Interactional operators in diamond semiotics

From polylogical transjunctions to polysemiotic interactions and reflections

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Abstract

Comparing polycontextural logics and semiotics, the idea of interactionality is introduced as a further step of interaction in embedded semiotics. To achieve interactionality/reflectionality for semiotics some new concepts had been introduced. For polylogical systems, transjunctional operators are defining interactions between logics. After a sketch of polysemiotics, poly-semiotic formulations of interaction and reflection operators are introduced.

1. Semiotics and polylogics

1.1. Motivation

Transjunction, as important operators of interaction, are well known in polycontextural logics. Semiotics offers a different approach to cognitive/volitive modeling. In this paper, some steps to sketch an interactional approach in semiotics along the experiences, models and formalizations of polycontextural logic, is undertaken.[1]

The semiotic matrix is introduced as the “Cartesian product” of sub-signs (Bense, Toth).

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

"A **sub-sign** is obtained by mapping the three sign relations (.1, .2, .3) into themeselves.”
"The rows are called **triadic values** and the colomns **trichotomic values** of the matrix. In order to build a a **sign class**, one sub-sign has to be taken out of each of the three rows, the rows thus being different."
"Therefore, **sign sets** like *(3.1 3.2 1.3), *(2.1 2.2 1.2), *(1.1 1.3 3.1) are not considered sign classes.” (Toth, Ghost, p.9)

Cartesian *products* as a conceptual point of contact.[2]
The aim of polycontextural semiotics is to design a dynamic sign theory without any fixation on a special or privileged n-ary and m-adic system. Another attempt to augment the structural and architectonic flexibility of semiotics is proposed by Toth’s approach to a 3- and 4-dimensional semiotics resulting in complex topological structures. (Cf. Transit-Korridor, 2009)

1.2. Is there a privileged number for dissemination?

An introduction of the topics of polycontextural formal systems, like polylogics, poly-arithmetic or polysemiotics, has to deal with the question of a *privileged number* of a possible extension of 2-valued logics, semiotics and arithmetic. This has been thematized at different places and can't be exposed *in extenso* in this *Short Study* to Polysemiotics.[3]

1.2.1. Gunther's approach to many-valued logics

In the advent of many-valued logics there was a big run to find a privileged number of truth-values, logical functions and their semantic interpretation.
Gunther's Program. Each single value and each single logical function is entitled to have a logical meaning.
It is absurd to chase for the meaning of logical values and functions for arbitrary many-valued systems. Special value classes of some interest had been studied by logicians for 2, 3, 4, and infinite.
Hence, a method, like the arithmetic position system which is able to determine arbitrary numbers on a finite base system, has to be invented. This was Gunther's approach to many-valued place-value systems (Stellenwertlogik).
Semiotics, today, is still in a pre-decompositional, i.e. conceptionally static state of research, not necessarily in the spirit of Peirce's 'speculations'.

1.2.2. Gunther's criticism of Peirce/Bense's trinitarism

Gunther has taken the opportunity to write down and publish, what was clear at least since the advent of his place-valued logics in the 50s. That the restriction of Peirce and his decade long friend Max Bense is a heritage of Western and Christian thinking, which was conceived by Gunther as dead, at least since Nietzsche and American Cybernetics.

1.2.3. Beyond Gunther's stance on numbers

Gunther repeated the argumentation of Aristotle against a privileged number, say for his m-valued polycontextural logic, but was nevertheless the only one who himself introduced a (Neo)Pythagorean concept and some formalism of transclassic numbers, called "Philosophical numbers" (Gattungszahlen).
In short: In polycontextural logic, no special number is privileged because each number has its own specific characteristics, hence its own privilege. With this paradoxical characterization of 'privileged'/'unprivileged numbers, the whole idea of a privileged number in the traditional sense is obsolete. But this polycontextural magnitude of de-privileged privileges is based on a strategy of a finite structure, the number 4 of '*tetraktomai*', i.e. of *doing* the tetraktys, also called proemial relationship or diamond strategies. Again, this number of the *praxis* of tetraktomai, i.e. diamodization, isn't a member of any arithmetical number system.[4]

1.2.4. Toth's criticism of Bense's triadic-trichotomic semiotics

"Um es kurz zu sagen: Bense hatte - es ist fast nicht zu glauben - *n-äre und n-adische* Logiken verwechselt: Obwohl die Peirce-Bense-Semiotik triadisch ist, bleibt sie dennoch binär, und das, obwohl sie einen zehnfach ausdifferenzierten Realitätsbegriff besitzt." (Toth, Semiotische Strukturen und Prozesse, 2008). This, and other ebooks by Alfred Toth at:

2. Dissemination of semiotics

Interaction between different logical or semiotic systems is depending on the *architectonics* of the framework. In the proposed case, only two cases are presented.

First an architectonics based on a decomposition of the system into (2, 2)-subsystem.
And *second*, an architectonics based on the decomposition of the system into (3, 2)-

and subsystems.

The decomposition into (2, 2)-subsystems of 3-contextural systems corresponds to the usual polycontextural approach as introduced by Gotthard Gunther for his *place-valued* logic. It can be understood as a dissemination of *contextures* towards polycontexturality as the base for polycontextural logics in general.

This strategy of decomposing Peirce/Bense/Toth-semiotics into its dyadic-dichotomic parts opens up the possibility for a *polycontextural* approach to a logic, arithmetic and categorification of semiotics as a mediation of semiotically, logically and categorically independent elementary contextures of a mediated compound. This approach is in strict contrast to a modeling of triadic-trichotomic semiotics with methods of classical relation, set and category theory.

The (3, 3)-subsystem decomposition of 4-contextural systems, albeit it goes back to my early studies of polycontexturality, has been introduced recently for a new formalization of semiotics towards polysemiotics.

Polysemiotics are disseminating, in a first step, classical triadic-trichotomic semiotics, $Sem^{(3,2)}$, over different kenomic places to build more complex configurations.

2.1. Contextural decomposition triadic systems

2.1.1. Unary matrix$^{(3,1)}$

$Sem^{(3,1)} = (Sem^1, Sem^2, Sem^3)$

Valuation (val) of Sem :

$val(Sem^1, Sem^2, Sem^3) = (1_{1.3}, 2_{1.2}, 3_{2.3})$

Hence,

$Sem^1 = Sem_{1.3}$

$Sem^2 = Sem_{1.2}$

$Sem^3 = Sem_{2.3}.$

Matching conditions :

$(1)_1 \cong (1)_3$

$(2)_1 \cong (2)_2$

$(3)_2 \cong (3)_3.$

2.1.2. Binary matrix$^{(3,2)}$ and scheme$^{(3,2)}$

$$Sem^{(3,2)} = Sem^{(3,1)} \times Sem^{(3,1)} = \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right) \times \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right)$$

$$Sem^{(3,2)} = \left[\left(Sem^1 \times Sem^1\right), \left(Sem^2 \times Sem^2\right), \left(Sem^3 \times Sem^3\right)\right]:$$

$$val\left(Sem^1 \times Sem^1\right) = (1, 2)_1 \times (1, 2)_1$$

$$val\left(Sem^2 \times Sem^2\right) = (2, 3)_2 \times (2, 3)_2$$

$$val\left(Sem^3 \times Sem^3\right) = (1, 3)_3 \times (1, 3)_3.$$

Matching conditions :

$$(1, 1)_1 \cong (1, 1)_3$$

$$(2, 2)_1 \cong (2, 2)_2$$

$$(3, 3)_2 \cong (3, 3)_3.$$

Scheme of $Sem^{(3,2)}$:

$$Semiotics^{(3,2)} = \begin{bmatrix} (1.1)_{1.3} \longrightarrow & (1.2)_1 & \longrightarrow & (1.3)_3 \\ \downarrow \quad \times & \downarrow & \times & \downarrow \\ (2.1)_1 \longrightarrow & (2.2)_{1.2} & \longrightarrow & (2.3)_2 \\ \downarrow \quad \times & \downarrow & \times & \downarrow \\ (3.1)_3 \longrightarrow & (3.2)_2 & \longrightarrow & (3.3)_{2.3} \end{bmatrix}$$

Sub – system decomposition of $Sem^{(3,2)}$:

$$sub-system_1 = \begin{bmatrix} (1.1) \longrightarrow & (1.2) \\ \downarrow \quad \times & \downarrow \\ (2.1) \longrightarrow & (2.2) \end{bmatrix}$$

$$sub-system_2 = \begin{bmatrix} (2.2) \longrightarrow & (2.3) \\ \downarrow \quad \times & \downarrow \\ (3.2) \longrightarrow & (3.3) \end{bmatrix}$$

$$sub-system_3 = \begin{bmatrix} (1.1) \longrightarrow & (1.3) \\ \downarrow \quad \times & \downarrow \\ (3.1) \longrightarrow & (3.3) \end{bmatrix}$$

3 – contextural semiotic matrix

$$\mathrm{Sem}^{(3,2)} = \begin{pmatrix} MM & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1}_{1.3} & \mathbf{1.2}_{1} & \mathbf{1.3}_{3} \\ 2_{1.2} & \mathbf{2.1}_{1} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3_{2.3} & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

The mediation scheme of Semiotics$^{(3,2)}$:

$$\mathrm{mediation}\left(\mathrm{Semiotics}^{(3,2)}\right) = \begin{bmatrix} (1.1)_1 \longrightarrow & (2.2)_1 & \square \\ \square & \updownarrow & \\ \square & (2.2)_2 \longrightarrow & (3.3)_2 \\ | & & | \\ (1.1)_3 \longrightarrow & \longrightarrow & (3.3)_3 \end{bmatrix}$$

Chiastic structure

$$\textit{Order}\ \text{relations} = \left\{ \begin{array}{l} \square(1.1)_1 \longrightarrow (2.2)_1, \\ (2.2)_2 \longrightarrow (3.3)_2, \\ (1.1)_3 \longrightarrow (3.3)_3 \end{array} \right\},$$

$$\textit{Exchange}\ \text{relation} = \left\{(2.2)_1 \updownarrow (2.2)_2\right\},$$

$$\textit{Coincidence}\ \text{relations} = \left\{ \begin{array}{l} (1.1)_1 - (1.1)_3, \\ (3.3)_2 - (3.3)_3 \end{array} \right\}.$$

For systems, $m = 3$, $n = 2$, the matrix$^{(3,2)}$ and scheme$^{(3,2)}$ representation coincide.

