Morphogrammatics of Change

A monomorphy based sketch of morphogrammatic transformations

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Abstract

Sketch of (descriptive) morphogrammatics based on monomorphies. Change of morphograms as evolutive, integrative and reconfigurative transformations. Positionality of morphograms and monomorphies.

1. Modi of change/changes of modi

Its all about change. From the Book of Change (I Ching) to the challenges of a change in politics.

How to change something?
What are the possibilities of a change of something?
How to be changed by changing something?
How can change happen without something being changed?
How can change be changed by change?
How can something change the changer of a change?
How can something change the changer of a change without being itself neither change nor something at all?

And what’s about the ***"Change we can believe in"***?

Is it not enough into what we believed in all the time, again and again?
Isn't it time to stop believing and to start to compute our beliefs in an arithmetic we have not to believe in, like we have to do with our *natural and universal* systems of computation nobody believes in because nobody even knows that their calculations are based on beliefs.

To study such difficult conceptual challenges it seems to be reasonable to study it with the most simple model possible.
There is nothing more elementary and well known than natural numbers, sign systems or the stroke calculus.

By adding a new stroke to a chain of strokes, or by adding a numeric unit to a number, or to add a sign to an existing sign sequence is the most elementary operation of change. As we know it until now. And this simple operation is secure. It is based on our fundamental intuition, initialized by education - and its axiomatics. And this simple operation can be repeated endlessly. Never ever encountering any obstacle. There are no limits in the resources of matter, time and space. And the poor guys who have to count. At least in this world of abstraction.

Slate : http://cartoonbox.slate.com/hottopic/?image=7&topicid=114

But is that enough?
In a non-notational scenario children or scientists are adding to their Lego blocks new Lego blocks to build an extension, prolongation, i.e. a change towards more exiting Lego constructions. Such an extension of a pattern can happen at all loci possible for continuation. No linearity has to be supposed. And for the metaphor we can forget the need for any atomicity of the added elements. In the same sense, the actor is changing his identity depending on what and how he or she or it is creating his constructions and how those interactions are changing simultaneously the definitions of the actors.

In an experimental scenario children or scientists might add at each possible location of their chemical formulas new elements to produce more complex chemical patterns. Or they may organize mutations to their organisms.

And obviously, this all started in the caves and ended with Paul Lorenzen's stroke calculus.

Nevertheless, there are different modi of change. Are there?

And why should we trust in numbers which don't know their past and are blind for their future - by principle?

There are no changes without new beginnings and no new beginnings without changes.

For real-world systems based on numbers there is necessarily only endless iteration of the same or fatal crash.

What has changed for the formal theory of change, keno- and morphogrammatics, in the last 40 years?

For a change I will sketch some ideas about elementary features of change in formal systems surpassing, as it will turn out, the principal limitations of known formalisms. This sketch of morphogrammatics is choosing thoroughly a *descriptive* approach.

Maybe there is still a way out of the cave of neolitic inscriptions and its culmination in the stroke calculus of digital speculations?

2. Modi of beginnings and transformations

Instead of a beginning with the statement *"Given X"*, the kenomic formulation might be *"Having encountered Y"*. That is, if having encountered Y, find an appropriate succession or predeccession of Y. Depending on the structural complexity of Y, different prolongations are opened up. Not all have to be realized. Hence, a decision for a specific prolongation (succession) has to be drawn.

Therefore, there is no beginning pre-given. Each situation encountered might be accepted as a beginning of an interaction. Complementary, there is no situation given which coulden' t be accepted as an end.

Semiotics, category theory and arithmetic are playing with a single ultimate beginning and are believing in endless continuation. *"One start, no end"* is the slogan of the dream. Until it gets stopped by a wee crash.

To begin with the simplest elements in a formalism is more a question of an economic or stylistic decision than a compulsory conceptual necessity. As much as we can agree to start a stroke calculus with a single elementary stroke as the first action of the calculus we can agree to accept to encounter a morphogram of whatever complexity and to start to interact with it on the level of its encountered complexity.

To get access to the complexity of an encountered situation, the situation might be confronted with the interaction of *decomposition*.

[1]Disremption

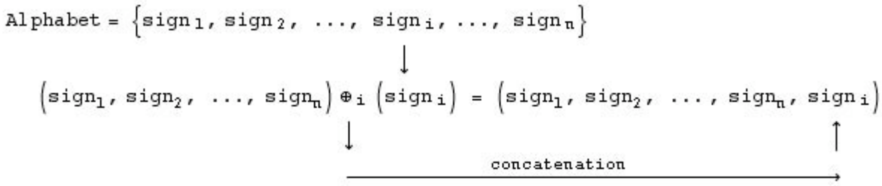
Atomic concatenation

The most secure mode of change is to add to an existing linear sequence of signs, numbers or marks an new one. This addition, called *concatenation*, is strictly independent of the pre-existing sequence and refers only to a pre-given sign repertoire, i.e. to its alphabet.
Its security is demanding to accept the linear order of the atomic signs and the rule not to intervene into the pre-given sign sequence.

Probably the most popular presentation of semiotic concatenation is given by the concept of lists and its manipulation in the programming language Lisp. Lisp was leading the advent and decline of AI (Artificial Intelligence) research.

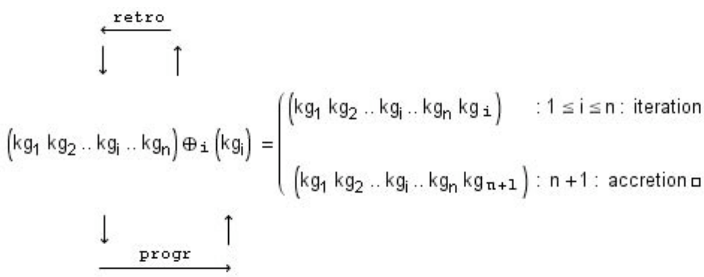
The concatenation scheme for signs:

If $\boxed{s_1\, s_2 \ldots s_n} \in$ SIGN and $\boxed{s_i} \in$ Alphabet, then $\boxed{\boxed{s_1\, s_2 \ldots s_n}\boxed{s_i}} \in$ SIGN.



Kenomic evolution

Kenogrammatic concatenation still relies to some degree on the linear order of its kenoms. But there is no need anymore for a pre-given alphabet and concatenation itself is only one of elementary operation of change. Further operations are *chaining* and different kinds of *fusion*. Without a pre-given alphabet the risk has to be taken to develop change out of the encountered kenogram sequence only. With that the abstractness of the semiotic concatenation is surpassed. There is not only no alphabet given, but the kenoms involved are semiotically indistinguishable. The operation of concatenation is defined by an interaction with the encountered kenogram sequence. Its range is determined by the occurring kenoms of the sequence which remains itself still untouched by the process of concatenation. Hence, kenogrammatic concatenation is not defined in an abstract way but retro-grade to the encountered kenomic pattern.



Lack of a pre-given alphabet
Because of the lack of an alphabet as a source for signs from the outside, i.e. from a lower level of the tectonics of the formal system, evolution of morphograms have to be constructed as extensions out of their inner structure.
This is a kind of an evolution of morphograms based on the mononomorphies of the morphogram.

