Triadic Diamonds

Robertson's algebra of triadic relations, Gunther's founding relations and diamond triads

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Abstract

Some further thematizations and formalizations of diamond topics, especially triads, are presented. Triads, and founded triads, are presented in the context of Gunther's epistemology, Toth's semiotics with the help of Robertson's "Algebra for triadic relations". It is proposed that until now founding relations had been thematized externally only. An implementation of founding strategies into the system to be founded by the diamond approach is realizing the simultaneity of construction (model) and verification (foundation) of the triad. This approach is open for arbitrary n-ads.

1. Beyond binary relations?

1.1. Triads, trilogs, triplets

Representamen

"My definition of a representamen is as follows:
A REPRESENTAMEN is a subject of a triadic relation TO a second, called its OBJECT, FOR a third, called its INTERPRETANT, this triadic relation being such that the REPRESENTAMEN determines its interpretant to stand in the same triadic relation to the same object for some interpretant." (Peirce)

Further Towards a Triadic Calculus

Christofer R. Longyear's reconstruction of Warren McCullochs hobbyhorse with Peircean triads.

Part 1-3:
http://www.vordenker.de/ggphilosophy/longyear-part_1.pdf

Triplets

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<rdf:RDF
  xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
  xmlns:terms="http://purl.org/dc/terms/">
    <rdf:Description rdf:about="urn:x-states:New%20York">
         <terms:alternative>NY</terms:alternative>
    </rdf:Description>
</rdf:RDF>
```

http://www.instructionaldesign.com.au/Academic/TechnicalTheme1.htm

Trilog

"Unary relations are obviously insufficient and quadratic (4-airy) relations provide only minimally more capacity than triadic relations. Hence the choice is between two and three. Tarski studied binary relations extensively, but relation names played a significant, distinct metadata role. Binary relations are sufficient to represent information in a fixed schema, *but the names of these relations are inaccessible* from the relation contents. Both a benefit and a disadvantage of binary relations is that they are inherently *closed* in an algebra of unary and binary operators."

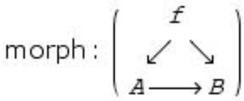
"Trilog is of course equivalent to the use of a fragment of *first order logic* to define ternary predicates, a fragment which has less convenient syntax and safety rules."
Edward L. Robertson, An Algebra for Triadic Relations, 2005
http://www.cs.indiana.edu/~edrbtsn/

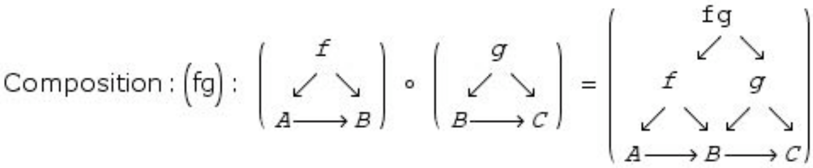
1.2. Morphisms as triads

1.2.1. Category theory

Binary:
morph= f(A, B), morph f: A-->B
morph: $A \xrightarrow{f} B$
composition: (fg): $A \xrightarrow{f} B$ o $B \xrightarrow{g} C$ ==> $A \xrightarrow{fg} C$

Ternary:
morph = (A, f, B), (A, f, B)⊆Morph

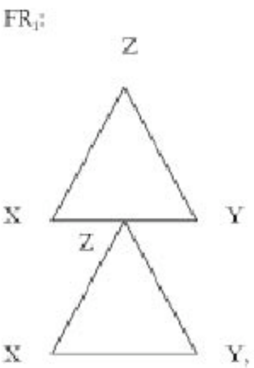




1.2.2. Semiotic foundation relation

Toth: "4. Starting with the geometric model of a sign class or reality thematic as an (equilateral) triangle, we notice that the semiotic foundational relations (FR) are *orthogonal* relations between the categories and the sign relations:
http://www.mathematical-semiotics.com/pdf/FoundRel.pdf

FR_1 := I <--> (M --> O) ≡ (.3.) <--> ((.1.) --> (.2.))
FR_2 := M <--> (O --> I) ≡ (.1.) <--> ((.2.) --> (.3.))
FR_3 := O <--> (M --> I) ≡ (.2.) <--> ((.1.) --> (.3.))



where X, Y, Z ∈ {.1., .2., .3.} and X, Y, Z are pairwise different, which means that for Z any
of the three prime-signs can be chosen, so that for FR_1 the following 6 relations are possible:

(3.a 2.b 1.c)	(3.a 2.b 1.c)	(3.a 2.b 1.c)	(3.a 2.b 1.c)
1.c	2.b	2.b	1.c
3.a	3.a	1.c	3.a
2.b	1.c	3.a	2.b

(3.a 2.b .... 1.c)
1.c
2.b
3.a

(3.a 2.b .... 1.c)
1.c
2.b
3.a

2. Diamond triads

2.1. Robertson's Trilog

[Obviously, the following presentation is not more than a *wee hint* to a promising direction. Especially, there is no need to over-interprete the triadicity of the triadic approach(es). All the restrictions here to triadicity are for '*didactical*' reasons only.]

Lower case letters (a, b, c, . . ., x, y, z) are used as variables over D.
The basic structures are sets of *triples* over D. We refer to these as *triadic* relations.
Triangular form of notation: (x, y, z) =: $\begin{smallmatrix} y \\ x\ z \end{smallmatrix}$ or $\left(\begin{smallmatrix} y \\ x\ z \end{smallmatrix}\right)$
"...the triple (x, *l*, z) indicates that the binary *l*(x, z) relationship holds."

3.1 Definition:
Let R, S, and T be triadic relations. The triadic *join* of R, S, and T, is defined

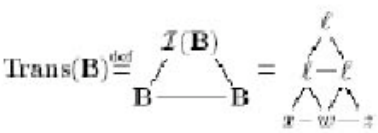
$$trijoin(R, S, T) \overset{def}{=} \left\{ \begin{smallmatrix} b \\ a\ c \end{smallmatrix} : \exists x, y, z \left[\begin{smallmatrix} x \\ a\ z \end{smallmatrix} \in \mathbf{R} \,\&\, \begin{smallmatrix} b \\ x\ y \end{smallmatrix} \in \mathbf{S} \,\&\, \begin{smallmatrix} y \\ z\ c \end{smallmatrix} \in \mathbf{T} \right] \right\}$$

An equivalent diagrammatic notation for **trijoin**(R, S, T) is $\begin{smallmatrix} S \\ R\ T \end{smallmatrix}$.