Sign classes for classical Semiotics

Sign classes are traditionally defined by :

$$ZR = (a, (a \Longrightarrow b), (a \Longrightarrow b \Longrightarrow c))$$

for

$$a = \{1.1,\ 1.2,\ 1.3\}$$

$$b = \{2.1,\ 2.2,\ 2.3\}$$

$$c = \{3.1,\ 3.2,\ 3.3\}$$

General sign relation :

$$ZR = \ <3.x,\ 2.y,\ 1.z> \text{ mit } x,\ y,\ z \in \{1,\ 2,\ 3\}$$

with $x \leq y \leq z$.

Resulting in the 10 sign classes :

3.1 2.1 1.1	3.1 2.3 1.3
3.1 2.1 1.2	3.2 2.2 1.2
3.1 2.1 1.3	3.2 2.2 1.3
3.1 2.2 1.2	3.2 2.3 1.3
3.1 2.2 1.3	3.3 2.3 1.3

Classical semiotics is not *mediating* its sub-systems, hence, no matching conditions are required. Therefore, classical semiotics is forced to introduce externally different *restriction* rules to determine the set of accepted sign classes.

Sign classes for $Sem^{(3,1,2)}$

$$\text{decomp}\left(\left[3.a,\ 2.b,\ 1.c\right]\right) = \begin{bmatrix} 3.x, & 2.y, & -- \\ -- & 2.y, & 1.z \\ 3.x, & -, & 1.z \end{bmatrix}$$

Examples

$$\text{decomp}\left(\left[3.2\ 2.1\ 1.1\right]\right) = \begin{bmatrix} 3.2, & 2.1, & -- \\ --, & 2.1, & 1.1 \\ \underline{3.2,} & \underline{--,} & \underline{1.1} \end{bmatrix}$$

$$\left[3.2\ \ 2.1\ \ 1.1\right] :: \left(\text{Sem}^2,\ \text{Sem}^1, \text{Sem}^3\right)$$

$$\text{decomp}\left(\left[3.1\ 2.2\ 1.2\right]\right) = \begin{bmatrix} 3.1, & 2.2, & -- \\ --, & 2.2, & 1.2 \\ \underline{3.1,} & \underline{--,} & \underline{1.2} \end{bmatrix}$$

$$\left[3.1\ \ 2.2\ \ 1.1\right] :: \left(\text{Sem}^2,\ \text{Sem}^1, \text{Sem}^3\right)$$

$$\text{decomp}\left(\left[3.2\ 2.2\ 1.2\right]\right) = \begin{bmatrix} 3.2, & 2.1, & -- \\ --, & 2.1, & 1.2 \\ \underline{3.2,} & \underline{--,} & \underline{1.2} \end{bmatrix}$$

$$\left[3.2\ \ 2.1\ \ 1.2\right] :: \left(\text{Sem}^2,\ \text{Sem}^1, \text{Sem}^3\right)$$

$$\text{decomp}\left(\left[2.2\ 2.2\ 1.2\right]\right) = \begin{bmatrix} 2.2, & 2.1, & -- \\ --, & 2.1, & 1.2 \\ \underline{2.2,} & \underline{--,} & \underline{1.2} \end{bmatrix}$$

$$\left[\left[2.2\ \ 2.1\ \ 1.2\right] :: \left(\text{Sem}^1, \text{Sem}^1, \text{Sem}^1\right)$$

Translation

$a.\left[3.1\ 2.1\ 1.1\right] \Longleftarrow; \left(\mathbf{3.1}_{3}\ \mathbf{2.1}_{1}\ \ \mathbf{1.1}_{1.3}\right)$

$\Longrightarrow \left[\left(\mathbf{3.1}_{3}\ \mathbf{xx}\ \mathbf{1.1}_{3}\right), \left(\mathbf{xx}\ \mathbf{2.1}_{1}\ \mathbf{1.1}_{1}\right)\right]$

$b.\left[3.2\ 2.1\ 1.1\right] \Longleftarrow; \left(\mathbf{3.2}_{2}\ \mathbf{2.1}_{1}\ \ \mathbf{1.1}_{1.3}\right)$

$\Longrightarrow \left[\left(\mathbf{3.2}_{2}\ \mathbf{xx}\ \ \mathbf{xx}\right), \left(\mathbf{xx}\ \mathbf{2.1}_{1}\ \mathbf{1.1}_{1}\right), \left(\mathbf{xx},\ \mathbf{xx},\ \mathbf{1.1}_{3}\right)\right],$

$c.\left[3.1\ 2.1\ 1.1\right] \Longleftarrow; \left(\mathbf{3.1}_{3}\ \mathbf{2.1}_{1}\ \ \mathbf{1.1}_{1.3}\right)$

$\Longrightarrow \left[\left(\mathbf{3.1}_{3}\ \mathbf{xx}\ \mathbf{1.1}_{3}\right), \left(\mathbf{xx}\ \mathbf{2.1}_{1}\ \mathbf{1.1}_{1}\right)\right]$

Super-operators for semiotic mappings

Independent of a specification of sign classes by accepting or abbolishing the *restriction* rules for semiotics (Toth, Ghost, p.9), mappings from sign class to sign class might be classified by the *super-operators* as they are defined in polycontextural logic:

Super – operators for semiotics

$$\text{Sem}^{(m,n)}: \left[\text{Sem}^{(m,n)}\right]_{\text{refl, act}} \xrightarrow[\text{sops}]{} \left[\text{Sem}^{(m,n)}\right]_{\text{refl, act}}$$

$$\text{id}(i,j): \quad \forall\, i, j \in s(m): \quad \left(\text{Sem}^{i.j}\right) \xrightarrow[\text{id}]{} \left(\text{Sem}^{i.j}\right)$$

$$\text{perm}(i,j): \forall\, i, j \in s(m): \left(\text{Sem}^{i}, \text{Sem}^{j}\right) \xrightarrow[\text{perm}]{} \left(\text{Sem}^{j}, \text{Sem}^{i}\right)$$

$$\text{red}(i,j): \forall\, i, j \in s(m): \left(\text{Sem}^{i}, \text{Sem}^{j}\right) \xrightarrow[\text{red}]{} \left(\text{Sem}^{i}, \text{Sem}^{i}\right)$$

$$\text{bif}(i,j): \forall\, i, j \in s(m): \left(\text{Sem}^{i}, \text{Sem}^{j}\right) \xrightarrow[\text{bif}]{} \left(\left(\text{Sem}^{i} \,\|\, \text{Sem}^{j}\right), \text{Sem}^{j}\right)$$

$$\text{repl}\,(i,j): \forall\, i, j \in s(m): \left(\text{Sem}^{i}, \text{Sem}^{j}\right) \xrightarrow[\text{repl}]{} \left(\left(\text{Sem}^{i} \,\middle|\, \text{Sem}^{i}\right), \text{Sem}^{j}\right)$$

$$\text{sops} = \{\text{id}, \text{perm}, \text{red}, \text{bif}, \text{repl}\}$$

http://www.thinkartlab.com/pkl/lola/ConTeXtures.pdf

http://www.thinkartlab.com/pkl/lola/From Ruby to Rudy.pdf § 11.3

Examples

$$\text{id}_{1.2.3}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix}$$

$$\text{repl}_{1.1\ .1\ .1}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & \square & \square \\ S_1 & S_2 & \square \\ S_{1.1} & \square & S_3 \end{pmatrix}$$

$$\text{perm}_{1-2}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_2 & \square & \square \\ \square & S_1 & \square \\ \square & \square & S_3 \end{pmatrix}$$

$$\text{red}_{1-2}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & \square & \square \\ \square & S_1 & \square \\ \square & \square & S_3 \end{pmatrix}$$

$$\text{bif}_{1-2}: \begin{pmatrix} S_1 & \square & \square \\ \square & S_2 & \square \\ \square & \square & S_3 \end{pmatrix} \Longrightarrow \begin{pmatrix} S_1 & S_2 & S_2 \\ S_2 & S_2 & \square \\ S_2 & \square & S_3 \end{pmatrix}$$

Considering the 3 principles of semiotic restrictions, i.e. triadic diversity, degenerative triadic order and trichotomic inclusion, *permutation* and *reduction* operations might add some more structure to semiotics without surpassing its general framework. The operation of permutation, which had a case as a *dualisation* (Bense) only, is complemented by Toth's concept of *transpositions*.