Self-generated alphabets
The wording that there is “no" alphabet means, there is no alphabet pre-given as the start of a kenogrammatic calculus. But what is not pre-given is not denied to exist in a different way. Hence, a positive wording concerning the alphabet of kenogrammatics might be turned into this: Encountered a morphopgram, a kenomic abstraction is collecting the kenoms involved into the morphogram. A successor operation then can rely on those kenograms to precede to the next morphogram, in an iterative or an accretive way.
Therefore, albeit there is no alphabet pre-given, kenogrammatic operations are producing situatively their own alphabet, i.e. set of kenoms, to proceed their operations.
Again, it is reasonable to speak about a parallelism or diamond movement of operators and

operands of kenomic operations. The kenomic alphabet has to be elicited. There is no need for a kenomic alphabet without the intended interactions with morphograms.

Despite the big difference between semiotic and kenogrammatic concatenation there are still some important similarities. Both share a kind of a linear order of their objects, signs and kenoms. And their units of iteration and concatenation are of similar structure. Semiotics depends on atomic signs, kenogrammatics on the other hand, on monadic kenoms.

The successor operation in kenogrammatics was up to now defined mainly as the *iterative* or *accretive* repetition of the kenoms in a kenogrammatic sequence. This approach is still supposing a kind of atomicity, i.e. of atomic separability of kenoms to be repeated. With this presumption, interesting results have been achieved.

Morphic evolution

The morphogrammatic approach to change is changing the presumption of a *linear* order of kenoms as it is supposed in the term “keno-sequence” and is emphasizing the *tabular pattern* structure of morphograms (morphe=pattern, Gestalt). As a consequence, changes in the sense of an evolution out of the pattern itself can happen at all *loci* of the pattern.

Hence, their is no need for a reduction to a successional prolongation. It can happen at all loci involved. Therefore, the encountered morphogram which is involved into a morphogrammatic concatenation operation is loosing its neutrality and gets itself involved into a change. This might be called an *interventional* evolution.
Kenogrammatic concatenation is played by a retro-grade self-referentiality, which has a *diamond* structure. To success, simultaneously, a retro-grade action happens. But the actand itself isn’t touched by this intriguing retro-grade interaction. It remains stable and is solely offering kenograms for further prolongations of the morphogram.

Such a change of the actand itself happens with the *morphic* evolution. The actand of change gets itself changed in the process of change.
This is realized with operation of *reconfiguration* (reconfigurative evolution, coalitions, composition).

Monomorphic concatenation

Morphograms are changed by the monomorphic concatenation according to their monomorphies. Monomorphies are patterns of kenoms and parts of the whole of the pattern-structure of the morphogram.
A new feature out of the morphic evolution is the change of the *actor* (operator) of the interaction.

Such an immanent evolution of morphograms is not changing the structure of the morphogram involved into the process of evolution. The structure of the original morphogram stays untouched. Despite the retro-grade movement of the kenomic successor operation to build successions the beginning morphogram is not involved in any change of the successor procedure.
The triviality of this observation gets a new turn with the tabular notational successor operation which is changing its beginning morphogram too. That is, to add something to a morphogrammatic structure might change the structure itself. Hence, two events happens, a) the succession of the morphogram and b) the ‘self'-transformation of the morphogram.
Therefore, an interaction with morphograms might emerge into a monomorphic evolution of the involved morphograms. Further interactions between morphograms are, e.g. concatenation, chaining and fusion.
That is, the progression or succession is not depending on any external objects, kenoms, to be added from the outside to the kenomic pattern but is fully defined by the structure of the morphogram involved into the interaction.
With that, a kind of a symmetry between the composition of morphograms and their decomposition into monomorphies is established.

Actional concatenation

A change of the actor in the process of interaction happens as a transformation of the actor “concatenation” into other evolutionary operators. It turns out that “concatenation” is only one interaction of a family of different interactions, like “chaining” and"fusion”.
Combinations of actors are involved into the *actional abstractions* responsible for the behavioral equality of different morphograms.

In an actor terminology we can say that change in the sense of morphogrammatics is changing all parts of interactivity, the *actor* and the *actands* and thus *interaction* as the operation.

Discontexturality

But with such a fulfillment of a change in the conceptual triadicity a new feature emerges.
Until now I stipulated only one encountered morphogram. Interactions happened with the morphogram which had been answered by a kind of a self-evolutionary process.

But what happens if two morphograms encounter? The same game might go on. In this case it doesn't make much a difference to the singular situation of self-evolution. We continue triadicity and silently suppose that there is no *discontextural* difference between morphograms. How can different morphograms interact if they are of different contextures, thus not only disjunct in their elements and operators but discontextural in their conceptionality?

With the introduction of a multitude of contextures, i.e. with polycontexturality, interactions between morphogrammatic systems are enabled which are surpassing the limits of operational triadicity by disseminating it.

To mention proudly, *"the sum is more than its parts"*, is supposing that a summation is possible and that the terms are commensurable. This innocent constellation might turn out as a fundamental limitation of the desire for change.

Summary of some new aspects of morphogrammatics

1. Shift of focus from kenoms to monomorphies in morphograms.
2. Transforming the concept of kenomic sequences to monomorphic patterns.
3. Understanding of the kenomic successor operation as a diamond structure of pro- and retro-gression.
4. New features of the succession operation on the base of the pattern and monomorphy approach (integration).
5. Chiastic definition of equivalence (dissimilarity) of morphograms (1994) as an abstraction over operations instead over operands.
6. Understanding of proemiality from an open to closed (cyclic) chiasms and finally to a complementary diamond formation.
7. Changing the idea of beginning(s) to an interactional concept of encounters.
8. For patterns there is no need to restrict to append a prolongation at the tail of a monomorphy. It can be the tail or the head of any monomorpy or other parts of a monomorphy if the monomorphy is not yet fully reduced.
9. Introduction of different kinds of positionality for morphograms.
10. Complementarity of kenomic beginnings in the sense of diamond category theory.

Metaphors and heuristics

Morphograms are considered as groups of monomorphies. A group, of whatever kind of objects or agents, might be in a situation where it has to change its constellation by growing or by self-differentiation. Also the group might encounter another group and strategies of co-operations, fusions or incorporation are occuring as necessary.

What are the structural possibilities for such a group to change?

The group may decide to not to grow, i.e. not to enlarge its domain with new positions but better to differentiate into a more complex structure or to reduce its complexity (complication) to a lower degree of differentiation.
The group is emanating between higher or lower complication and keeping its complexity stable.

This shall be called an *emanative* change of the group.

Emanative developments are preserving the structural complexity of the actional system. Hence, it easily reaches its limits.
A new strategy is called for. The group might extend its complexity by divesting parts of it. Every part might be divested and helping the group to evolve. Such evolvement by divestment is not outsourcing its agencies but is repeating and adding its existing agencies of the group to the group as a whole.

This is a relatively secure procedure but nevertheless it is augmenting the structural capacity of the group (organization, company, organism, chemism, etc.). Because such a divestment is purely structural it is not a simple repetive addition of existing faculties but an augmentation of the structural complexity of the whole.

This shall be called *iterative* transformation (change, disremption, prolongation, augmentation, etc.).