3.2 Definition:

$I(R)$ def $= \left\{ \begin{smallmatrix} x \\ x\ x \end{smallmatrix} : x \text{ occurs in the active domain of } R \right\}$

$D^3(R)$ def $= \left\{ \begin{smallmatrix} y \\ x\ z \end{smallmatrix} : x, y, \text{ and } z \text{ occur in the active domain of } R \right\}$



Rotation

Definition : The (clockwise) rotation operator ρ is defined over triadic relations in the

expected way :

$$\rho(R) \overset{\text{def}}{=} \left\{ \begin{matrix} & a & \\ c & & b \end{matrix} : \begin{matrix} & b & \\ a & & c \end{matrix} \in R \right\}$$

Robertson, An Algebra for Triadic Relations, 2005

2.2. Triadic diamonds

Lower case letters (a, b, c, ..., x, y, z) are used as variables over D.
Lower case letters ($\overline{a, b, c}$, ..., $\overline{x, y, z}$) are used as variables over $\overline{D}$.
The basic structures are bi-sets of diamond-triples over (D, $\overline{D}$).
We refer to these as *triadic diamond* relations.
Acceptional *triadic* relations are called **R**, **S**, **T**.
The complementary rejectional *dyadic* relations are called **r**, **s**, **t**.

Note:
The triadic rejectional relations (**r, s, t**) occur as *complementary* relations to ternary acceptional relations (**R, S, T**). Complementarity in diamond theory is based on an abstraction of the *compositions* of morphisms and is not to confuse with a categorical dualization of *morphisms*. Complementarily, categorical composition of morphism is possible only iff the criteria of saltatorial saltisition is realized.

This is not in conflict with the fact that category theory exists easily without any saltatory theory. Simply because saltatorical conditions of categories are implicitly used in the presupositions and not yet set from the *'mind to the blackboard'* (B. Brecht).

$$\textit{trijoin}_{\text{diam}}((R, r), (S, s), (T, t)) \text{ def} =$$

$$\left\{ \left(\begin{matrix} & b & \\ a & & c \end{matrix} \middle\| \begin{matrix} d_1 \\ d_2 \end{matrix} \right) : \frac{\exists\, x, y, z}{\overline{\exists\, x, y, z}} \left[\left(\begin{matrix} & x & \\ a & & z \end{matrix} \middle\| \begin{matrix} a_1 \\ a_2 \end{matrix} \right) \in \in (\mathbf{R}, \mathbf{r}) \;\&\&\; \left(\begin{matrix} & b & \\ x & & y \end{matrix} \middle\| \begin{matrix} b_1 \\ b_2 \end{matrix} \right) \in \in (\mathbf{S}, \mathbf{s}) \;\&\&\; \left(\begin{matrix} & y & \\ z & & c \end{matrix} \middle\| \begin{matrix} c_1 \\ c_2 \end{matrix} \right) \in \in (\mathbf{T}, \mathbf{t}) \right] \right\}$$

An equivalent diamond diagrammatic notation for

$\textit{trijoin}_{\text{diam}}((R, r), (S, s), (T, t))$ is $\left(\begin{matrix} & (\mathbf{S}, \mathbf{s}) & \\ (\mathbf{R}, \mathbf{r}) & & (\mathbf{T}, \mathbf{t}) \end{matrix} \right)$, with distributivity : $\left(\begin{matrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \end{matrix} \right) \Big\| \left(\begin{matrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \end{matrix} \right)$, hence

$$\textit{trijoin}_{\text{diam}}((R, r), (S, s), (T, t)) = \textit{trijoin}_{\text{diam}}\left[\left(\begin{matrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \end{matrix} \right) \Big\| \left(\begin{matrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \end{matrix} \right) \right].$$

$$\textit{trijoin}_{\text{diam}}\left(\left(\begin{matrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \end{matrix} \right) \Big\| \left(\begin{matrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \end{matrix} \right) \right) =$$

$$\left[\begin{matrix} \left(\begin{matrix} & b & \\ a & & c \end{matrix} : \exists\, x, y, z \right) : \\ \left[\begin{matrix} & x & \\ a & & z \end{matrix} \in \mathbf{R} \;\&\; \begin{matrix} & b & \\ x & & y \end{matrix} \in \mathbf{S} \;\&\; \begin{matrix} & y & \\ z & & c \end{matrix} \in \mathbf{T} \right] \right\} \end{matrix} \Bigg\| \begin{matrix} \left(\begin{matrix} & b & \\ a & & c \end{matrix} : \overline{\exists\, x, y, z} \right) : \\ \left[\begin{matrix} \bar{x} \\ \bar{a} \end{matrix} \in \boldsymbol{r} \;\&\; \begin{matrix} \bar{b} \\ \bar{x} \end{matrix} \in \boldsymbol{s} \;\&\; \begin{matrix} \bar{y} \\ \bar{c} \end{matrix} \in \boldsymbol{t} \right] \right\} \end{matrix} \right]$$

