2\} \in C^i \\ 4.\ \beta \in S^i \Rightarrow S^i \cup \{\beta_1\} \in C^i \text{ or } S^i \cup \{\beta_2\} \in C^i \end{array}\right\} \text{junctions}$$

$$\left.\begin{array}{l} 5.\ \delta \in S^j \Rightarrow S^j \cup \{\delta_1, \delta_2\} \in C^j \\ 6.\ \gamma \in S^j \Rightarrow S^j \cup \{\gamma_1, \gamma_2\} \in C^j \text{ or } S^j \cup \{\gamma_3, \gamma_4\} \in C^j \end{array}\right\} \text{total transjunctions}$$

$$\left.\begin{array}{l} 5'.\ \delta' \in S^j \Rightarrow S^j \cup \{\delta_1, \delta_2\} \in C^j \text{ or } S^j \cup \{\gamma_1, \gamma_2\} \in C^j \\ 6'.\ \gamma' \in S^j \Rightarrow S^j \cup \{\gamma_3, \gamma_4\} \in C^j \\ 5''.\ \delta'' \in S^j \Rightarrow S^j \cup \{\delta_1, \delta_2\} \in C^j \text{ or } S^j \cup \{\gamma_3, \gamma_4\} \in C^j \\ 6''.\ \gamma'' \in S^j \Rightarrow S^j \cup \{\gamma_1, \gamma_2\} \in C^j \end{array}\right\} \text{partial transjunctions}$$

1.2.5 Conjugations

Symmetric conjugations

Between α-terms and β-terms we observe a nice and simple conjugation. Both, α- and β-terms, are dual.

Conjugation rules

$$\rho(\alpha) = (\beta)$$
$$\rho(\alpha) \Leftrightarrow (\rho(\alpha_1) \text{ and } \rho(\alpha_2))$$
$$\rho(\beta) \Leftrightarrow (\rho(\beta_1) \text{ or } \rho(\beta_2))$$
$$\rho(\alpha) \Leftrightarrow (\beta_1 \text{ or } \beta_2)$$

Asymmetric conjugations

This elegant symmetric duality is excluding logical function like equivalence and contra-valence. This two functions are not symmetric in respect of their possible α-terms and β-terms. Thus, they are treated as derived functions, composed of implication and conjunction. But nevertheless, the tableaux rules for the equivalence shows a clear asymmetry between the two parts, the truth-part and the false-part.

Composed conjugation for equivalence :

$$X \leftrightarrow Y = (X \rightarrow Y) \wedge (X \leftarrow Y)$$

$$If\ t_i\ (\alpha_1 \vee \alpha_2) \xleftrightarrow[dual]{} f_i\ (\beta_1 \wedge \beta_2),$$

$$then\ \alpha_1 \xleftrightarrow[dual]{} \beta_1,\ \ \alpha_2 \xleftrightarrow[dual]{} \beta_2$$

$t_i\ X \leftrightarrow Y$		$f_i\ X \leftrightarrow Y$	
$t_i\ X$	$f_i\ X$	$t_i\ X$	$f_i\ X$
$t_i\ Y$	$f_i\ Y$	$f_i\ Y$	$t_i\ Y$

This solution is perfect for classical logic because there are only a few asymmetric cases but in polylogical constellations they are quantitatively and from their significance in the vast majority. Morphogrammatically it looks more balanced, from the 15 basic morphograms for polylogical functions, 8 are junctional and 7 are transjunctional. But this balance disappears quickly because from the 8 junctional morphograms only 4 are relevant, producing the 8 classical logical functions. On the other hand, the 7 transjunctional morphograms are basic and all full in the game additionally, the deliver the structure for different logical interpretations, mirrored as different realizations of semantic mappings into the morphograms.

To restore a reasonable conjugation theory for asymmetric situations too, we have to involve the fact that transjunctional functions are better understood not as total but as combinations of partial functions. Thus, transjunctions can be introduced by applying the methods used by defining derived functions. Transjunctions are not only compositions of partial functions but are the functions which are reflecting the environment of the logical place they are set. This two observations, partiality and environment, gives a clue how to construct a general setting for asymmetric conjugations.