The group might decide to augment its complexity with a structural risk. The risk for the new to be taken by the group is transforming the complexity of the group by accepting to evolve into an unknown domain (contexture), creating a structurally new position. Again, the degree of the risk is ruled by the structure of the group. The new, added to the group, is new only in respect to the existing constellation of the group. Hence, there is nothing hazardous involved into this risk of extending the complexity into new dimensions. What's new is new solely in respect to the historically developed structuration of the group (organization).

This shall be called *accretive* transformation (metamorphosis, change).

Hence, *iteration* and *accretion* are the two modi of change which are augmenting the complexity of the group (whole).
Gotthard Gunther calls this two complementary modi of transformation, *evolutive* change.

Both, *evolution* and *emanation* together, are designing the framework of structural change of organizations (groups, wholes, etc.), i.e. the *morphogenesis* of structuration. This kind of double structuration shall be called *disremption*.
Disremption is understood as the keno- and morphogrammatic opposite to the semiotic operation of concatenation.

Hence, a group inscribed as a morphogram is embedded into a complementarity of evolutive (iteration/accretion) and emanative (differentiation/reduction) transformations.

Because the whole is build by its parts, those strategies of evolution and emanation, are applicable to the single parts as well as to the whole as such.

Such an understanding of the structuration of change is not depending on any identities, objects, agents, processes, information, etc in the known sense.

Therefore, this strategy and theory of change (structuration) is called *morphogrammatics*.

Morphogrammatics is independent of any system and complexity theory. Its material resource are kenograms, i.e. the place-holders for the parts of morphograms (groups, constellations) created in the process of structuration. The parts of morphograms are called the *monomorphies* of the morphogram.

Morphogrammatics of change sounds extremely simple.

There are no strange attractors, chaos theory, maturation and adaption, autopoiesis and homeostasis, etc. involved at all. Neither any logical systems, multiple-valued, modal, paraconsistent, etc. nor terms like paradox, circularity, antinomy, etc. nor information processing, computability, diagonalization, etc. and so on.

But there is a morphogrammatics of logic and arithmetics, mono- and polycontextural. With this turn, logic - and formal systems in general - are appearing as maximally reductionist theories of change, i.e. as stable theories and formalisms of zero structural change.

3. Morphograms as patterns of monomorphies

3.1. Positionality of morphograms

Morphograms are conceived as patterns of *monomorphies*. This perception of morphograms as patterns of monomorphies is realized by the operation of decomposition.
To be able to interact with an encountered morphogram it has to be decomposed into its monomorphies.

A decomposition of a morphogram into its monomorphies gives the key to understanding its behavior. Its behavior is essentially characterized by two dimensions: *evolution* and *emanation*.

Evolution happens as *iterative* and *accretive* repetitions of monomorphies.

Emanation happens as *differentiation* and *reduction* of monomorphies of a morphogram.
Differentiation is augmenting, reduction reducing the number of monomorphies in a morphogram.

Morphograms, therefore, are placed in a *grid* of emanative and evolutive iterability.

Monomorphies are build by kenograms, called *kenoms*.
Kenoms might represent classes of signs.

Each monomorphy of a morphogram is positioned at a place in the morphogram.

Change happens as evolutive, integrative and reconfigurative transformations of morphograms.
Differentiation of change happens as emanative transformations.

This leads to the positional matrix for morphograms:|

Position $MG^{(m, n)}$	
$MG^{(m)}$	locus
$Dec(MG^{(m)})$	monomorphy
$Ken(MG^{(m)})$	kenom

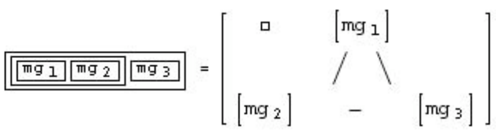
Example

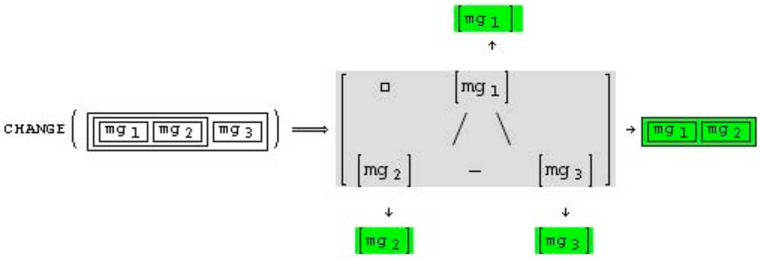
MG	loc_1	loc_2	loc_3	loc_4	loc_5
Dec	mg_1	mg_2	mg_3	mg_1	mg_5
Ken	*a*	*b*	*c*	*a*	*b*
	a	*b*	*c*	*a*	*b*
	a	ø	*c*	*a*	*b*
	a	ø	ø	*a*	*b*
	ø	ø	ø	ø	ø
	x	*y*	*z*	*u*	*v*

Positionality of morphograms: <Position, Locality, Place>.
Position of the morphogram in a morphogrammatic system defined by emanation and evolution.
Locality of the monomorphies in a morphogram; loci are offering place for different monomorphies.
Monomorphies might be reduced to homogeneous patterns or they might keep some structuration.
Place of a kenom in a monomorphy depending on the length of the monomorphy.

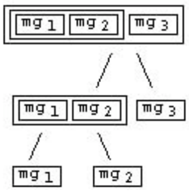
3.2. Pattern structure of morphograms

Pattern structure



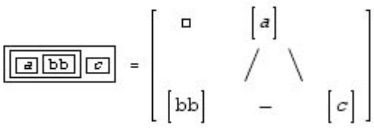


Tree structure

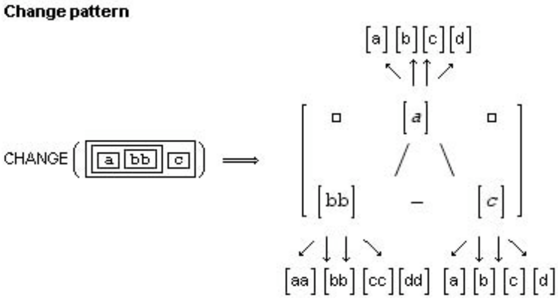


Kenomic representation

Kenomic representation with mg_1 = [a], mg_2 = [bb], mg_3 = [c]:

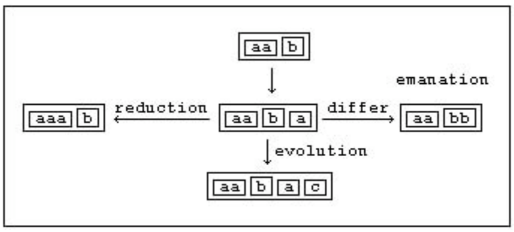


Change pattern



Morphograms in the grid of evolution and emanation

Example

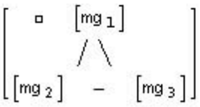


3.2.1. Notations

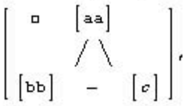
Depending on the usage, different representations for morphograms can be applied : kenomic, linear and pattern, numeric, alphabetic, symbolic, graphic.

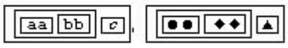
Example

MG = (mg_1, mg_2, mg_3) with kenom(mg_1) = [aa], kenom(mg_2) = [bb], kenom(mg_3) = [c].