$$trijoin_{\text{diam}}\left(\begin{matrix} & S & \\ R & & T\end{matrix}\right) \Big\| \left(\begin{matrix} & s & \\ r & & t\end{matrix}\right) =$$

$$\left[\begin{matrix} \left(\begin{matrix} & b & \\ a & & c\end{matrix} : \exists\, x, y, z\right) \\ \left[\begin{matrix} & x & \\ a & & z\end{matrix} \in \mathbf{R}\right] \\ \left[\begin{matrix} & b & \\ x & & y\end{matrix} \in \mathbf{S}\right] \\ \left[\begin{matrix} & y & \\ z & & c\end{matrix} \in \mathbf{T}\right] \end{matrix}\right] \Big\| \left[\begin{matrix} \left(\begin{matrix} & b & \\ a & & c\end{matrix} : \exists\, \overline{x, y, z}\right) \\ \left[\begin{matrix} \bar{x} \\ \bar{a}\end{matrix} \in \mathbf{r}\right] \\ \left[\begin{matrix} \bar{b} \\ \bar{x}\end{matrix} \in \mathbf{s}\right] \\ \left[\begin{matrix} \bar{y} \\ \bar{c}\end{matrix} \in \mathbf{t}\right] \end{matrix}\right]$$

$x, y, z \in D,\ x, y, z \in \bar{D},\ \text{trijoin} \subseteq \left(D \shortparallel \bar{D}\right)$

$$trijoin_{\text{diam}}\left(\begin{matrix} & S & \\ R & & T\end{matrix}\right) \Big\| \left(\begin{matrix} & s & \\ r & & t\end{matrix}\right) =$$

$$\left[\begin{matrix} \left(\begin{matrix} & b & \\ a & & c\end{matrix} : \exists\, x, y, z;\ \exists\, \overline{x, y, z}\right) : \\ \left[\begin{matrix} & x & \\ a & & z\end{matrix} \in \mathbf{R}\right] \Big\| \left[\begin{matrix} \bar{x} \\ \bar{a}\end{matrix} \in \mathbf{r}\right] \\ \left[\begin{matrix} & b & \\ x & & y\end{matrix} \in \mathbf{S}\right] \Big\| \left[\begin{matrix} \bar{b} \\ \bar{x}\end{matrix} \in \mathbf{s}\right] \\ \left[\begin{matrix} & y & \\ z & & c\end{matrix} \in \mathbf{T}\right] \Big\| \left[\begin{matrix} \bar{y} \\ \bar{c}\end{matrix} \in \mathbf{t}\right] \end{matrix}\right]$$

$x, y, z \in D,\ x, y, z \in \bar{D},\ \text{trijoin} \subseteq \left(D \shortparallel \bar{D}\right)$

2.2.1. Diamond triad rotation

Diamond triad rotation

$$\left(\rho \shortparallel \bar{\rho}\right)\left(R \shortparallel r\right) \overset{\text{def}}{=} \rho R \shortparallel \bar{\rho} r \overset{\text{def}}{=} \left\{ \begin{matrix} & a & \\ c & & b \end{matrix} \Big\| \begin{matrix} x \\ y \end{matrix} : \begin{matrix} & b & \\ a & & c \end{matrix} \in R \Big\| \begin{matrix} y \\ x \end{matrix} \in r \right\}$$

2.2.2. From triads to n-ads

2.3. Founded triads modeled by triadic diamonds

2.3.1. Definition of founded triads

Founding relations of $\left(S^s, S^o, O\right)$

$$\left(\begin{matrix} \square & O & \square \\ \square & \swarrow\ \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{matrix}\right) \Longrightarrow \left(\begin{matrix} S^S\, r^F\left(O \longrightarrow S^O\right) \\ O\, r^F\left(S^O \longleftrightarrow S^S\right) \\ S^O\, r^F\left(S^S \longrightarrow O\right) \end{matrix}\right)$$

Founding relations constists of a relation, r^F, between monadic instances, S^S, S^O, O and dyadic relations, $\left(O \longrightarrow S^O\right)$, $\left(S^O \longleftrightarrow S^S\right)$ and $\left(S^S \longrightarrow O\right)$.

Table of founding relations

$S^S\, r^F\left(O \longrightarrow S^O\right)$	$O\, r^F\left(S^O \longleftrightarrow S^S\right)$	$S^O\, r^F\left(S^S \longrightarrow O\right)$
$\left(\begin{matrix} \square & S^S & \square \\ \square & \downarrow & \square \\ O & \longrightarrow & S^o \end{matrix}\right)$	$\left(\begin{matrix} \square & O & \square \\ \square & \downarrow & \square \\ S^o & \longleftrightarrow & S^s \end{matrix}\right)$	$\left(\begin{matrix} \square & S^O & \square \\ \square & \downarrow & \square \\ S^s & \longrightarrow & O \end{matrix}\right)$

Context – valued logic modeling of the founding relation

The unary founding relations r^F are modeled into a contextualized (parametrized) binary function.

$$r_{binary}{}^F : (S^S, S^o, O) \times (S^S, S^o, O) \longrightarrow \begin{pmatrix} S^S; (S^S, S^o, O) \\ S^o; (S^S, S^o, O) \\ O; (S^S, S^o, O) \end{pmatrix}$$

$$r_{binary}{}^F(S^S, S^o, O) = \begin{pmatrix} r_{S^S}{}^F : (S^S, S^o, O) \times (S^S, S^o, O) /_{S^S} \longrightarrow (S^S; (S^S, S^o, O)) \\ r_{S^o}{}^F : (S^S, S^o, O) \times (S^S, S^o, O) /_{S^o} \longrightarrow (S^o; (S^S, S^o, O)) \\ r_O{}^F : (S^S, S^o, O) \times (S^S, S^o, O) /_O \longrightarrow (O; (S^S, S^o, O)) \end{pmatrix}$$

In a ternary functional setting, binary operations, additional to the unary, are easily introduced.