2.1.3. Ternary matrix$^{(3,3)}$ and scheme$^{(3,3)}$

$$\text{Sem}^{(3,3)} = \left(\text{Sem}^{(3,1)} \times \text{Sem}^{(3,1)} \times \text{Sem}^{(3,1)}\right) = \begin{bmatrix} \left(\text{Sem}^1 \times \text{Sem}^1 \times \text{Sem}^1\right), \\ \left(\text{Sem}^2 \times \text{Sem}^2 \times \text{Sem}^2\right), \\ \left(\text{Sem}^3 \times \text{Sem}^3 \times \text{Sem}^3\right) \end{bmatrix}$$

$$\text{val}\left(\text{Sem}^{(3,3)}\right) = \begin{bmatrix} \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right) \times \\ \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right) \times \\ \left(1_{1.3}, 2_{1.2}, 3_{2.3}\right) \end{bmatrix}:$$

$$\text{val}\left(\text{Sem}^1 \times \text{Sem}^1 \times \text{Sem}^1\right) = (1, 2)_1 \times (1, 2)_1 \times (1, 2)_1$$
$$\text{val}\left(\text{Sem}^2 \times \text{Sem}^2 \times \text{Sem}^2\right) = (2, 3)_2 \times (2, 3)_2 \times (2, 3)_2$$
$$\text{val}\left(\text{Sem}^3 \times \text{Sem}^3 \times \text{Sem}^3\right) = (1, 3)_3 \times (1, 3)_3 \times (1, 3)_3$$

Matching conditions:

$(1, 1, 1)_1 \cong (1, 1, 1)_3$

$(2, 2, 2)_1 \cong (2, 2, 2)_2$

$(3, 3, 3)_2 \cong (3, 3, 3)_3.$

Sign classes Sem$^{(3,3)}$

$(3.1\ 2.2\ 1.3) \in \text{Sem}^{(3,2)}$

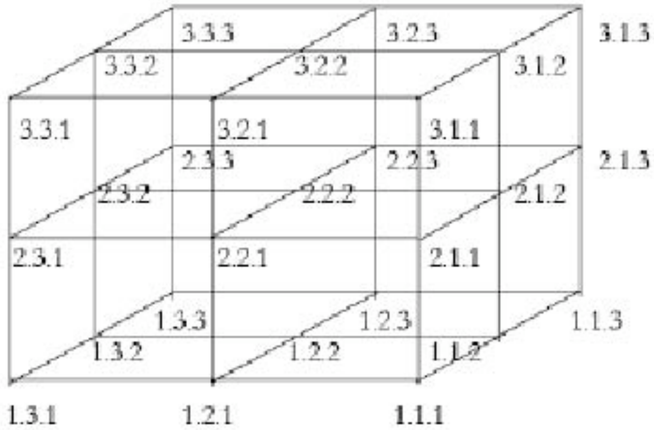
$(3.1\ .3\ \ 2.1\ .3\ \ 1.3\ .3) \in \text{Sem}^{(3,3)}$

3 – contextural semiotic matrix $^{(3,3)}$

$$\text{Sem}^{(3,3)} = \begin{bmatrix} \begin{pmatrix} \text{MM} & .1_{1.3} & .2_{1.2} & .3_{2.3} \\ 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix} \\ \begin{pmatrix} 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix} \\ \begin{pmatrix} 1_{1.3} & \mathbf{1.1_{1.3}} & \mathbf{1.2_1} & \mathbf{1.3_3} \\ 2_{1.2} & \mathbf{2.1_1} & \mathbf{2.2_{1.2}} & \mathbf{2.3_2} \\ 3_{2.3} & \mathbf{3.1_3} & \mathbf{3.2_2} & \mathbf{3.3_{2.3}} \end{pmatrix} \end{bmatrix}$$

$\sum = 27$

Different presentation of the matrix$^{(3,3)}$
(Toth, Strukturen + Processe, *p*.36, 2008)



Semiotics scheme for Sem$^{(3,3)}$

Sem$^{(3,3)}$ – scheme

$$\begin{bmatrix} (1.1.1)_{1.3} & \longrightarrow & (1.1.2) & \longrightarrow & (1.2.1) & \longrightarrow & (1.2.2) & \longrightarrow & (1.1.3) & \longrightarrow & (1.3.1) & \longrightarrow & (1.3.3) \\ \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow \\ (2.1.1) & \longrightarrow & (2.1.2) & \longrightarrow & (2.2.1) & \longrightarrow & (2.2.2)_{1.2} & \longrightarrow & (2.2.3) & \longrightarrow & (2.3.2) & \longrightarrow & (2.3.3) \\ \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow & \square & \downarrow \\ (3.1.1) & \longrightarrow & (3.1.3) & \longrightarrow & (3.3.1) & \longrightarrow & (3.2.2) & \longrightarrow & (3.2.3) & \longrightarrow & (3.3.2) & \longrightarrow & (3.3.3)_{2.3} \end{bmatrix}$$

$\sum = 21$

Combinatorics

Matrix: $3\mathrm{x}3\mathrm{x}3 = 27$

Scheme: $(3\mathrm{x}3\mathrm{x}3)_{/MC} = (3\mathrm{x}3\mathrm{x}3) - 6 = 21$

$\{1, 2, 3\} = \{(1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)\}$

For $m = 3,\ n = 2$: $|\text{matrix}| = |\text{scheme}|$

In general: $m \geq 2,\ n \geq 3$: $|\text{matrix}| > |\text{scheme}|$.

Sub – system decomposition of Sem$^{(3, 3)}$:

sub – system$_1$ =

$(1, 1)\mathrm{x}(1) = (1.1.1)$

$(1, 1)\mathrm{x}(2) = (1.1.2)$

$(1, 2)\mathrm{x}(1) = (1.2.1)$

$(1, 2)\mathrm{x}(2) = (1.2.2)$

$(2, 1)\mathrm{x}(1) = (2.1.1)$

$(2, 1)\mathrm{x}(2) = (2.1.2)$

$(2, 2)\mathrm{x}(1) = (2.2.1)$

$(2, 2)\mathrm{x}(2) = (2.2.2)$.

The same for sub – system$_2$ and sub – system$_3$.

sub – systems Sem$^{(3, 3)}$:

$$\text{sub-system}_1 = \begin{bmatrix} (1.1.1) \longrightarrow & (1.1.2) & \longrightarrow & (1.2.1) & \longrightarrow & (1.2.2 \\ \downarrow \quad x & \downarrow & x & \downarrow & x & \downarrow \\ (2.1.1) \longrightarrow & (2.1.2) & \longrightarrow & (2.2.1) & \longrightarrow & (2.2.2) \end{bmatrix}$$

$$\text{sub-system}_2 = \begin{bmatrix} (2.2.2) \longrightarrow & (2.2.3) & \longrightarrow & (2.3.2) & \longrightarrow & (2.3.3 \\ \downarrow \quad x & \downarrow & x & \downarrow & x & \downarrow \\ (3.2.2) \longrightarrow & (3.2.3) & \longrightarrow & (3.3.2) & \longrightarrow & (3.3.3) \end{bmatrix}$$

$$\text{sub-system}_3 = \begin{bmatrix} (1.1.1) \longrightarrow & (1.1.3) & \longrightarrow & (1.3.1) & \longrightarrow & (1.3.3 \\ \downarrow \quad x & \downarrow & x & \downarrow & x & \downarrow \\ (3.1.1) \longrightarrow & (3.1.3) & \longrightarrow & (3.3.1) & \longrightarrow & (3.3.3) \end{bmatrix}$$

Possible constellation of the *matrix* of Sem$^{(3, 3)}$, like (1, 2, 3), (1, 3, 2), that is, constellations with (i, j, k), i!=j!=k, for the ternary function AxBxC of the matrix are not decomposable into Sem$^{(2, 2)}$ – subsystems of the semiotic *scheme*.

For systems m, n>=3, well known combinatorial problems of decomposition into

sub-systems have to be solved (Kaehr, Mahler, § 9, 1993). http://www.thinkartlab.com/pkl/media/mg-book.pdf
As a consequence of the matching conditions of decomposition, the semiotic system $\text{Sem}^{(3,3)}$ is not delivering $3^9 = 19683$ different decomposable semiotic functions as demanded by (Toth, Gost, p. 9, 2008).