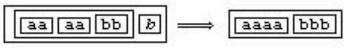
Morphogram MG = 

kenomic sequence: alphabethic: [aabbc], numeric: [1, 1, 2, 2, 3], symbolic: [●, ●, ◆, ◆, ▲]

kenomic pattern: ,

graphic representation: 

numeric representation: $\left(1_1^2,\ 2_2^2,\ 3_3^1\right)$

unified representation: 

1_1^1	2_2^1	1_3^1
1_1^1	∅	1_3^1

$\xrightarrow{\text{valuation}}$

a	*b*	*a*
a	∅	*a*

$\xrightarrow{\text{linear}}$ [aabaa] $\xrightarrow{\text{numeric}}$ [1, 1, 2, 1, 1] $\xrightarrow{\text{decomp}}$ [[aa], [b], [aa]]

mg_1 = [aa], mg_2 = [b], mg_3 = [aa]

3.2.2. Tectonics

Morphogrammatics : $\mathbf{MG}^{(m,\,n)} = \left[\mathrm{MG}^{(m,\,n)},\ \mathrm{Ops}\right]$

Morphograms : $\mathrm{MG}^{(m,\,n)}$

Monomorphies: mg_i^j

Kenomic interpretations of monomorphies : [kseq]

Semiotic interpretations of kenoms: (sign – seq)

3.3. Epsilon/Nu-structure of morphograms

3.3.1. Kenomic equality

A way to characterize and compare morphograms without a direct recurs to semiotic signs is introduced by the ∈/∨-structure of morphograms. A comparison of kenoms in the morphogram is defined as a relation between each tupel of kenoms as *equal* (∈) or *non-equal* (∨), hence the procedure is called (∈,∨)-procedue and is delivering the (∈,∨)-structure of the morpgogram. This relational characterization of morphograms is defining morphograms as (∈,∨)-structures. It is not depending on the identity of the kenoms or signs (marks) but on the differences between the kenoms. With that a further abstraction from any concrete signs as notational conventions is achieved. What counts are the relations of equality and non-equality of the marks and not the identity of the signs involved. Therefore, the alphabetic signs and their lexical order "a", "b", "c", etc. are used for conventional reasons only.

Also the ∈/∨-structure of morphograms was introduced quite early it got a first mathematical and programming treatement by Wolfgang Niegel and his students. The final formalization, implementation and bibliography, again, can be found at ThinkArt Lab [2] "*MorphoLab 1.0*".

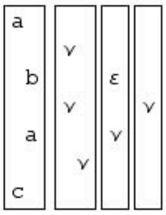
Nevertheless, this approach is equivalent to other more set theoretically oriented approaches which are building equivalence classes over sign sequences to define morphograms.

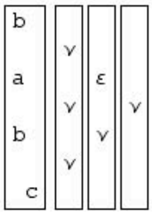
While the decompositional interaction with morphograms is asking for its monomorphies, the ∈/∨-procedure is asking for its internal structure concerning the differences between the kenoms of the morphogram.

Two morphograms as keno-sequences are kenogrammatically equal iff they have the same (ε, ν)-structure.

Example.

$[abac]\ :\ (\varepsilon, \nu)-\text{comp}([abac]) = (\nu\nu\varepsilon\nu\nu)$ $[babc]\ :\ (\varepsilon, \nu)-\text{comp}([abac]) = (\nu\nu\varepsilon\nu\nu) \Longrightarrow [abac] =_{kg} [babc]$





$\Longrightarrow [abac] =_{kg} [babc]$

As a ML – module :

```
datatype EN = E | N
fun delta (i, j) z = if (pos i z) = (pos j z)
then (i, j, E)
else (i, j, N);
type enstruc = (int * int * EN) list list;
```

Because the ϵ/V-test is applied to all kenoms of a string of kenoms, it seems to be more reasonable to understand ϵ/V-structures not simply as linear strings of kenoms, like it happens for sign sequences, but as tabular complexions of kenoms. The tabular notational system for morphogram, introduced in this paper, is stressing this point further and new possibilities of defining kenomic and morphogrammatic operations will be sketched.

Again, in contrast, two sign sequences A and B are equal iff they are equal in all their atomic signs and of the same length.

$A = (a_1, a_2, \ldots, a_n),\ B = (b_1, b_2, \ldots, b_m)$

$A =_{sem} B$ iff

1) $m = n$

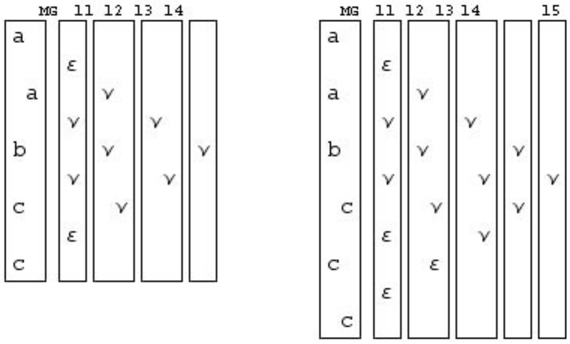
2) $\forall i, 1 \le i \ge m, n : a_i =_{graph} b_i$.

On the base of the ϵ/V-procedure for morphograms it is straight forward to define operations over morphograms in analogy to recursive word arithmetics. Hence, *successor*, *concatenation*, '*multiplication*', *reflector*, etc. operations are naturally introduced.

3.3.2. Decompositions

Decompositions[3]of morphograms into monomorphies might be defined by the ϵ/V-structure of the decomposable morphogram. *Homogeneous* monomorphies are characterized by ϵ-structures in V-environments of the ϵ/V-structure of level l1 of the morphogram.

Examples
Dec([aabcc]) = {[aa], [b], [cc]} and Dec([aabccc]) = {[aa], [b], [ccc]}



Relativity of monomorphies

The concept of monomorphies is relative to the *degree* of decomposition. It depends on the degree of decomposition from the morphogram into its decomposable parts as what a monomorphy is conceived. Monomorphies might be distinguished as *homogeneous* or *heterogeneuos* monomorphies. Homogeneuos monomorphies are decomposed into monads.
Thus, e.g. the decomposition of [abc] with $[abc] \longrightarrow ([ab], [c]) \longrightarrow ([a], [b], [c])$, can be considered as $([ab], [c])$, consisting on one *heteromorph* monomorphy [ab], or as ([a], [b], [c]), consisting of homogeneous monads.
This might have interesting consequences for further formalizations.