Founding relation in 'trijoin'

Founding relation in epistemic trilog

$$\mathbf{r}^F - trijoin_{epistem}(\boldsymbol{R}, \boldsymbol{S}, \boldsymbol{T}) \stackrel{def}{=}$$

$$\left\{ \begin{matrix} S^s & \\ S^o & O \end{matrix} : \exists\, x, y, z \left[\begin{matrix} x & \\ S^o & z \end{matrix} \Big/ _{(S^0)} \in \mathbf{R} \;\&\; \begin{matrix} S^s & \\ x & y \end{matrix} \Big/ _{(S^s)} \in \mathbf{S} \;\&\; \begin{matrix} y & \\ z & O \end{matrix} \Big/ _{(O)} \in \mathbf{T} \right] \right\}$$

Dyadic diamond foundations of triads

$$\begin{pmatrix} \square & O & \square \\ \square & \swarrow \; \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} S^S\, r^F \left(O \longrightarrow S^O\right) \\ S^O\, r^F \left(S^S \longrightarrow O\right) \\ O\, r^F \left(S^O \longleftrightarrow S^S\right) \end{pmatrix} \right. \Longrightarrow \begin{pmatrix} \square & O & \square \\ \square & \swarrow \; \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix} \right.$$

Monadic diamond foundations of triads

$$\begin{pmatrix} \square & O & \square \\ \square & \swarrow \; \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} S^S\, r^F \left(O \longrightarrow S^O\right) \\ S^O\, r^F \left(S^S \longrightarrow O\right) \\ O\, r^F \left(S^O \longleftrightarrow S^S\right) \end{pmatrix} \right. \Longrightarrow \begin{pmatrix} \square & O & \square \\ \square & \swarrow \; \nwarrow & \square \\ S^o & \longleftrightarrow & S^s \end{pmatrix} \left\| \begin{pmatrix} (S^s{}_{1.2}) \\ (S^o{}_{1.2}) \\ (O_{1.2}) \end{pmatrix} \right.$$

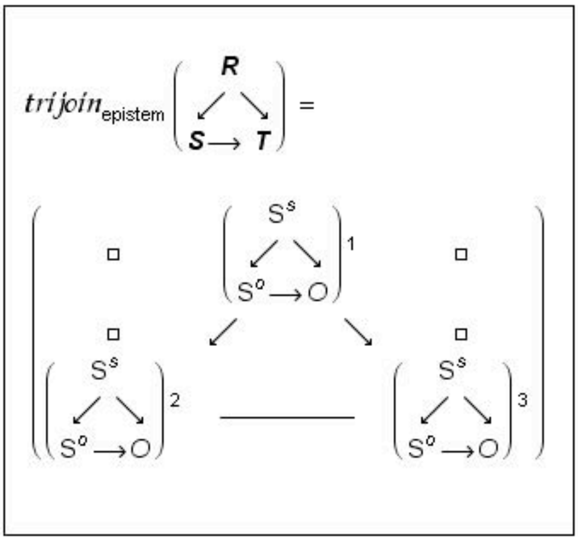
2.3.2. Composition of epistemic triads

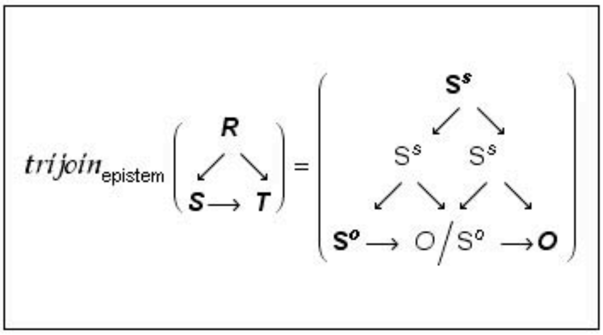
$$trijoin_{epistem}(\boldsymbol{R}, \boldsymbol{S}, \boldsymbol{T}) \stackrel{def}{=} \left\{ \begin{matrix} S^s & \\ S^o & O \end{matrix} : \exists\, x, y, z \left[\begin{matrix} x & \\ S^o & z \end{matrix} \in \mathbf{R} \;\&\; \begin{matrix} S^s & \\ x & y \end{matrix} \in \mathbf{S} \;\&\; \begin{matrix} y & \\ z & O \end{matrix} \in \mathbf{T} \right] \right\}$$

Substitution for $\begin{pmatrix} & b & \\ a & & c \end{pmatrix} \Longrightarrow \begin{pmatrix} & S^s & \\ S^o & & O \end{pmatrix}$, $R, S, T \subseteq U$, $r, s, t \subseteq \overline{U}$

$$\boldsymbol{R} = \begin{pmatrix} & S^s & \\ \swarrow & & \searrow \\ S^o & \longrightarrow & O \end{pmatrix}^1, \; \boldsymbol{S} = \begin{pmatrix} & S^s & \\ \swarrow & & \searrow \\ S^o & \longrightarrow & O \end{pmatrix}^2, \; \boldsymbol{T} = \begin{pmatrix} & S^s & \\ \swarrow & & \searrow \\ S^o & \longrightarrow & O \end{pmatrix}^3$$

$$\boldsymbol{r} = \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix}^1, \; \boldsymbol{s} = \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix}^2, \; \boldsymbol{t} = \begin{pmatrix} (S^o, S^s) \\ (S^o, O) \\ (S^s, O) \end{pmatrix}^3$$





2.3.3. Composition of founded epistemic triads

Monadic foundations of epistemic triads

$$\begin{pmatrix} \square & & O & & \square \\ \square & \swarrow & & \nwarrow & \square \\ S^o & & \longleftrightarrow & & S^s \end{pmatrix} \Bigg\| \begin{pmatrix} \left(S^s{}_{1.2}\right) \\ \left(S^o{}_{1.2}\right) \\ \left(O_{1.2}\right) \end{pmatrix}$$

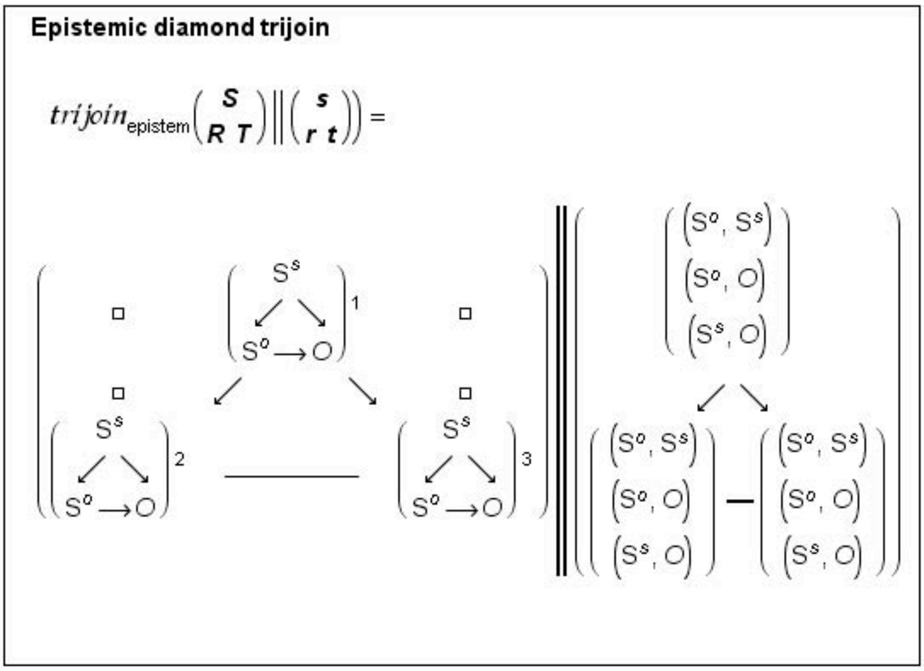
Dyadic foundations of epistemic triads

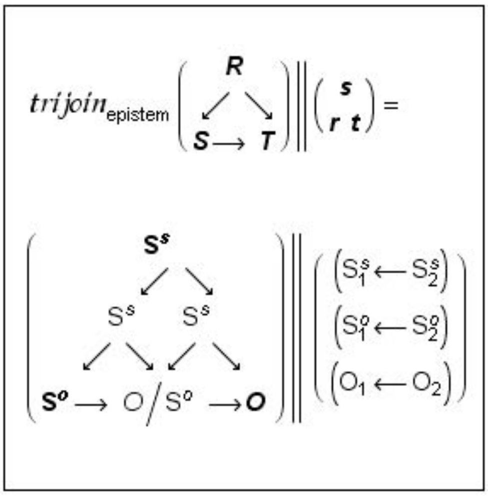
$$\begin{pmatrix} \square & & O & & \square \\ \square & \swarrow & & \nwarrow & \square \\ S^o & & \longleftrightarrow & & S^s \end{pmatrix} \Bigg\| \begin{pmatrix} \left(S^o, S^s\right) \\ \left(S^o, O\right) \\ \left(S^s, O\right) \end{pmatrix}$$

$$trijoin_{\text{diam}}\left(\begin{matrix} \boldsymbol{S} \\ \boldsymbol{R}\ \boldsymbol{T} \end{matrix}\right) \Big\| \left(\begin{matrix} \boldsymbol{s} \\ \boldsymbol{r}\ \boldsymbol{t} \end{matrix}\right) =$$