$\text{Sem}^{(4,3)}$ – scheme

(1.1.1)	→	(1.1.2)	→	(1.2.1)	→	(1.2.2)	→	(1.1.3)	→	(1.3.1)	→	(1.3.3)	→	(1.1.4)	→	(1.4.1)	→	(1.4
↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓
(2.1.1)	→	(2.1.2)	→	(2.2.1)	→	(2.2.2)	→	(2.2.3)	→	(2.3.2)	→	(2.3.3)	→	(2.2.4)	→	(2.4.2)	→	(2.4
↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓
(3.1.1)	→	(3.1.3)	→	(3.3.1)	→	(3.2.2)	→	(3.2.3)	→	(3.3.2)	→	(3.3.3)	→	(3.3.4)	→	(3.4.3)	→	(3.4
↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓	▫	↓
(4.1.1)	→	(4.1.4)	▫	(4.4.1)	→	(4.2.2)	→	(4.2.4)	→	(4.4.2)	→	(4.4.3)	→	(4.3.4)	→	(4.4.3)	→	(4.4

$\sum$ scheme = 40, matrix = 4 x4x4 = 64

2.2. Semiotic decomposition of tetradic systems

2.2.1. Unary tetradic matrix

$$\text{Sem}^{(4,1,3)} = \left(\text{Sem}^1, \text{Sem}^2, \text{Sem}^3, \text{Sem}^4\right)$$

$$\text{val}\left(\text{Sem}^{(4,1,3)}\right) = \boxed{\begin{array}{l} 1_1 \longrightarrow 2_1 \longrightarrow 3_1 \longrightarrow x \\ x \longrightarrow 2_2 \longrightarrow 3_2 \longrightarrow 4_2 \\ 1_3 \longrightarrow 2_3 \longrightarrow x \longrightarrow 4_3 \\ 1_4 \longrightarrow x \longrightarrow 3_4 \longrightarrow 4_4 \end{array}}$$

$$\text{val}\left(\text{Sem}^{(4,1,3)}\right) = \left(1_{1.3.4},\ 2_{1.2.3},\ 3_{1.2.4},\ 4_{2.3.4}\right).$$

2.2.2. Binary tetradic matrix

$Sem^{(4,2,3)} = Sem^{(4,1,3)} \times Sem^{(4,1,3)} =$
$[(Sem^1 \times Sem^1), (Sem^2 \times Sem^2), (Sem^3 \times Sem^3), (Sem^4 \times Sem^4)].$

$val(Sem^{(4,1,3)} \times Sem^{(4,1,3)}) =$

$(1_{1.3.4}, 2_{1.2.3}, 3_{1.2.4}, 4_{2.3.4}) \times (1_{1.3.4}, 2_{1.2.3}, 3_{1.2.4}, 4_{2.3.4})$

with :

$val(Sem^1 \times Sem^1) = (1, 2, 3)_1 \times (1, 2, 3)_1$

$val(Sem^2 \times Sem^2) = (2, 3, 4)_2 \times (2, 3, 4)_2$

$val(Sem^3 \times Sem^3) = (1, 2, 4)_3 \times (1, 2, 4)_3$

$val(Sem^4 \times Sem^4) = (1, 3, 4)_4 \times (1, 3, 4)_4.$

As presented at " Semiotics in Diamonds " *a* coloring of the subsystems might emphazise

http : // www.thinkartlab.com / pkl / lola / Semiotics – in – Diamonds / Semiotics – in – Diamonds.html

$$val\left(Sem^{(4,1,3)} \; x \; Sem^{(4,1,3)}\right) =$$

$$sem^1 \; x \; sem^1 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3.4} & 1.2_1 & 1.3_1 & 1.4 \\ 2 & 2.1_1 & 2.2_{1.2.3} & 2.3_1 & 2.4 \\ 3 & 3.1_1 & 3.2_1 & 3.3_{1.2.4} & 3.4 \\ 4 & 4.1 & 4.2 & 4.3 & 4.4 \end{pmatrix}$$

$$sem^2 \; x \; sem^2 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_1 & 1.2_1 & 1.3_1 & 1.4 \\ 2 & 2.1_1 & 2.2_{1.2} & 2.3_{1.2} & 2.4_2 \\ 3 & 3.1_1 & 3.2_{1.2} & 3.3_{1.2} & 3.4_2 \\ 4 & 4.1 & 4.2_2 & 4.3_2 & 4.4_2 \end{pmatrix}$$

$$sem^3 \; x \; sem^3 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3} & 1.2_{1.3} & 1.3_1 & 1.4_3 \\ 2 & 2.1_{1.3} & 2.2_{1.2.3} & 2.3_{1.2} & 2.4_{2.3} \\ 3 & 3.1_1 & 3.2_{1.2} & 3.3_{1.2} & 3.4_2 \\ 4 & 4.1_3 & 4.2_{3.2} & 4.3_2 & 4.4_{3.2} \end{pmatrix}$$

$$sem^4 \; x \; sem^4 = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3.4} & 1.2_{1.3} & 1.3_{1.4} & 1.4_{3.4} \\ 2 & 2.1_{1.3} & 2.2_{1.2.3} & 2.3_{1.2} & 2.4_{2.3} \\ 3 & 3.1_{1.4} & 3.2_{1.2} & 3.3_{1.2.4} & 3.4_{2.4} \\ 4 & 4.1_{3.4} & 4.2_{3.2} & 4.3_{2.4} & 4.4_{2.3.4} \end{pmatrix}$$

4 – contextural semiotic matrix

$$Sem^{(4,2,3)} = \begin{pmatrix} MM & 1 & 2 & 3 & 4 \\ 1 & 1.1_{1.3.4} & 1.2_1 & 1.3_{1.4} & 1.4_{3.4} \\ 2 & 2.1_1 & 2.2_{1.2.3} & 2.3_2 & 2.4_{2.3} \\ 3 & 3.1_{1.4} & 3.2_2 & 3.3_{1.2.4} & 3.4_{2.4} \\ 4 & 4.1_{3.4} & 4.2_{3.2} & 4.3_{2.4} & 4.4_{2.3.4} \end{pmatrix}$$

Sign classes for $Sem^{(4,1,2)}$

$ZR^{(3,2)} = \langle 3.x,\ 2.y,\ 1.z \rangle$ with $x,\ y,\ z \in \{1,\ 2,\ 3\}$

$ZR^{(4,2)} = (ZR^1,\ ZR^2,\ ZR^3,\ ZR^4)$:

$ZR^1 = \langle 3.x,\ 2.y,\ 1.z,\ -- \rangle$

$ZR^2 = \langle --,\ 3.x,\ 2.y,\ 1.z \rangle$

$ZR^3 = \langle 3.x,\ 2.y,\ --,\ 1.z \rangle|$

$ZR^4 = \langle 3.x,\ --,\ 2.y,\ 1.z \rangle$

$$\text{decomp}(ZR^{(4,2)}) = \begin{bmatrix} 3.x, & 2.y, & 1.z, & -- \\ --, & 3.x, & 2.y, & 1.z \\ 3.x, & 2.y, & --, & 1.z \\ 3.x, & --, & 2.y, & 1.z \end{bmatrix}$$

$$\text{comp}\left(\begin{bmatrix} 3.x, & 2.y, & 1.z, & -- \\ --, & 3.x, & 2.y, & 1.z \\ 3.x, & 2.y, & --, & 1.z \\ 3.x, & --, & 2.y, & 1.z \end{bmatrix}\right) = [4.a,\ 3.b,\ 2.c,\ 1.d]$$

Matching conditions

$(3.x)_1 \cong (3.x)_3 \cong (3.x)_4$

$(2.y)_1 \cong (3.x)_2 \cong (2.y)_3$

$(1.z)_1 \cong (2.y)_2 \cong (2.y)_4$

$(1.z)_2 \cong (1.z)_3 \cong (1.z)_4$.

Each sign class of $Sem^{(4,2)}$ is decomposable into its 4 $Sem^{(3,2)}$ sign classes.

Example

$[4.3\ 3.2\ 2.1\ 1.1] \in Sem^{(4,2)}$

$$\text{decomp}([4.3\ 3.2\ 2.1\ 1.1]) = \begin{bmatrix} (4.3,\ 3.2,\ 2.1,\ --) \in Sem^1 \\ (--,\ 3.2,\ 2.1,\ 1.1) \in Sem^2 \\ (4.3,\ 3.2,\ --,\ 1.1) \in Sem^3 \\ (4.3,\ --,\ 2.1,\ 1.1) \in Sem^4 \end{bmatrix}$$

$$\overline{}$$

$[4.3\ 3.2\ 2.1\ 1.1] \in Sem^{(4,2)}$

2.2.3. (Some) Sign classes for $Sem^{(4,2)}$

Class *A* = (4.1)

```
4.1 3.1 2.1 1.1
4.1 3.1 2.1 1.2
4.1 3.1 2.1 1.3
4.1 3.1 2.1 1.4
4.1 3.1 2.2 1.2
4.1 3.1 2.2 1.3
4.1 3.1 2.2 1.4

4.1 3.2 2.2 1.2
4.1 3.2 2.2 1.3
4.1 3.2 2.2 1.4

4.1 3.2 2.2 1.2
4.1 3.2 2.2 1.3
4.1 3.2 2.2 1.4

4.1 3.2 2.3 1.3
4.1 3.2 2.3 1.4
4.1 3.3 2.3 1.4

4.1 3.2 2.4 1.4
4.1 3.3 2.4 1.4
4.1 3.4 2.4 1.4
```