Example
1. Two-level decomposition

$Succ(Dec_2([abc])) =$
$Succ([a], [b], [c]) =$
$\{([aa], [b], [c]), ([ab], [b], [c]),$
$([a], [ba], [c]), ([a], [bb], [c]), ([a], [bc], [c]),$
$([a], [b], [ca]), ([a], [b], [cb]), ([a], [b], [cc]), ([a], [b], [cd])\}.$

$\sum |Succ([a], [b], [c])| = 9.$

One-level decomposition

$Succ(Dec_{1.2}([abc]) =$
$Succ([ab], [c]) =$
$\{([aba], [c]), ([abb], [c]), ([abc], [c]),$
$([ab], [ca]), ([ab], [cb]), ([ab], [cd])\}.$

$\sum |Succ_{1.2}([ab], [c])| = 6.$

$\Longrightarrow Succ_2([a], [b], [c]) \neq Succ_{1.1}([ab], [c])$
$\Longrightarrow Succ(Dec_2([abc]) \neq Succ(Dec_1([abc])$

$Succ(Dec_{1.1}([abc]) =$
$Succ([a], [bc]) =$
$\{([aa], [bc]), ([ab], [bc]),$
$([a], [bca]), ([a], [bcb]), ([a], [bcc]), ([a], [bcd])\}.$

$\sum |Succ_{1.1}([ab], [c])| = 6.$

$Succ(Dec_{1.1}([abc]), Succ(Dec_{1.1}([abc]) \subset Succ(Dec_2([abc]).$

3.4. Monomorphic evolution of morphograms

3.4.1. Evolution schemes

Morphograms are holistic patterns. Any change of a whole has to take into account that a whole is composed by parts. Hence, the concept of change for morphograms has to consider all possibility of changing a whole by its parts and by changing the parts in respect to the whole.

Tail and head appendices
As a consequence of the pattern structure of morphograms, operations like successor operation, have to applied to each monomorphy of the morphogram. As a further consequence,

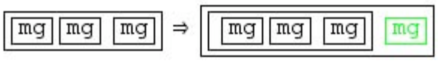
successor operations can be applied as prolongations not only at the tail of the monomorphy but dually also at the head of a monomorphy.
An addition to the body can be seen as an prolongation into the future, i.e. evolving into new complexity of the new.
Additions to the head can be seen as a retro-directed prolongation into the past, i.e. as a re-definition of what happened before.

Backwards prolongations
A backward prolongation is re-interpreting the history of the object in a *productive* way. A decomposition of a morphogram into its monomorphies is de-composing the morphogram into its constituents which led to the actual structure of the morphogram. Hence, it is a reconstruction of the past of the morphogram in a *re-productive* way. And it is not involved into any semantic re-interpretation of the existing morphogram.

Change as evolution happens in different ways.

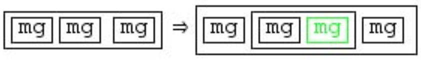
Different modi of evolution

1. Evolution of morphograms:



Evolution of a morphogram happens as a kenomic concatenation of a monomorphy with the morphogram.

2. Evolution of a monomorphy

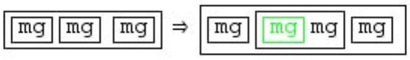


Evolution of a monomorphy of a morphogram happens as a kenomic concatenation of a new monomorphy with a monomorphy of the morphogram.

3. Reconfiguration of morphograms:

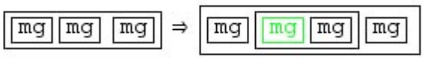
Reconfiguration of a morphogram happens as a concatenation of a monomorphy with the head of a morphogram.

4. Reconfiguration of a monomorphy



Reconfiguration of a monomorphy happens as a concatenation of a monomorphy with the head of a monomorphy.

5. Integration of a monomorphy into the morphogram.



An integration of a monomorphy into a morphogram happens as a concatenation of a monomorphy with a monomorphy.

Evolution, integration and reconfiguration schemes

$\forall\, i,\ 1 \le i \geqslant n:$

$$\text{Evol}_{mg_i}\left(\left[[mg_1], [mg_2], \ldots, [mg_n]\right]\right) = \text{kadd}_i\left([mg_1], [mg_2], \ldots, [mg_n], \langle [mg_i] \rangle\right)$$

$$\text{Integr}_{mg_i}\left(\left[[mg_1], [mg_2], \ldots, [mg_n]\right]\right) = \text{kadd}_i\left(\langle [mg_1], [mg_2], [mg_i], [mg_i] \rangle, \ldots, [mg_n]\right)$$

$$\text{Reconfig}_{mg_i}\left(\left[[mg_1], [mg_2], \ldots, [mg_n]\right]\right) = \text{kadd}_i\left([mg_i], \langle [mg_1], [mg_2], \ldots, [mg_n] \rangle\right)$$

3.4.2. A kenomic interpretation of monomorphies

A kenomic interpretation of the schemes of changes as given by the example has to consider the *kenomic context rule*. A self-evolution of a kenomic pattern is strictly ruled by the kenomic distinction of sameness and differentness of the encountered kenograms in the morphogram. The *range* of the kenograms is given by the morphogram and is applied to its monomorphies.

For the morphogram MG = ([mg 1], [mg 2]) with its kenomic interpretation ([a], [bb]) = $\begin{array}{|c|c|}\hline a & b \\ \hline \emptyset & b \\ \hline\end{array}$, the range of kenoms is {a, b} and its accretion is {c}, hence the kenomic operational range of the morphogram is {a, b, c}.

Evolution:

Evol$_{mg_1}$: Evol $_{mg_{2.1}}$ (MG) = ([mg 1], [mg 2], **⟨[mg 1]⟩**)

The pattern $\begin{array}{|c|c|}\hline a & b \\ \hline \emptyset & b \\ \hline\end{array}$ forms (develops, looms, evolves, emerges) for $Evol_1$ in the mode of *sameness* to $\begin{array}{|c|c|c|}\hline a & b & a \\ \hline \emptyset & b & \emptyset \\ \hline\end{array}$,

and in the mode of *differentness* to $\begin{array}{|c|c|c|}\hline a & b & b \\ \hline \emptyset & b & \emptyset \\ \hline\end{array}$ and $\begin{array}{|c|c|c|}\hline a & b & c \\ \hline \emptyset & b & \emptyset \\ \hline\end{array}$.

Evol$_{mg_2}$: Evol $_{mg_{2.2}}$ (MG) = ([mg 1], [mg 2], **⟨[mg 2]⟩**)

A similar development happens for $Evol^2$:

The pattern $\begin{array}{|c|c|}\hline a & b \\ \hline \emptyset & b \\ \hline\end{array}$ evolves in the mode of *sameness* to: $\begin{array}{|c|c|c|}\hline a & b & a \\ \hline \emptyset & b & a \\ \hline\end{array}$,

and in the mode of *differentness* to: $\begin{array}{|c|c|c|}\hline a & b & b \\ \hline \emptyset & b & b \\ \hline\end{array}$ and $\begin{array}{|c|c|c|}\hline a & b & c \\ \hline \emptyset & b & c \\ \hline\end{array}$.