$$\left[\begin{matrix} \left(\begin{matrix} S^s \\ S^o\ O \end{matrix} : \exists\, x, y, z\right) \\ \left[\begin{matrix} x \\ S^o\ z \end{matrix} \in \mathbf{R}\right] \\ \left[\begin{matrix} S^s \\ x\ y \end{matrix} \in \mathbf{S}\right] \\ \left[\begin{matrix} y \\ z\ O \end{matrix} \in \mathbf{T}\right] \end{matrix}\right] \Bigg\| \left[\begin{matrix} \left(\begin{matrix} S^s \\ S^o\ O \end{matrix} : \exists\, \overline{x, y, z}\right) \\ \left[\begin{matrix} \bar{x} \\ \bar{S^o} \end{matrix} \in \mathbf{r}\right] \\ \left[\begin{matrix} \bar{S^s} \\ \bar{x} \end{matrix} \in \boldsymbol{s}\right] \\ \left[\begin{matrix} y \\ \bar{O} \end{matrix} \in \boldsymbol{t}\right] \end{matrix}\right]$$

$x, y, z \in D,\ \ x, y, z \in \bar{D},$ trijoin $\subseteq \left(D \amalg \bar{D}\right)$





3. Gunther's founding relation

3.1. The idea of founding relations

Diamonds might be considered as implementations of Gunther's founding relation for reflectional triads. In other words, Gunther's founding relation might get a formalization as a special triadic diamond in general diamond theory.

Triads, like (sS, oS, oO), introduced by Gunther as the framework of a new epistemological paradigm, are reflected from each knot of the graph: sS(oS, oO), oS(sS, oO) and oO(sS, oS). Those relations are interpretations, respectively foundations of the triad as a whole from each of its monadic instances. Hence there are two modelings of reflection mapped into one triad: the *triad* and its *foundations*.

The foundations, realized from each standpoint of the triad, are delivering the 3 dyadic relations which are constituting the whole. Those parts, the dyadic relations, might be mirrored "outside" the whole as constitutive parts. Hence, the whole is a triad mirrored, inside-out, by its constitutive *simultaneous* dyads. Furthermore, the dyads of the triad are obtained from the unary elements, hence from monads. The whole construction for triadic reflectionality entails self-reflectionally a triad, consisting of a triad, 3 dyads and 3 monads.

"an *exchange* relation between logical positions
an *ordered* relation between logical positions
a *founding* relation which holds between the member of a relation and a relation itself."

TRIAD =(triad, dyad, monad).

But the wording of the construction suggests a *simultaneity* of the reflectional triad and its foundation by the founding relations of the triadic relation.
That is, Gunther's trans-classical model of subjectivity is developed in three steps:
1. The stipulation of the *triadic model* as such,
2. The analysis of the triad by the new idea of *founding relations* and
3. The *composition* of the specific founding relations together to the founded triadic model.

This might be interpreted as a diamond construction with:

(oO)==>(sS, oS, oO)||(oS,sS)
(sS)==>(sS, oS, oO)||(oS,oO)
(oS)==>(sS, oS, oO)||(sS,oO)

The unary positions (oO), (sS) and (oS) are all thematizing the corresponding dyads. Hence, the unary monads are invoveld into two aspects based on the binarity of the founded dyads of the triad. Therefore, the monads shall be indexed by the index set={1, 2}.

Gunther's concept of founding relations found some application in general systems theory (Alfred Locker). The formalism might have its own value from a descriptive viewpoint but is not well prepared for *operative* transformations. One attempt to formalize the epistemic model one step further happend with the application of Gunther's *Kontextwertlogic* (Contextvalued logic), as opposed to Stellenwertlogik (place-valued logic) (cf. Kaehr 1978, Baldus 1982, Grochowiak 1979).