Class *B* = (4.2)

```
4.2 3.1 2.1 1.1
4.2 3.1 2.1 1.2
4.2 3.1 2.1 1.3
4.1 3.1 2.1 1.4
4.2 3.1 2.2 1.2
4.2 3.1 2.2 1.3
4.2 3.1 2.2 1.4

4.2 3.2 2.2 1.2
4.2 3.2 2.2 1.3
4.2 3.2 2.2 1.4

4.2 3.2 2.2 1.2
4.2 3.2 2.2 1.3
4.2 3.2 2.2 1.4

4.2 3.2 2.3 1.3
4.2 3.2 2.3 1.4
```

Class *C* = (4.3)

```
4.3 3.3 2.3 1.3
4.3 3.3 2.3 1.4
4.3 3.3 2.4 1.4
4.3 3.4 2.4 1.4
```

Class *D* = (4.4)

```
4.4 3.4 2.4 1.4
```

Combinatorics

```
|ClassA| = 5 x3 = 15
|Class B| = 6 x3 + 1 = 19
|Class C| = 4
|Class D| = 1
Total = 1 + 15 + 19 = 35
```

2.3. Interplay of semiotics, logics and arithmetic

A study of polycontextural semiotics, focused on semiotics alone, is not yet guaranteeing its polycontexturality. The logical and arithmetical status of semiotics, mono- and polycontextural, remains undetermined if its corresponding logics are not determined.
There are many ways open to formalize, logically and arithmetically, semiotics and polysemiotics. Good candidates are the logics from the modal logic pool. Nevertheless, they have all to be classified as mono-contextural.

For the purpose of this introductory sketch of a *descriptive* characterization of the idea of poly-semiotics, it might be sufficient to hint to the decision to use 3-contextural subsystems of 4-contextural logics and arithmetics. Instead of the usual decomposition into elementary contextures.

As a consequence, it turns out that the apparatus of classical category theory is not adequate to formalize semiotics and polysemiotics.

Hence, from a 4-contextural logic, $Log^{(4)}$, with its six 2-contextures, $Log^{(4,2)}$, its four 3-contextures, $Log^{(4,3)}$, only the four 3-contxtural subsystems are in direct correspondence to the 4-contextural (poly)semiotics, decomposed into its 3-contextural semiotic parts.

$$\mathbf{Graphematics}^{(4,3,2)} = \left(Sem^{(4,2)},\ Log^{(4.2)},\ Arith^{(4,2)}\right)$$

with

$$\mathbf{Sem}^{(4,2)} = \left(Sem^{(3,1)},\ Sem^{(3,2)},\ Sem^{(3,3)},\ Sem^{(3,4)}\right)$$

$$\mathbf{Log}^{(4.2)} = \left(Log^{(3,1)},\ Log^{(3,2)},\ Log^{(3,3)},\ Log^{(3,4)}\right)$$

$$\mathbf{Arith}^{(4,2)} = \left(Arith^{(3,1)},\ Arith^{(3,2)},\ Arith^{(3,3)},\ Arith^{(3,4)}\right)$$

$Sem^{(4,2)}$ is a 4 – contextural semiotics, which is realizing the paradigmatic and conceptual transformations of the 4-contextural logics $Log^{(4)}$ and arithmetic $Arith^{(4)}$.

$Log^{(4,2)}$ is a 4 – contextural logic, which is realizing the structural and deductional transformations of the 4-contextural semiotics $Sem^{(4)}$ and arithmetic $Arith^{(4,2)}$.

$Arith^{(4,2)}$ is a 4 – contextural arithmetic, which is realizing the structural and computational transformations of the 4-contextural semiotics $Sem^{(4)}$ and logics $Log^{(4,2)}$.

$Graphematics^{(4,3,2)}$ is the interplay of semiotics, logics and arithmetic of complexity $compl^{(4,2)}$.

2.4. Multi-dimensional and polycontextural semiotics

2.4.1. Toth’s multi-dimensional semiotics

Scheme of *a* 3 – dimensional semiotics

Toth introduced in (Transit – Korridor, 2009) *a* 3 – dimensional sign relation

$$3-ZKL = ((a.3.b)(c.2.d)(e.1.f))$$

with its 27 variations.

Examples

(1.3.3 1.2.2 1.1.1) (1.3.1 1.2.2 1.1.3)

(2.3.3 2.2.2 2.1.1) (2.3.1 2.2.2 2.1.3)

(3.3.3 3.2.2 3.1.1) (3.3.1 3.2.2 3.1.3).

Scheme of a Transit – Korridor :

$TK = \{<a.3.3.b \;\; c.2.2.d \;\; e.1.1.f>\}$

with $a, c, e \in \{1, 2, 3\}$ and $b, d, f \in \{1, 2, 3, 4\}$.

In general (I guess) :

$m - ZKL =$

$((a.3.3 \ldots 3 b_1 b_2 \ldots b_m)(c.2.2 \ldots 2 d_1 d_2 \ldots d_m)(e.1.1 \ldots 1 f_1 f_2 \ldots f_m))$

with $a, c, e \in \{1, 2, 3\}$ and $b, d, f \in \{1, 2, \ldots, m\}$.

2.4.2. Polycontextural (uni-dimensional) semiotics

$Sem^{(4,2)} - scheme = [4.a, 3.b, 2.c, 1.d]$

with $a, b, c, d \in \{1, 2, 3, 4\}$.

Sign classes for $Sem^{(4)}$ are defined by :

$ZR = (a, (a \Longrightarrow b), (a \Longrightarrow b \Longrightarrow c), (a \Longrightarrow b \Longrightarrow c \Longrightarrow d))$

for

$a = \{1.1, 1.2, 1.3, 1.4\}$

$b = \{2.1, 2.2, 2.3, 2.4\}$

$c = \{3.1, 3.2, 3.3, 3.4\}$

$d = \{4.1, 4.2, 4.3, 4.4\}$

General sign relation for $ZR^{(4)}$:

$ZR^{(4)} = <4.u, 3.x, 2.y, 1.z>$ with $u, x, y, z \in \{1, 2, 3, 4\}$

and $u \leq x \leq y \leq z$

In general :

$Sem^{(m,2)} = [m.a_m, m-1.a_{m-1}, \ldots, 2.a_2, 1.a_1]$

2.4.3. Comparisons

$2 - ZKL = <3.x, 2.y, 1.z>$

with $x, y, z \in \{1, 2, 3\}$ and $x \leq y \leq z. \Longrightarrow$ 10 sign classes

$3 - ZKL = ((a.3.b)(c.2.d)(e.1.f)) \Longrightarrow$ 27 sign classes

$Sem^{(4,2)} - scheme = [4.a, 3.b, 2.c, 1.d] \Longrightarrow 35 (??)$

3. Interactivity in poly-semiotics

3.1. Interactions between (2,2)-subsystems of $Sem^{(3,2)}$

Interactional semiotic functions

Interactions, in the form of transjunctions are of great importance in polycontextural logic
In fact, from a combinatorial point of view, most polylogical functions are transjunctional.
Therefore, they should deserve a prominent place in a polycontextural semiotics.
At this place, not more than a short hint can be given.

$(\text{transjunction, junction, junction})_{\text{Polylogic}}$
$\Longrightarrow$
$(\text{interaction, action, action})_{\text{Polysemiotics}}$

$$Sem^{(3,2)}_{(inter,\ act,\ act)} = \left(Sem^{(3,1)} \times Sem^{(3,1)}\right)_{(inter,\ act,\ act)} =$$

$$\left[\left(\left(Sem^{1} \times Sem^{1}\right) \| \left(Sem^{2.3} \times Sem^{2.3}\right)\right), \left(Sem^{2} \times Sem^{2}\right), \left(Sem^{3} \times Sem^{3}\right)\right]:$$

$$val\left(inter_1\left(Sem^{1} \times Sem^{1}\right)\right) = (1,1)_1 \times (2,2)_1 \| (2,3)_{2.3} \times (3,2)_{2.3},$$
$$val\left(act_2\left(Sem^{2} \times Sem^{2}\right)\right) = (2,3)_2 \times (2,3)_2,$$
$$val\left(act_3\left(Sem^{3} \times Sem^{3}\right)\right) = (1,3)_3 \times (1,3)_3,$$
$$\text{with } (2,3)_{2.3} \times (3,2)_{2.3} = \left(2_2, 3_{2.3}\right) \times \left(3_{2.3}, 2_2\right)$$

Operational notation

$$Op_{(inter,\ act,\ act)} : \left[\left(Sem^{1} \times Sem^{1}\right), \left(Sem^{2} \times Sem^{2}\right), \left(Sem^{3} \times Sem^{3}\right)\right]$$
$$\Longrightarrow$$
$$\left[\left(\left(Sem^{1} \times Sem^{1}\right) \| \left(Sem^{2.3} \times Sem^{2.3}\right)\right), \left(Sem^{2} \times Sem^{2}\right), \left(Sem^{3} \times Sem^{3}\right)\right]$$

or short :

$$Op_{(inter,\ act,\ act)} : Sem^{(3,2)} \Longrightarrow bif_{1.2.3}\left(id_{2.3}\left(Sem^{(3,2)}\right)\right)$$

$$[\text{inter, act, act}] \equiv [\blacklozenge, \circ, \circ]$$

$$Sem^{(3,2,2)}_{(inter,\ act,\ act)} = \begin{pmatrix} [\blacklozenge, \circ, \circ] & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{2.3}_{2.3} & \mathbf{1.3}_{3} \\ 2 & \mathbf{3.2}_{2.3} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3 & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