Integration

Integr$_{mg_1}$: Integr $_{mg_{1.1}}$ (MG) = ([mg 1], ⟨**[mg 1]**, [mg 2]⟩)

The pattern $\begin{array}{|c|c|}\hline a & b \\ \hline \emptyset & b \\ \hline\end{array}$ evolves *integratively* in the mode of *sameness* to: $\begin{array}{|c|c|c|}\hline a & a & b \\ \hline \emptyset & \emptyset & b \\ \hline\end{array}$ and in the mode of differentness to $\begin{array}{|c|c|c|}\hline a & b & b \\ \hline \emptyset & \emptyset & b \\ \hline\end{array}$ and $\begin{array}{|c|c|c|}\hline a & c & b \\ \hline \emptyset & \emptyset & b \\ \hline\end{array}$ = $\begin{array}{|c|c|c|}\hline a & b & c \\ \hline \emptyset & \emptyset & c \\ \hline\end{array}$

Integr$_{mg_2}$: Integr $_{mg_{1.2}}$ (MG) = ([mg 1], ⟨ **[mg 2]**, [mg 2]⟩)

The pattern $\begin{array}{|c|c|}\hline a & b \\ \hline \emptyset & b \\ \hline\end{array}$ evolves *integratively* in the mode of *sameness* to: $\begin{array}{|c|c|c|}\hline a & a & b \\ \hline \emptyset & a & b \\ \hline\end{array}$ and $\begin{array}{|c|c|c|}\hline a & b & b \\ \hline \emptyset & b & b \\ \hline\end{array}$ and in the mode of differentness to $\begin{array}{|c|c|c|}\hline a & c & b \\ \hline \emptyset & c & b \\ \hline\end{array}$ = $\begin{array}{|c|c|c|}\hline a & b & c \\ \hline \emptyset & b & c \\ \hline\end{array}$.

Reconfiguration

$Reconfig_{mg_1}: \text{Reconfig}_{mg_{1.1}}(\text{MG}) = (\langle [\mathbf{mg}_1], [\text{mg}_1]\rangle, [\text{mg}_2])$

The pattern $\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array}$ evolves *reconfiguratively* in the mode of *sameness* to : $\begin{array}{|c|c|c|}\hline a & a & b \\ \hline \varnothing & \varnothing & b \\ \hline\end{array}$

and in the mode of *differentness* to $\begin{array}{|c|c|c|}\hline b & a & b \\ \hline \varnothing & \varnothing & b \\ \hline\end{array} = \begin{array}{|c|c|c|}\hline a & b & a \\ \hline \varnothing & \varnothing & a \\ \hline\end{array}$ and $\begin{array}{|c|c|c|}\hline c & a & b \\ \hline \varnothing & \varnothing & b \\ \hline\end{array} = \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & \varnothing & c \\ \hline\end{array}$.

$Reconfig_{mg_2}: \text{Reconfig}_{mg_{2.2}}(\text{MG}) = (\langle [\text{mg}_1], [\mathbf{mg}_2]\rangle, [\text{mg}_2])$

The pattern $\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array}$ evolves *reconfiguratively* in the mode of *sameness* to : $\begin{array}{|c|c|c|}\hline a & a & b \\ \hline \varnothing & a & b \\ \hline\end{array}$ and $\begin{array}{|c|c|c|}\hline a & b & b \\ \hline \varnothing & b & b \\ \hline\end{array}$

and in the mode of *differentness* to $\begin{array}{|c|c|c|}\hline a & c & b \\ \hline \varnothing & c & b \\ \hline\end{array} = \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & b & c \\ \hline\end{array}$.

Summary

$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Evol}(mg_1)} \left\{ \begin{array}{|c|c|c|}\hline a & b & a \\ \hline \varnothing & b & \varnothing \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & b \\ \hline \varnothing & b & \varnothing \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & b & \varnothing \\ \hline\end{array} \right\}$$

$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Evol}(mg_2)} \left\{ \begin{array}{|c|c|c|}\hline a & b & a \\ \hline \varnothing & b & a \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & b \\ \hline \varnothing & b & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & b & c \\ \hline\end{array} \right\}.$$

$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Integr}(mg_1)} \left\{ \begin{array}{|c|c|c|}\hline a & a & b \\ \hline \varnothing & \varnothing & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & b \\ \hline \varnothing & \varnothing & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & \varnothing & c \\ \hline\end{array} \right\}$$

$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Integr}(mg_2)} \left\{ \begin{array}{|c|c|c|}\hline a & a & b \\ \hline \varnothing & a & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & b \\ \hline \varnothing & b & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & b & c \\ \hline\end{array} \right\}.$$

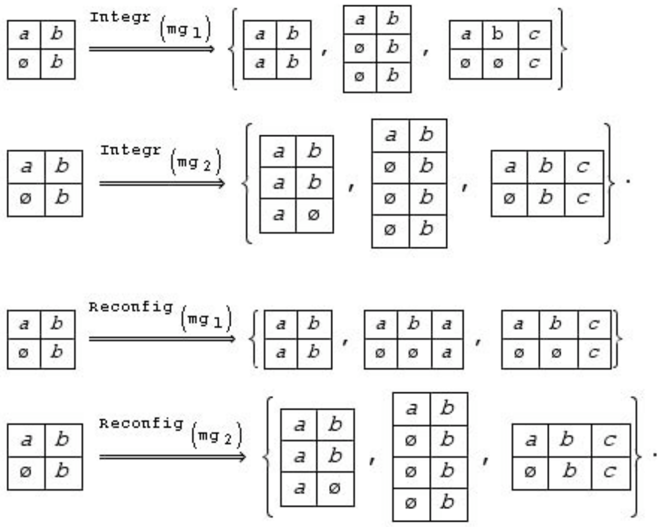
$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Reconfig}(mg_1)} \left\{ \begin{array}{|c|c|c|}\hline a & a & b \\ \hline \varnothing & \varnothing & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & a \\ \hline \varnothing & \varnothing & a \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & \varnothing & c \\ \hline\end{array} \right\}$$

$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Reconfig}(mg_2)} \left\{ \begin{array}{|c|c|c|}\hline a & a & b \\ \hline \varnothing & a & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & b \\ \hline \varnothing & b & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & b & c \\ \hline\end{array} \right\}.$$

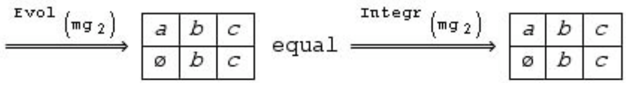
Unified interpretation

$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Evol}(mg_1)} \left\{ \begin{array}{|c|c|c|}\hline a & b & a \\ \hline \varnothing & b & \varnothing \\ \hline\end{array}, \begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline \varnothing & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & b & \varnothing \\ \hline\end{array} \right\}$$

$$\begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline\end{array} \xrightarrow{\text{Evol}(mg_2)} \left\{ \begin{array}{|c|c|c|}\hline a & b & a \\ \hline \varnothing & b & a \\ \hline\end{array}, \begin{array}{|c|c|}\hline a & b \\ \hline \varnothing & b \\ \hline \varnothing & b \\ \hline \varnothing & b \\ \hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c \\ \hline \varnothing & b & c \\ \hline\end{array} \right\}.$$



Comments



3.5. Monomorphic devolution of morphograms

Predecessor and subtraction functions are naturally defined in primitive recursive arithmetic. How are they represented in morphogrammatics?

Devolution scheme

$\forall i,\ 1 \leq i \geq n:$

$\text{Devol}_{mg_i}\left(\left[mg_1\right], \left[mg_2\right], \ldots, \left[mg_n\right]\right) = \text{pred}_i\left(\left[\left[mg_1\right], \left[mg_2\right], \ldots, \left[mg_n\right]\right], \left[mg_i\right]\right)$

The predecessor function in arithmetics acts as the opposite of the successor function and is recursively defined by the rules:

$\text{pred}(0) = 0,$
$\text{pred}(n+1) = n$

What's the analogon of the predecessor function in morphogrammatics? Is the decomposition function an analogon to the predecessor function?
Obviously, the predecessor of a monad is a monad:
Dec(monad) = monad, hence in analogy: pred(monad) = monad
But what's a monad?
Trivially, it is a 1-kenomic pattern: [a]. Thus, Dec([a]) = [a] and analogous: pred([a]) = [a].
On the other hand we also learned, that 1-kenomic pattern of higer complication are not decomposable into monomorphies.
Dec([aaaa]) = [aaaa]. But this is not in an analogy to pred([aaaa]) = [aaa].