A new attempt to formalize the idea of founding relations is proposed by the *diamond* approach which takes into account the *simultaneity* of the model and its foundation. It also reflects the fact, that a foundation of an operation is localized on a different level of abstraction. The activity of modeling and the activity of founding are complementary activities demanding different kinds of abstractions. Hence, any applicative iteration of the model on itself is not fulfilling the criteria of foundation.

3.1.1. Chinese Ontology and Diamonds

The idea of in-sourcing *the matching conditions into the definition of diamonds tries to realize the two postulates of* Chinese Ontology*, the permanent* change *of things and the endness or* closeness *of situations. That is, diamonds should be designed as structural explications of the* happenstance *of compositions and not as a succession of events (morphisms). More exactly, diamond are contemplating the interplay of acceptional and rejectional thematizations. Thus, morphisms with their matching conditions and composability are in fact of secondary order for the understanding of diamonds.*

The complementarity of construction *and* verification*, which is happening at once and not in a temporal delay, is a consequence of the finiteness and dynamics postulate of* *polycontextural "ontology". This simultaneous interplay is based on the insight that a delayed verification (or testing in programming) would not necessarily verify the construction in question because, at least, the context will have changed in-between. Delayed verification is possible only in the very special case of frozen dynamics.*

In other words, in a changing open/closed world, the activities of construction and verification (of correctness and relevance) have to happen at once. Otherwise, because the conditions might have changed, the *relevancy* of the construction to be verified would have to be verified itself, again, and this ad nauseam. Obviously, the statement is not about/against the *stability* of the construction (program, system, agreement, contract), this might be rock solid, but about the *relevancy* of the rock solid construction.
(In therapy, even by constructivists, this delayed checks are called "reality check". Nearly everytime, such a reality to be checked has escaped any relevance.)

In-sourcing the matching conditions
Diamond strategies are offering a fundamentally different approach.
Each step in a diamond world has simultaneously its counter-step. Hence, each operation
has an environment in which a legitimation of it can be stated. The legitimation
is not happening before or after the step is realized but immediately in parallel to it.

Morphisms are representing mappings between objects, seen as domains and
codomains of the mapping function.
Hetero-morphisms are representing the conditions of the possibility (Bedingungen
der Möglichkeit) of the composition of morphisms. That is, the conditions, expressed
by the matching conditions, are reflected at the place of the heteromorphisms.
Hetero-morphisms as reflections of the matching conditions of composition
are therefore second-order concepts realized "inside" the diamond system.

Morphisms and their composition are first-order concepts, which have to match
the matching conditions defined by the axiomatics of the categorical composition
of morphisms. But these matching conditions are not explicit in the composition of
morphism but implicit, defined "outside" of the compositional system.
Hence, in diamonds, the matching conditions of categories are explicit, and moved from the "outside" to the inside of the system.
In this sense, the rejectional system of hetero-morphisms is a reflectional system,
reflecting the interactions of the compositions of the acceptional system. Heteromorphisms are, thus, the "morphisms" of the matching conditions for morphisms.

3.1.2. Duplicity of reflection

This approach, to model the founding relation in the framework of diamond theory might be achieved with a decomposition of the founding relation into its parts: *monadic* and *dyadic* relations as the rejectional parts of the diamond interplay between model and foundation, i.e. acceptional (categorical) and rejectional (saltatorical) thematizations.
The founding operation itself, r_F, which has unary and binary operands, e.g. $O \in$ unary, $S^S \longleftrightarrow S^O \in$ binary, is not implemented and involved into the definition of the triadic model itself as it is introduced by Gunther.

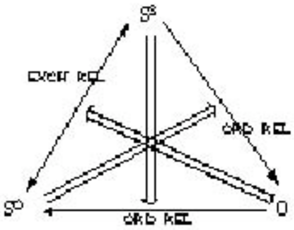
Funding relations and structural relations are complementary and antidromic in their orientation. If there is something like a *"Duplizität des Ich"* (Fichte) (A duplicity of the ego) then such a duplicity is of interest only if this duplicity is a simultaneous duplicity. A successive hierarchy of different levels of epistemic reflection, as it is supposed in Anglo-Saxon philosophy and computational reflection, belongs to a strictly different paradigm of thinking. Similar intricate situations of duplicity of consciousness had been discovered by Edmund Husserl with his distinction of *retention* and *protention* of the temporal structure of reflectional acts.

3.1.3. Subjectivity as a founded triadic model

"What we still have to consider is the relation any of the three terms S^s, O and S^O may assume to the relation which holds between the other two terms. From a purely combinational view point three possibilities exist for a demanded relation ... r^F ... they are:

S^S r^F (O -> S^O)
O r^F (S^O<-> S^S)
S^O r^F (S^S-> O)

Formally speaking it is the relation any of the two realizations of S , namely S^S or S^O, may have toward the connection of the other S and O. We call this the founding relation (r^F) because by it, and only by it, a self reflective subject separates itself from the whole Universe which thus becomes the potential contents of the consciousness of a Self gifted with awareness. In contrast to it the classic relation O r^F(S^O<-> S^S) is still a founding relation but not for consciousness.



But this claim also extracts from the "outside" observer, S^S an interesting admission. He will state that, seen from his vantage point, the inclusive disjunction does not only hold in the case of:

(1) $S^S r^F$(O->S^O) .∨. $S^O r^F$(S^S->O) but also in the other two cases:
(2) $S^O r^F$(S^S->O) .∨. Or^F(S^S<->S^O) (3) SSrF(O->SO) .∨. OrF(SS<->SO) provided, of course, that he uses a two-valued logic.

But in doing so he realizes by self-reflection that he has committed a momentous logical mistake. Since in classic logic only two values are available for the determination of the distinction between subject and object, it is impossible to describe the *triadic* relation between the subjective subject; the objective subject and the object.

This investigation intends only to show that the concept of *Totality* or *Ganzheit* is closely linked to the problem of subjectivity and trans-classic logic and that it is based on three basic structural relations:
an *exchange* relation between logical positions
an *ordered* relation between logical positions
a *founding* relation which holds between the member of a relation and a relation itself.