Different modi of interaction with Sem^1 :

$$\begin{pmatrix} [\blacklozenge_1, \circ, \circ] & 1 & 2 & 3 \\ 1 & id_{1.3} & \alpha_{2.3} & \alpha_3 \\ 2 & \alpha^\circ{}_{2.3} & id_{1.2} & \alpha_2 \\ 3 & \alpha^\circ{}_3 & \alpha^\circ{}_2 & id_{2.3} \end{pmatrix} \begin{pmatrix} [\blacklozenge_2, \circ, \circ] & 1 & 2 & 3 \\ 1 & id_{1.3} & \alpha^\circ{}_{2.3} & \alpha_3 \\ 2 & \alpha_{2.3} & id_{1.2} & \alpha_2 \\ 3 & \alpha^\circ{}_3 & \alpha^\circ{}_2 & id_{2.3} \end{pmatrix}$$

$$\begin{pmatrix} [\blacklozenge_3, \circ, \circ] & 1 & 2 & 3 \\ 1 & id_{1.3} & \alpha^\circ{}_{2.3} & \alpha_3 \\ 2 & \alpha^\circ{}_{2.3} & id_{1.2} & \alpha_2 \\ 3 & \alpha^\circ{}_3 & \alpha^\circ{}_2 & id_{2.3} \end{pmatrix} \begin{pmatrix} [\blacklozenge_4, \circ, \circ] & 1 & 2 & 3 \\ 1 & id_{1.3} & \alpha_{2.3} & \alpha_3 \\ 2 & \alpha_{2.3} & id_{1.2} & \alpha_2 \\ 3 & \alpha^\circ{}_3 & \alpha^\circ{}_2 & id_{2.3} \end{pmatrix}$$

General distribution tables for [inter, act, act]

$[\blacklozenge, \circ, \circ]$	O_1	O_2	O_3
M_1	sem_1	x	x
M_2	$trans_2$	sem_2	x
M_3	$trans_3$	x	sem_3

3.2. Interactions between (3, 3)-subsystems of $\text{Sem}^{(4,2,3)}$

$$Sem^{(4,2,3)}_{(inter, act, act, act, act, inter)} =$$

$$\left[\left(\left(Sem^{1}\ x\ Sem^{1}\right) \| \left(Sem^{2.3}\ x\ Sem^{2.3}\right)\right),\right.$$
$$\left(Sem^{2}\ x\ Sem^{2}\right),\ \left(Sem^{3}\ x\ Sem^{3}\right),$$
$$\left.\left(\left(Sem^{4}\ x\ Sem^{4}\right) \| \left(Sem^{2.3}\ x\ Sem^{2.3}\right)\right)\right].$$

$$val\left(Sem^{(4,1,3)}\ x\ Sem^{(4,1,3)}\right) =$$
$$\left(1_{1.3.4},\ 2_{1.2.3},\ 3_{1.2.4},\ 4_{2.3.4}\right) x \left(1_{1.3.4},\ 2_{1.2.3},\ 3_{1.2.4},\ 4_{2.3.4}\right)$$

with:

$$val\left(Sem^{1}\ x\ Sem^{1}\right) = (1, 2, 3)_1\ x\ (1, 2, 3)_1 \| \left((2, 3, 4)_2\ x\ (2, 3, 4)_2,\ (1, 2, 4)_3\ x\ (1, 2, 4)_3\right)$$
$$val\left(Sem^{2}\ x\ Sem^{2}\right) = (2, 3, 4)_2\ x\ (2, 3, 4)_2$$
$$val\left(Sem^{3}\ x\ Sem^{3}\right) = (1, 2, 4)_3\ x\ (1, 2, 4)_3$$
$$val\left(Sem^{4}\ x\ Sem^{4}\right) = (1, 3, 4)_4\ x\ (1, 3, 4)_4 \| \left((2, 3, 4)_2\ x\ (2, 3, 4)_2,\ (1, 2, 4)_3\ x\ (1, 2, 4)_3\right)$$

$$Op_{(inter, act, act, act, act, inter)} : Sem^{(4,2)} \Longrightarrow bif_{1.2.3}\left(id_{2.3.4.5}\left(bif_{6.3.2}\left(Sem^{(4,2)}\right)\right)\right)$$

4 – contextural semiotic matrix

$Sem^{(4,2,3)} =$

MM	1	2	3	4
1	$1.1_{1.3.4}$	$\mathbf{2.3_{2.3}}$	$1.3_{1.4}$	$1.4_{3.4}$
2	$\mathbf{3.2_{2.3}}$	$2.2_{1.2.3}$	2.3_{2}	$2.4_{2.3}$
3	$3.1_{1.4}$	3.2_{2}	$3.3_{1.2.4}$	$\mathbf{2.3_{2.3}}$
4	$4.1_{3.4}$	$4.2_{3.2}$	$\mathbf{3.2_{2.3}}$	$4.4_{2.3.4}$

[◆, ○, ○, ○, ○, ◆]	O_1	O_2	O_3	O_4
M_1	sem_1	x	x	x
M_2	$trans_2$	sem_2	x	$trans_2$
M_3	$trans_3$	x	sem_3	$trans_3$
M_4	x	x	x	sem_4

3.3. Interaction and reflectionality

Following the concepts and methods developed in *"ConTeXtures. Programming Dynamic Complexity"* (Kaehr, 2005), short hints of their application to disseminated semiotics are given. Both, the bracket and the table notation are emphazing the architectonic structure of reflection and interaction.

Reflections in Sem$^{(3,2)}$

$$\left[\begin{array}{l}\left[\begin{array}{l} O_1 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \quad (Sem_{110})\end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_2 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \quad (Sem_{222})\end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_3 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \quad (Sem_{033})\end{array}\right)\end{array}\right]\end{array}\right]$$

[reflection]	O_1	O_2	O_3
M_1	sem_1	sem_2	x
M_2	sem_1	sem_2	sem_3
M_3	x	sem_2	sem_3

Iterative reflection in Sem$^{(3,2)}$

$$\left[\begin{array}{l}\left[\begin{array}{l} O_1 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ (Sem_{110}) \left(\begin{array}{l} M_1\ M_2\ M_3 \\ (Sem_{110}) \left(\begin{array}{l} M_1\ M_2\ M_3 \\ (Sem_{110}) \left(\begin{array}{l} M_1\ M_2\ M_3 \\ (Sem_{110})\end{array}\right)\end{array}\right)\end{array}\right)\end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_2 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \quad (Sem_{222})\end{array}\right)\end{array}\right] \\ \left[\begin{array}{l} O_3 \\ \left(\begin{array}{l} M_1\ M_2\ M_3 \\ \quad (Sem_{033})\end{array}\right)\end{array}\right]\end{array}\right]$$

[reflections]	O_1	O_2	O_3
M_1	$sem_{1.1.1.1}$	sem_2	x
M_2	$sem_{1.1.1.1}$	sem_2	sem_3
M_3	x	sem_2	sem_3

Interplay between interactionality and reflectionality in $Sem^{(3,2)}$

$$\left[\begin{matrix} \left[\begin{matrix} O_1 \\ \left(\begin{matrix} M_1\ M_2\ M_3 \\ \left(Sem_{111}\right) \end{matrix}\right) \end{matrix}\right] \\ \left[\begin{matrix} O_2 \\ \left(\begin{matrix} M_1 \left(\begin{matrix} \left(Sem_{100}\right) \\ \left(\left(M_2 \left(\begin{matrix} M_3 \\ \left(Sem_{003}\right) \end{matrix}\right)\right)\right) \end{matrix}\right) \\ \left(Sem_{222}\right) \end{matrix}\right) \end{matrix}\right] \\ \left[\begin{matrix} O_3 \\ \left(\begin{matrix} M_1\ M_2\ M_3 \\ \left(Sem_{033}\right) \end{matrix}\right) \end{matrix}\right] \end{matrix}\right]$$

[interact, reflect]	O_1	O_2	O_3
M_1	sem_1	$sem_{2.1}$	x
M_2	sem_1	$sem_{2.0}$	sem_3
M_3	sem_1	$sem_{2.3}$	sem_3

Reflections in $Sem^{(4,2)}$

[reflection]	O_1	O_2	O_3	O_4
M_1	sem_1	x	x	x
M_2	sem_1	sem_2	x	sem_4
M_3	sem_1	x	sem_3	sem_4
M_4	x	x	x	sem_4

Interactions and reflections in $Sem^{(4,2)}$

[inter, refl]	O_1	O_2	O_3	O_4
M_1	sem_1	x	sem_3	x
M_2	$trans_2$	sem_2	sem_3	$trans_2$
M_3	$trans_3$	sem_2	sem_3	$trans_3$
M_4	x	x	x	sem_4

4. Logification of semiotics

On the base of the introduced concepts for semiotic interactions interesting operations, rules and transformations (deductions) might be studied. Much of the work in semiotics and pre-semiotics is mainly *descriptive*, introducing its concepts and demonstrating some transformations. But there is nearly no work done for a kind of a deductive treatment in the semiotic field. This goes back mainly to the fact that semiotics in general has not yet accepted the concept of a polycontextural deductional system. On the other hand, a logical and deductive treatment of a genuine triadic-trichotomic semiotics by a *classical* logical approach goes hand in hand with a *reduction* procedure of the triadic-trichotomic complexity of classical semiotics to a dyadic-dichotomic model.