Therefore, the terminology of predecessor and successor gets a reasonable application for the sequence-approach of kenogrammatics. Kenomic sequences are decomposable into monadic kenoms, hence the analogy to the predecessor function applies.

$$Dec(MG) = (mg_1, mg_2, ..., mg_n),$$

$$pred(Dec(MG)) =$$
$$(mg_1, mg_2, ..., mg_{n-1})\ pred([mg_1, mg_2, ..., mg_n]\ kadd_i\ [mg_{n+1}]) = [mg_1, mg_2, ..., mg_n]$$

$$pred_i(succ_j(MG)) = succ_j(pred_i(MG)),\ i = j$$

3.6. Morphogrammatic coalitions

The modi of morphogrammatic coalitions have to reflect and implement the different modi of evolution into their definitions. Hence, coalitions appears as *evolutive*, *integrative* and *reconfigurative* coalitions.

3.6.1. Change as coalitions

$$coal\left(\left([mg_{i_1}], [mg_{i_2}], ..., [mg_{i_n}]\right), \left([mg_{j_1}], [mg_{j_2}], ..., [mg_{j_n}]\right)\right) =$$

$$coal_1: \left([mg_{i_1}], [mg_{i_2}], ..., [mg_{i_n}], [mg_{j_1}]\right)$$
$$coal_2: \left([mg_{i_1}], [mg_{i_2}], ..., [mg_{i_n}], [mg_{j_2}]\right)$$
$$\cdots$$
$$coal_n: \left([mg_{i_1}], [mg_{i_2}], ..., [mg_{i_n}], [mg_{j_n}]\right)$$

Example

$$MG_1 = [mg_{i_1}],\ MG_2 = \left([mg_{j_1}], [mg_{j_2}]\right)$$

$$coal_1\left(\left([mg_{i_1}], \left([mg_{j_1}], [mg_{j_2}]\right)\right)\right) =$$
$$kadd_1\left([mg_{i_1}], \left([mg_{j_1}], [mg_{j_2}]\right)\right) = \left([mg_{j_1}], [mg_{j_2}], [mg_{i_1}]\right)$$

$$coal_2\left(\left([mg_{i_1}], \left([mg_{j_1}], [mg_{j_2}]\right)\right)\right) =$$
$$kadd_2\left([mg_{i_1}], \left([mg_{j_1}], [mg_{j_2}]\right)\right) = \left([mg_{j_1}], [mg_{i_1}], [mg_{j_2}]\right)$$

3.6.2. Evolutive coalitions

1. Example

$$\text{coal}_{\text{evol}}\left(\begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}, \begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}\right) = \left\{\begin{array}{|c|c|}\hline a & a\\\hline b & b\\\hline\end{array}, \begin{array}{|c|c|}\hline a & a\\\hline b & c\\\hline\end{array}, \begin{array}{|c|c|}\hline a & b\\\hline b & a\\\hline\end{array}, \begin{array}{|c|c|}\hline a & b\\\hline b & c\\\hline\end{array}\right\}$$

2. Example

$$\text{coal}_{\text{evol}}\left(\begin{array}{|c|}\hline a\\\hline a\\\hline\end{array}, \begin{array}{|c|c|}\hline a & b\\\hline \emptyset & b\\\hline\end{array}\right) = \left\{\begin{array}{|c|c|c|}\hline a & b & a\\\hline \emptyset & b & a\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & b\\\hline \emptyset & b & b\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c\\\hline \emptyset & b & c\\\hline\end{array}\right\}$$

3. Example

$$\text{coal}_{\text{evol}}\left(\begin{array}{|c|c|}\hline a & b\\\hline a & \emptyset\\\hline\end{array}, \begin{array}{|c|c|}\hline a & b\\\hline \emptyset & b\\\hline\end{array}\right) =$$

$$\left\{\begin{array}{|c|c|c|c|}\hline a & b & a & b\\\hline a & \emptyset & \emptyset & b\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline a & b & a & c\\\hline a & \emptyset & \emptyset & c\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline a & b & b & a\\\hline a & \emptyset & \emptyset & a\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline a & b & c & a\\\hline a & \emptyset & \emptyset & a\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline a & b & c & b\\\hline a & \emptyset & \emptyset & b\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline a & b & c & d\\\hline a & \emptyset & \emptyset & d\\\hline\end{array}\right\}$$

$\text{coal}_{1.2}: \begin{array}{|c|c|c|c|}\hline a & b & a & b\\\hline a & \emptyset & \emptyset & b\\\hline\end{array}$ $\quad$ $\text{coal}_{1.3}: \begin{array}{|c|c|c|c|}\hline a & b & a & c\\\hline a & \emptyset & \emptyset & c\\\hline\end{array}$

$\text{coal}_{2.1}: \begin{array}{|c|c|c|c|}\hline a & b & b & a\\\hline a & \emptyset & \emptyset & a\\\hline\end{array}$ $\quad$ $\text{coal}_{3.1}: \begin{array}{|c|c|c|c|}\hline a & b & c & a\\\hline a & \emptyset & \emptyset & a\\\hline\end{array}$

$\text{coal}_{3.2}: \begin{array}{|c|c|c|c|}\hline a & b & c & b\\\hline a & \emptyset & \emptyset & b\\\hline\end{array}$ $\quad$ $\text{coal}_{3.4}: \begin{array}{|c|c|c|c|}\hline a & b & c & d\\\hline a & \emptyset & \emptyset & d\\\hline\end{array}$.

Numeric representation

$\text{add}_{1.2}\left(\boxed{1_1^2, 2_2^1}, \boxed{1_1^1, 2_2^2}\right) = \boxed{1_1^2, 2_2^1, 1_3^1, 2_4^2}$

$\text{add}_{1.3}\left(\boxed{1_1^2, 2_2^1}, \boxed{1_1^1, 2_2^2}\right) = \boxed{1_1^2, 2_2^1, 1_3^1, 3_4^2}$

$\text{add}_{2.1}\left(\boxed{1_1^2, 2_2^1}, \boxed{1_1^1, 2_2^2}\right) = \boxed{1_1^2, 2_2^1, 2_3^1, 1_4^2}$

$\text{add}_{3.1}\left(\boxed{1_1^2, 2_2^1}, \boxed{1_1^1, 2_2^2}\right) = \boxed{1_1^2, 2_2^1, 3_3^1, 1_4^2}$

$\text{add}_{3.2}\left(\boxed{1_1^2, 2_2^1}, \boxed{1_1^1, 2_2^2}\right) = \boxed{1_1^2, 2_2^1, 3_3^1, 2_4^2}$

$\text{add}_{3.4}\left(\boxed{1_1^2, 2_2^1}, \boxed{1_1^1, 2_2^2}\right) = \boxed{1_1^2, 2_2^1, 3_3^1, 4_4^2}$.