We are now able to establish the fundamental law that governs the connections between exchange, ordered and founding relation.

Thus we may say: the founding-relation is an exchange relation based on an ordered relation. But since the exchange relations can establish themselves only between ordered relations we might also say: the founding-relation is an ordered relation based

on the succession of exchange-relations. When we stated that the founding relation establishes subjectivity we referred to the fact that a self-reflecting system must always be:
self-reflection of (self- and hetero-reflection)."
(Gunther, Formal Logic, Totality and The Super-additive Principle, 1965) in:
http://www.thinkartlab.com/pkl/archive/Cyberphilosophy.pdf

3.1.4. Semiotic triads

Semiotic triads occur as morphisms between the instances I, O, M and their combinations, called graph theoretic sign models.

1. (I-->O -->M)
2. (M-->O-->I)
3. (I-->M--> O)
4. (O-->M-->I)
5. (I-->M-->O), (M-->I-->O)
6. (O-->I-->M).

"In 1966, Günther showed that the three reflexive categories of a three-valued logic, objective subject (oS), object (O) and subjective subject (sS) correspond (in this order) with the
semiotic categories of medium (firstness), object (secondness) and interpretant (thirdness)."
(cf. Toth 2008, p. 64):
oS <==> M ≡ (.1.)
O <==> O ≡ (.2.)
sS <==> I ≡ (.3.)
http://www.mathematical-semiotics.com/pdf/Obj.andrefl.existence.pdf

3.1.5. Gunther's journeys

Triadic semiotics (Bense, Toth) and triadic epistemology (Gunther). Also Gunther's approach and semiotic triads are fitting, at least at a first glance, well together, Gunther's epistemological triadism shouldn't be taken too seductively, because (t)his obsession lasted only for a short and specific time of Gunther's speculations. In the early 60s, the dialogical concept was replaced by a much more socialist distribution of subjectivity over a mass of 'subject centres' (Chinese Cultural Revolution).

"To sum it up:
A non *Aristotelian* or trans-classical logic is a system of distributed rationality. Our traditional (two valued) logic presents human rationality in a non distributed form. This means: the tradition recognizes only one single universal subject as the carrier of logical operations.

A *non-Aristotelian* logic, however, takes into account the fact that subjectivity is ontologically distributed over a plurality of subject centres. And since each of them is entitled to be the subject of logic human rationality must also be represented in a distributed form. The means to do this is to interpret many valued structures as place-value systems of our two valued logic." (Gunther 1962)
http://www.vordenker.de/ggphilosophy/gg_tradition-of-logic.pdf

In a German paper, 1965, Gunther writes: "Wir sind zum Übergang zu einer vierwertigen Logik genötigt, in der nicht nur Subjekt-überhaupt und Objekt-überhaupt durch logische Werte vertreten sind, sondern in der U sowohl als S_1, S_2, S_3 ontologische Instanzen repräsentieren, von denen jede Vertretung duch einen eigenen Wert beansprucht."
http://www.vordenker.de/ggphilosophy/gg_problem-trans-klass-logik.pdf

Some years later in 1968, Gunther developed a general theory of mediation where anthropological roots had been erased in favor of the history of the world as such - with or without human beings.
http://www.vordenker.de/ggphilosophy/gg_struk-min-theor-obj-geist.pdf

Furthermore , 2 years later, in his "*Die historische Kategorie des Neuen*", in Moscow in favor for Change, his theory of polycontexturality is strictly neutral to any specific

interpretation. Neither I, Thou, It or other subjectivity constructs are appearing in the historical development of the New.
http://www.vordenker.de/ggphilosophy/gg_category.pdf

The whole conceptual story is well sketched at:
Ditterich, Kaehr: Einübung in eine andere Lektüre
http:///www.vordenker.de/ggphilosophy/kaehr_einuebung.pdf

Nevertheless, it seems that even today it would be a revolution to realize a working 3-contextural scientific paradigm and technology (of computing and social organizations).

Again, the proposed classical theory of triads is fomualted in the framework of a binary and dichotomous First-Order Logic., i.e. n-ary relations are *logically* reducible to binary relations. Hence, it achieves a simulation of trichotomic logic and never a realization. (Cf. Ternary Computers, Moscow)

What do we learn? As Max Bense mentioned correctly, Gunther was a Laborphilosoph (lab philosopher) and not a Kathederphilosoph (lectem philosopher) - this is true, despite the fact that each attempt to his philosophy, albeit only a fragment, was declared with absoluteness. Asked some month later about his theoretical advances and some immanent problems of it, he even didn't remember it. This surely was an exaggeration, but he was, again, some steps further on. And so on.

3.1.6. Toth's founding strategy

Toth's semiotic modeling of Gunther's founding relation is of importance, not only for systematic semiotics alone but for applications in computational semiotics and the triple-approach for semantic implementations in the project of a Semantic Web or Web 3.0.

Nevertheless, both, Gunther and Toth, are stressing on the successive, iterative or orthogonal structure of the idea of founding logic and semiotics. Because the basic triadic structure to be founded remains untouched, the whole approach might be more an *application* then a foundation.

This analysis is confirmed by Toth' s statements:
"In accordance with Günther (1991), these *superizations* [based on categories and relations] are based on semiotic orthogonality." (Toth 2008)

Superization, obviously, is a special application and is immanent of semiotics, not changing anything in its fundamental definition.

"From the logical standpoint, the latter means that the "Thou" founds the order relation
between an "I" and an "It" (OS --> (SS --> O)), that an "I" founds the order relation between an "It" and a "Thou" (SS --> (O --> OS)), and finally, that an "It" founds the exchange relation between an "I" and a "thou" (O --> (SS <-->OS))." (Toth 2008)
http://www.mathematical-semiotics.com/pdf/FoundRel.pdf

All parts of the founding relations are unchanged parts of the logical or epistemic model. The different *roles* of the instances might be used but not represented. That is, e.g. "It" as a founding point of view and "It" as a founded element of the model are technically not distinguished in the semiotic approach. This fact is based on the triviality that the model and its founding relations don' t have the necessary structural complexity. That is, at least two variables would be needed to model the different roles of the elements depending on their context. Context-value logic (Kontextwertlogik) was introduced just for this purpose (Gunther 1968).