1.3 General rules and tableaux rules

Only for the junctional situation we have a one-to-one correspondence between the general unification rules and the tableaux rules connected to their sub-systems.

Even between the general rule for total transjunction and its tableaux their is some degree of differentiation. In the case of reductional transjunctions the general rules for the transjunction has not to be applied to all its sub-systems. In other words, we can compose full transjunctions as conjunctions of conditional transjunctions. This applies to total as well as to conditional transjunctions.

The general rules are defined for "isolated" sub-systems. Between the α–β-rules there is a duality, and for the case of only one sub-system, that is, for i=1, the whole logical system is defined. There is no ambiguity of differentiation in that. Polylogical systems in contrast have to take their whole constellation into account and have therefore to be characterized by the tableaux and the rules for the whole operators. They can not be determined by the local characteristics only. But this difference between the general rules and the tableaux rules is, conceptionally, also at place for the junctional situation of classical logic. The only difference is that in the classical case there is a coincidence between the uniqueness of the logical system and the uniqueness of the rules. Nevertheless, they have to fit together.

Even for the classical case, some conventions, which could be questioned, are necessary to adapt of the logical operators to their unificational rules. Examples for a need of some kind technical conventions are negation and logical equivalence.

$\dfrac{t_1\ X\ J < J\ Y}{t_1\ J}$	$\dfrac{f_1\ X\ J < J\ Y}{f_1\ J}$	$\dfrac{t_1\ X\ J > J\ Y}{t_1\ J\ \Vert\ \gamma}$	$\dfrac{f_1\ X\ J > J\ Y}{f_1\ J\ \Vert\ \delta}$
$\dfrac{t_2\ X\ J < J\ Y}{t_2\ \delta}$	$\dfrac{f_2\ X\ J < J\ Y}{f_2\ \delta}$	$\dfrac{t_2\ X\ J > J\ Y}{t_2\ \delta}$	$\dfrac{f_2\ X\ J > J\ Y}{f_2\ \delta}$
$\dfrac{t_3\ X\ J < J\ Y}{t_3\ J\ \Vert\ \gamma}$	$\dfrac{f_3\ X\ J < J\ Y}{f_3\ J\ \Vert\ \delta}$	$\dfrac{t_3\ X\ J > J\ Y}{t_3\ J}$	$\dfrac{f_3\ X\ J > J\ Y}{f_3\ J}$

The operator J represents junctions as α– or β-parts.

1.4 Patterns of unification

Patterns of unification correspond the application of super-operators to standard (id, id, id)-patterns of logical operators.

1.4.1 General classisfication

Junctional patterns: {a, b } –> {id, red, perm}

Transjunctional patterns: (a, b) –> {id, red, perm, bif}

1.4.2 Special classification

Depending on the distribution of the super-operators {id, perm, red, bif} different classification systems can be established.

$$\boxed{\begin{array}{l} perm : Z^{i.j} \in S^{(3)} \Rightarrow (Z^{j.i}) \in S^{(3)} \\ red : \quad Z^{i.j} \in S^{(3)} \Rightarrow (Z^{i.i}) \in S^{(3)} \end{array}}$$

$$red(\alpha^1\alpha^2\alpha^3) \Rightarrow \{(\alpha^1\alpha^1\alpha^3), (\alpha^1\alpha^3\alpha^1), (\alpha^1\alpha^3\alpha^3), (\alpha^1\alpha^1\alpha^1)\}$$

1.5 Contradiction and incompatibility

A branch δ of a tableau T is called closed by contradiction if both X and non-X occurs on δ for some propositional formula X, or if F occurs on δ.

A branch δ of a tableau T is called *closed by incompatibility* iff X, Y and Z occurs on δ for some propositional formula X with val(X) /= val(Y) /=val(Z). Such a branch can be eliminated, cutted away, from the tree.

Branches can be eliminated if they contain at least 3 different variables with 3 different values (for the case of m=3).