Logification of semiotics becomes relevant if we want to study semiotic operations in poly-semiotic systems. Like for logical systems, we can ask for

a specific state of the system in transformation.
Transformations might produce conflicting results, similar to contradictions in logic. Such irregularities can be easily detected by the *tableaux* method for decomposed semiotic constellations. Therefore, deductive aspects, semiotic model theory ('semantics'), proof theory, etc. of semiotic systems are accessible to be studied for their specific characteristics.

$$Sem^{(3,2,2)}_{(inter,\,act,\,act)} = \begin{pmatrix} [\blacklozenge, \circ, \circ] & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{2.3}_{2.3} & \mathbf{1.3}_{3} \\ 2 & \mathbf{3.2}_{2.3} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3 & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{pmatrix}$$

$$\overset{\text{logification}}{\Longrightarrow} \begin{pmatrix} [\blacklozenge, \vee, \wedge] & T_{1.3} & F_{1.2} & \mathbf{F}_{2.3} \\ T_{1.3} & T_{1.3} & \mathbf{F}_{2.3} & \mathbf{F}_{3} \\ F_{1.2} & \mathbf{F}_{2.3} & F_{\mathbf{1.2}} & F_{\mathbf{2}} \\ \mathbf{F}_{2.3} & \mathbf{F}_{3} & F_{\mathbf{2}} & \mathbf{F}_{2.3} \end{pmatrix}$$

$$\log\left(Sem^{(3,2,2)}_{[\blacklozenge,\,\vee,\,\wedge]}\right)$$

with:

$T_{1.3} \equiv 1.1_{1.3} \equiv t_1, t_3$

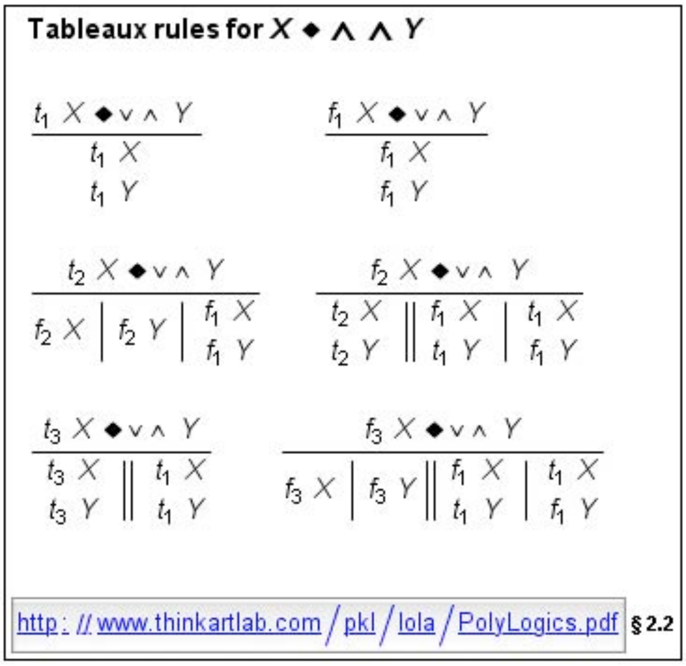
$F_{1.2} \equiv 2.2_{1.2} \equiv f_1, t_2$

$\mathbf{F}_{2.3} \equiv 3.3_{2.3} \equiv f_2, f_3$

Example

$$\frac{F_2\left(Sem^{(3,2,2)}_{(inter,\,act,\,act)}\right)}{F_2\, X \;\Big|\; F_2\, Y} \qquad \frac{\mathbf{F}_2\left(Sem^{(3,2,2)}_{(inter,\,act,\,act)}\right)}{\begin{array}{c}\mathbf{F}_2\, X \\ \mathbf{F}_2\, Y\end{array}}$$

$$\frac{t_2\left(Sem^{(3,2,2)}_{[\blacklozenge,\,\vee,\,\wedge]}\right)}{t_2\, X \;\Big|\; t_2\, Y} \qquad \frac{f_2\left(Sem^{(3,2,2)}_{[\blacklozenge,\,\vee,\,\wedge]}\right)}{\begin{array}{c}f_2\, X \\ f_2\, Y\end{array}}$$



With such a mapping of semiotics onto logics, the whole machinery of combinatorics as studied earlier, might be directly applied (Kaehr, Mahler, 1993).

5. Interactions in diamonds

5.1. Interactions in diamonds

The new distinctions for diamonds between semiotic *systems* and their *environments* are allowing new kinds of interactions. Additionally, *anchored* semiotics and diamonds might be involved into even more radical interactions , like interventions and metamorphosis.

In general, it seems not to be realistic to deal with multi-leveled autonomous systems, say polysemiotics, in their isolation, without considering their complex interactions, e.g. *interpenetrations* (Luhmann), between heterarchically distributed sub-systems.

Interpenetration
"First, interpenetration is not a general relation between system and environment but an intersystem relation between systems that are environments for each other. In the domain of intersystem relations, the concept of interpenetration indicates a very specific situation, which must be distinguished above all from input/output relations (performances). We speak of "penetration" if a system makes its own complexity (and with it indeterminancy, contingency, and the pressure to select) available for constructing another system." (Niklas Luhmann)

Mediation scheme for semiotic diamond$^{(3,2)}$

$$\text{Diamond}^{(3,2)} = \left[\begin{array}{ccccccc} \square & \square & (2.2)_4 & \longleftarrow & (2.2)_4 & \square & \square \\ \square & \square & \updownarrow & \square & \updownarrow & \square & \square \\ (1.1)_1 & \longrightarrow & (2.2)_1 & \diamond & (2.2)_2 & \longrightarrow & (3.3)_2 \\ \mid & \square & & \square & \square & \square & \mid \\ (1.1)_3 & \longrightarrow & - & - & - & \longrightarrow & (3.3)_3 \end{array}\right] \equiv \begin{pmatrix} \square & S_4 & \square \\ S_1 & \square & S_2 \\ \square & S_3 & \square \end{pmatrix}$$

Correspondences for the diamond semiotics Sem_4 : $\delta(2.2)_2 \equiv (2.2)_4,\ \delta(2.2)_1 \equiv (2.2)_4$.

Semiotic diamond scheme for interaction

$$\text{Diamond}^{(3,2,2)}_{(inter,act,act,act)} = \left[\left[\begin{array}{cccc} [\blacklozenge, \circ, \circ] & 1 & 2 & 3 \\ 1 & \mathbf{1.1}_{1.3} & \mathbf{2.3}_{2.3} & \mathbf{1.3}_{3} \\ 2 & \mathbf{3.2}_{2.3} & \mathbf{2.2}_{1.2} & \mathbf{2.3}_{2} \\ 3 & \mathbf{3.1}_{3} & \mathbf{3.2}_{2} & \mathbf{3.3}_{2.3} \end{array}\right]\right] \Bigg\| \left[(2.2)_{(4.4)}\right]$$

A *polylogical* modeling of a semiotic diamond, $\text{Diamond}^{(4,2)}_{(act,inter,act,act)}$, as $\text{Diamond}^{(4,2)}_{(\vee\blacklozenge\vee\wedge)}$ with interaction, transjunction, in sub-system$_2$ and its *interference* in the environmental sub-system$_4$, conjunction, gives some insight into the internal structure of a diamond with a *weak interaction* with sub-system$_4$.