There is up to now no equivalent at all to find in Bensean semiotics.

Some more approximations

$$trijoin_{\text{diam}}\Big(trijoin_{\text{diam}}\big((R,r),(S,s),(T,t)\big),\ trijoin_{\text{diam}}\big((R,r),(S,s),(T,t)\big),\ trijoin_{\text{diam}}\big((R,r),(S,s),(T,t)\big)\Big)$$

$$= trijoin_{\text{diam}}\left(trijoin_{\text{diam}}\left[\begin{pmatrix} S \\ R\ T \end{pmatrix}\middle\|\begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\ trijoin_{\text{diam}}\left[\begin{pmatrix} S \\ R\ T \end{pmatrix}\middle\|\begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\ trijoin_{\text{diam}}\left[\begin{pmatrix} S \\ R\ T \end{pmatrix}\middle\|\begin{pmatrix} s \\ r\ t \end{pmatrix}\right]\right)$$

$$= trijoin^{(3,3)}{}_{\text{diam}}\left(\left[\begin{pmatrix} S \\ R\ T \end{pmatrix}\middle\|\begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\left[\begin{pmatrix} S \\ R\ T \end{pmatrix}\middle\|\begin{pmatrix} s \\ r\ t \end{pmatrix}\right],\left[\begin{pmatrix} S \\ R\ T \end{pmatrix}\middle\|\begin{pmatrix} s \\ r\ t \end{pmatrix}\right]\right)$$

$$= trijoin_{\text{iter}}{}^{(3,3)}{}_{\text{diam}}\left[\left[\begin{pmatrix} \begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix}\begin{pmatrix} S \\ R\ T \end{pmatrix} \end{pmatrix}\middle\|\begin{pmatrix} \begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix}\begin{pmatrix} s \\ r\ t \end{pmatrix} \end{pmatrix}\right]\right].$$

$$trijoin_{\text{iter}}{}^{(3,4)}{}_{\text{diam}}\left(\left[\left[\begin{pmatrix} \begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix}\begin{pmatrix} S \\ R\ T \end{pmatrix} \end{pmatrix}\middle\|\begin{pmatrix} \begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix}\begin{pmatrix} s \\ r\ t \end{pmatrix} \end{pmatrix}\right]\right]\right) = \left[\left[\begin{pmatrix} \begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix}\begin{pmatrix} S \\ R\ T \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \end{pmatrix} \end{pmatrix}\middle\|\begin{pmatrix} \begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix}\begin{pmatrix} s \\ r\ t \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \end{pmatrix} \end{pmatrix}\right]\right].$$

$$4 - join_{\text{acc}}{}^{(4,3)}{}_{\text{diam}}\left(\left[\left[\begin{pmatrix} \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix}\begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \end{pmatrix}\middle\|\begin{pmatrix} \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix}\begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \end{pmatrix}\right]\right]\right.$$

$$4 - join_{\text{acc}}{}^{(4,4)}{}_{\text{diam}}\left[\left[\begin{pmatrix} \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix}\begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \\ \begin{pmatrix} S \\ R\ T \\ U \end{pmatrix} \end{pmatrix}\middle\|\begin{pmatrix} \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix}\begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \\ \begin{pmatrix} s \\ r\ t \\ u \end{pmatrix} \end{pmatrix}\right]\right].$$

4 – join diamond approx

$$4 - join_{\text{diam}}\left(\begin{pmatrix} & \boldsymbol{S} & \\ \boldsymbol{R} & & \boldsymbol{T} \\ & \boldsymbol{U} & \end{pmatrix} \Bigg|\Bigg|\Bigg| \begin{pmatrix} & \boldsymbol{s} & \\ \boldsymbol{r} & & \boldsymbol{t} \\ & \boldsymbol{u} & \end{pmatrix}\right) =$$

$$x, y, z, u \in D,\; x, y, z, u \in \bar{D},\; 4 - \text{join} \subseteq \left(D \amalg \bar{D}\right)$$

$$\left[\begin{matrix} \begin{pmatrix} & b & \\ a & & c : \exists\, x, y, z, u \\ & d & \end{pmatrix} \\ \begin{bmatrix} & x & \\ a & & z \in \mathbf{R} \\ & u & \end{bmatrix} \\ \begin{bmatrix} & b & \\ x & & y \in \mathbf{S} \\ & u & \end{bmatrix} \\ \begin{bmatrix} & y & \\ z & & c \in \mathbf{T} \\ & u & \end{bmatrix} \\ \begin{bmatrix} & y & \\ x & & z \in \mathbf{U} \\ & d & \end{bmatrix} \end{matrix}\right] \Bigg|\Bigg|\Bigg| \left[\begin{matrix} \begin{pmatrix} & b & \\ a & & c : \exists\, \overline{x, y, z, u} \\ & d & \end{pmatrix} \\ \begin{bmatrix} & x & \\ a & & z \in \mathbf{r} \end{bmatrix} \Big|\Big|\Big| \begin{bmatrix} \bar{x} & \\ \bar{a} & \in \mathbf{r} \end{bmatrix} \\ \begin{bmatrix} & b & \\ x & & y \in \mathbf{s} \end{bmatrix} \Big|\Big|\Big| \begin{bmatrix} \bar{b} & \\ \bar{x} & \in \boldsymbol{s} \end{bmatrix} \\ \begin{bmatrix} & y & \\ z & & c \in \mathbf{t} \end{bmatrix} \Big|\Big|\Big| \begin{bmatrix} \bar{y} & \\ \bar{c} & \in \boldsymbol{t} \end{bmatrix} \\ \begin{bmatrix} & y & \\ x & & d \in \mathbf{u} \end{bmatrix} \Big|\Big|\Big| \begin{bmatrix} \bar{y} & \\ \bar{d} & \in \boldsymbol{u} \end{bmatrix} \end{matrix}\right]$$