Weak interaction in a diamond with sub − system$_4$

$$\mathrm{val}\left(X^{(4)}\left(\begin{matrix}\wedge\\ \vee\,\blacklozenge\\ \vee\end{matrix}\right)Y^{4)}\right) \equiv \mathrm{val}(X \vee \blacklozenge \vee \wedge Y)$$

$$\frac{t_1\ X \vee\blacklozenge\vee\wedge Y}{t_1\ X \,\Big|\, t_1\ Y \,\Big\|\, \begin{matrix} f_2\ X \\ t_2\ Y\end{matrix} \,\Big|\, \begin{matrix} t_2\ X \\ f_2\ Y\end{matrix}} \qquad \frac{f_1\ X \vee\blacklozenge\vee\wedge Y}{\begin{matrix} f_1\ X \\ f_1\ Y\end{matrix} \,\Big\|\, \begin{matrix} f_4\ X \\ f_4\ Y\end{matrix} \,\Big\|\, \begin{matrix} t_2\ X \\ t_2\ Y\end{matrix}}$$

$$\frac{t_2\ X \vee\blacklozenge\vee\wedge Y}{\begin{matrix} t_2\ X \\ t_2\ X\end{matrix} \,\Big|\, \begin{matrix} t_4\ X \\ t_4\ X\end{matrix}} \qquad \frac{f_2\ X \vee\blacklozenge\vee\wedge Y}{\begin{matrix} f_2\ X \\ f_2\ Y\end{matrix}}$$

$$\frac{t_3\ X \vee\blacklozenge\vee\wedge Y}{t_3\ X \,\Big|\, t_3\ Y \,\Big\|\, \begin{matrix} f_2\ X \\ t_2\ Y\end{matrix} \,\Big|\, \begin{matrix} t_2\ X \\ f_2\ Y\end{matrix}} \qquad \frac{f_3\ X \vee\blacklozenge\vee\wedge Y}{\begin{matrix} f_3\ X \\ f_3\ Y\end{matrix} \,\Big\|\, \begin{matrix} f_2\ X \\ f_2\ Y\end{matrix}}$$

$$\frac{t_4\ X \vee\blacklozenge\vee\wedge Y}{\begin{matrix} t_2\ X \\ t_2\ Y\end{matrix} \,\Big\|\, \begin{matrix} t_4\ X \\ t_4\ Y\end{matrix}} \qquad \frac{f_4\ X \vee\blacklozenge\vee\wedge Y}{\begin{matrix} f_1\ X \\ f_1\ Y\end{matrix} \,\Big\|\, f_4\ X \,\Big|\, f_4\ Y}$$

5.2. Interactions between diamonds

As introduced in *Diamond Text Theory*, special interactions between diamonds are building networks of textemes. In this case, interaction between semiotic systems happens mediated by their neighboring environments.

Notes

1 *Computational* semiotics is interested in modeling interactions in computational scenarios. As much as there is no proper logic of interaction there is even much less development in computational semiotics. There is not even an awareness about the conceptual lack of interactivity constructs in theoretical semiotics. Despite the many *applicative* approaches to semiotic interactions, e.g. in human-computer interface research, it seems, that theoretical and foundational research for a semiotic theory of interaction and reflection is not supported.

Christopher R. Longyear, Further Towards a Triadic Calculus (Part 1, 2, 3)
http://www.vordenker.de/ggphilosophy/longyear-part_1.pdf

2 Independent of later steps of *abolishing* restrictions in the traditional definition of sign classes by Toth's studies, the concept of a Cartesian product remains a fundamental construction to build up a semiotic system.

This fact allows to study the semiotic matrix under a different angle: the *polycontextural* approach of dissemination, i.e. distribution and mediation, of sub-systems as a mechanism to construct and to deconstruct the semiotic matrix. In this sense, an extension of the semiotic matrix for complex sign systems, called polysemiotics, is introduced.

To use Cartesian products doesn't mean that they will remain stable in the development of a general theory of polylogics and polysemiotics. As shown at other places, what was a good starting point, became the main obstacle for further developments. Here again, the abstract mathematical frame (set and category theory) is not always adequate for the project of formalizing transclassical approaches.

This disseminative approach to the semiotics matrix allows to introduce a comparison of semiotic and logical constructions. As main operators of logical interaction, the polylogical *transjunctions* had been studied *in extenso.(Kaehr, 1978, 2005)*

In analogy and translation or transposition from the polycontextural to the semiotic topics, semiotic interactions between semiotic sub-systems shall be introduced. Semiotic sub-systems are a result of a decomposition of the semiotic matrix into its sub-systems. Such a decomposition is dynamic, depending on the complexity of the semiotic matrix. In this paper, only two cases are introduced. The decomposition into (2, 2)-subsystems, with $S_1 = \{1, 2\}$, $S_2 = \{2, 3\}$, $S_3 = \{1, 3\}$. And the decomposition into (3, 3)-subsystems of a polysemiotic system $Sem^{(4,2)}$.

3 Nevertheless, a specific redundancy has to be repeated because of its established and deep-rooted sheepishness and stultifying ignorance. The more or less only answer or ‘feed-back’ I got, when I was emphasizing the importance of a number, e.g. 4, was, *“Why an extension to 4 and not to 7 or 13 or 5112?”* Nobody ever questioned the fact that their response is based on the number 2 (TWO). And surely I never privileged a single natural number of the established number system.

A criticism of such an idea of a privilege of a single natural number was perfectly done long before by Aristotle with his refutation of Pythagorean number theory.
It seems to be better to live and to die with the number TWO than to question it.
As far, it was an important scientific step by Peirce to introduce his triadic-trichotomic semiotics and first sketches to a trichotomic mathematics.

4 "Die systematische Auszeichnung der 4 mag willkürlich erscheinen; warum nicht die 3 oder die 11 und warum eine und nicht mehrere oder gar alle Zahlen?
Die Kritik Aristoteles' an der pythagoräischen Auszeichnung der 4 bzw. der 10 setzt die Linearität der natürlichen Zahlen und das Prinzip der potentiellen Realisierbarkeit voraus. Erst dann entstehtein Konflikt zwischen der Reihe der natürlichen Zahlen, d.h. einer beliebigen Zahl und der Auszeichnung einer Zahl dieser Reihe als Gattungszahl der Reihe selbst.
Wird jedoch unter der 4 die '*Gattungszahl*' der 4 Schrifttypen der Graphematik verstanden, also das Geviert der geschlossenen Proemialität, dann entsteht kein Widerspruch zwischen Auszeichnung einer Zahl und der Zahlenreihe selbst. Die 4 eröffnet die Vielfalt der Zahlensysteme der Polykontexturalität, liegt jedoch als solche nicht in der Reihe der natürlichen Zahlen einer beliebigen Kontextur. Aristoteles lehnt die Auszeichnung der 4 (und mit ihr die der 10) ab, ist aber selbst gezwungen, die 1 auszuzeichnen. Denn die Uni-Linearität der Reihe der natürlichen Zahlen setzt die 1 als Maß der Zahlen und als unum der Unizität der Reihe voraus. Die Auszeichnung der 4 unter der Voraussetzung der Uni-Linearität heißt, daß die vertikale Sprachachse der Graphematik auf die horizontale Linie der natürlichen Zahlen projiziert wird.
Der Widerspruch zwischen 'Gattungszahl' und 'Reihenzahl' ist somit das Produkt einer Verdeckung, einer Koinzidenz der beiden 'Zahlenachsen'. Dabei wird auch stillschweigend vorausgesetzt, daß die Zahlziffern selbst eindeutig und nicht einer Überdetermination ausgesetzt sind. Aristoteles' Kritik verfängt auch dann nicht, wenn sich die 4 vertikalen Sprachschichten nicht legitimieren lassen und ihre Anzahl vergrößert oder verkleinert werden muß.
Die Kritik an der Auszeichnung einer bestimmten Zahl vor der anderen durch die transklassische Arithmetik, kann sich jedoch nicht auf Aristoteles berufen, denn seine Kritik umfaßt generell die Mehrlinigkeit der platonischen Zahlen und diese wiederum ist ein wesentlicher Charakter der transklassischen Zahlentheorie.
So argumentiert Günther: *„Aristoteles ist im Recht. Es ist notwendig, konsequent zu sein. Entweder sehen wir uns gezwungen, nicht nur der Monas, der Dyas, der Triade usw., kurz jeder pythgagoräischen n-Zahl den Rang einer ontologischen Idealität zuzubilligen oder aber die ganze Problemsicht ist verfehlt und keine Zahl hat die Würde einer Idee-außer vielleicht die Einheit und die aoristos duas, die man aber beide nicht als Zahlen zu betrachten braucht. Daß die zweite Auffassung nicht haltbar ist, lehrt die Geistesgeschichte vergangener Epochen."*
Günther insistiert also auf der Auszeichnung jeder Zahl und nicht nur der pythagoräischen Tetraktys. D.h. jede Zahl hat die Würde einer Idee und erhält somit eine logisch-strukturelle Relevanz in der Polykontexturalitätstheorie. Dort entspricht jeder natürlichen Zahl m eine bestimmte irreduzible m-kontexturale Qualität.
Damit geht aber die Idee der Auszeichnung, des Abschlusses und die Dialektik von offenem und geschlossenem System, wie sie sonst in der Kenogrammatik von Relevanz ist, verloren. Läßt sich keine Zahl auszeichnen, sondern müssen umgekehrt alle Zahlen einer Auszeichnung würdig sein, so führt sich die Idee der Auszeichnung ad absurdum. Daß alle natürlichen Zahlen logisch-strukturell ausgezeichnet werden können, ist aber das Resultat einer vollständigen Dekonstruktion der Konzeption der uni-linearen aristotelischen Arithmetik wie sie in der Kenogrammatik und der Polykontexturalitätstheorie vollzogen wurde. Mit der isolierten Thematisierung der Iterierbarkeit der

m-kontexturalen Zahlensysteme wird das wenig dialektische Moment der schlechten Unendlichkeit zugelassen." (Kaehr, Einschreiben in Zukunft, § 6,1981)
http://www.thinkartlab.com/pkl/media/DISSEM-final.pdf