Null

3.6.3. Integrative coalitions

$$\text{coal}_{\text{mg 1 integr}}\left(\begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}, \begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}\right) = \left\{\begin{array}{|c|c|c|}\hline a & a & a\\\hline b & \emptyset & b\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & a\\\hline b & \emptyset & b\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & a\\\hline b & \emptyset & c\\\hline\end{array}\right\}$$

$$\text{coal}_{\text{mg 2 integr}}\left(\begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}, \begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}\right) = \left\{\begin{array}{|c|c|c|}\hline a & a & a\\\hline b & \emptyset & b\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & a\\\hline b & \emptyset & b\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & a\\\hline b & \emptyset & c\\\hline\end{array}\right\}$$

$$\text{coal}_{\text{mg 1 integr}}\left(\begin{array}{|c|}\hline a\\\hline a\\\hline\end{array}, \begin{array}{|c|c|}\hline a & b\\\hline \emptyset & b\\\hline\end{array}\right) = \left\{\begin{array}{|c|c|c|}\hline a & a & b\\\hline \emptyset & a & b\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & a\\\hline \emptyset & b & a\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c\\\hline \emptyset & b & c\\\hline\end{array}\right\}$$

$$\text{coal}_{\text{mg 2 integr}}\left(\begin{array}{|c|}\hline a\\\hline a\\\hline\end{array}, \begin{array}{|c|c|}\hline a & b\\\hline \emptyset & b\\\hline\end{array}\right) = \left\{\begin{array}{|c|c|}\hline a & b\\\hline a & b\\\hline a & \emptyset\\\hline\end{array}, \begin{array}{|c|c|}\hline a & b\\\hline \emptyset & b\\\hline \emptyset & b\\\hline \emptyset & b\\\hline\end{array}, \begin{array}{|c|c|c|}\hline a & b & c\\\hline \emptyset & b & c\\\hline\end{array}\right\}$$

3.6.4. Reconfigurative coalitions

$$\mathrm{coal}_{mg_1\,recon}\left(\begin{array}{|c|}\hline a\\\hline b\\\hline\end{array},\ \begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}\right)=\left\{\begin{array}{|c|c|c|}\hline a&a&a\\\hline \varnothing&b&b\\\hline\end{array},\ \begin{array}{|c|c|c|}\hline b&a&a\\\hline \varnothing&b&b\\\hline\end{array},\ \begin{array}{|c|c|c|}\hline c&a&a\\\hline \varnothing&b&b\\\hline\end{array}\right\}$$

$$\mathrm{coal}_{mg_2\,recon}\left(\begin{array}{|c|}\hline a\\\hline b\\\hline\end{array},\ \begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}\right)=\left\{\begin{array}{|c|c|c|}\hline a&a&a\\\hline \varnothing&b&b\\\hline\end{array},\ \begin{array}{|c|c|c|}\hline b&a&a\\\hline \varnothing&b&b\\\hline\end{array},\ \begin{array}{|c|c|c|}\hline c&a&a\\\hline \varnothing&b&b\\\hline\end{array}\right\}$$

$$\mathrm{coal}_{mg_1\,recon}\left(\begin{array}{|c|}\hline a\\\hline a\\\hline\end{array},\ \begin{array}{|c|c|}\hline a&b\\\hline \varnothing&b\\\hline\end{array}\right)=\left\{\begin{array}{|c|c|c|}\hline a&a&b\\\hline a&\varnothing&b\\\hline\end{array},\ \begin{array}{|c|c|c|}\hline a&b&a\\\hline a&\varnothing&a\\\hline\end{array},\ \begin{array}{|c|c|c|}\hline a&b&c\\\hline a&\varnothing&c\\\hline\end{array}\right\}$$

$$\mathrm{coal}_{mg_2\,recon}\left(\begin{array}{|c|}\hline a\\\hline a\\\hline\end{array},\ \begin{array}{|c|c|}\hline a&b\\\hline \varnothing&b\\\hline\end{array}\right)=\left\{\begin{array}{|c|c|}\hline a&b\\\hline a&b\\\hline a&\varnothing\\\hline\end{array},\ \begin{array}{|c|c|}\hline a&b\\\hline \varnothing&b\\\hline \varnothing&b\\\hline \varnothing&b\\\hline\end{array},\ \begin{array}{|c|c|c|}\hline a&b&c\\\hline \varnothing&b&c\\\hline\end{array}\right\}$$

3.6.5. Properties of coalitions

Non-Commutativity of coalitions, super-additivity

3.7. Morphogrammatic multiplication

Again, the modi of morphogrammatic multiplications have to reflect and implement the different modi of change into their definitions. Hence, multiplications appears as *evolutive*, *integrative* and *reconstructive* compositions.
A multiplication mxn is understood as an n-fold addition of m. Hence, the concept of multiplication is inheriting the features of addition, i.e. *evolutive*, *integrative* and *reconfigurative* transformations and their retro-grade definition of the range of the operations.
The definition and programming of the construct “kmul” as *evolutive (concatenational) multiplication* goes back to Thomas Mahler (Morphogrammatik 1993, pp 78 - 82).[4]

3.7.1. Evolutive multiplication

$$\mathrm{kmul}_{evol}=\left(MG_1,MG_2\right)=\left(mg_{1_1}\times\left[mg_{2_1},\ mg_{2_2}\right],\ mg_{1_2}\times\left[mg_{2_1},\ mg_{2_2}\right]\right)$$

Context - Rule

$\forall\, i \in \mathrm{Dec}(MG_{i+1}),\ mg_i \in MG_1,\ mg_i \in MG_2:$

$$\mathrm{kmul}(MG_1,\ MG_2) \text{ iff } \begin{pmatrix} \mathrm{head}(mg)_i \neq \mathrm{head}(mg)_{i+1} \\ \mathrm{body}(mg)_i \neq \mathrm{body}(mg)_{i+1} \end{pmatrix}$$

Head = first kenom of *a* monomorphy
Body = the rest.

Order

numerical lexical order(MG) = 1 < 2 < 3 < ...
alphabetical lexical order(MG) = *a* < *b* < *c* < ...

Multiplication tables for kmult ([1], [1, 2]) and kmult ([1, 2], [1])

kmul	1	2
1	1	2
∅	∅	∅

kmul	1
1	1
2	2

$$\mathrm{kmul}\left(\boxed{1},\ \begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array}\right) = \begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array} = \mathrm{kmul}\left(\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array},\ \boxed{1}\right)$$

In arithmetic we distinguish between a zero – and a unit – element.
This distinctions are necessary to run recursive functions.

Identity elements " *zero* " and " *one* ".
addition:
a + 0 = 0 + a = a
multiplication:
a x1 = 1 x a = a
a x 0 = 0 x a = 0

```
[ ] a  =  [ [ ] ]
b [ [ ] ]  =  [ [ ] ]
a [1]  =  [ a ]
[1] b  =  [ b ]
```

Multiplication tables for kmul([1, 2], [1, 2])

kmul	1	2
1	1	*x*
2	2	*y*

kmul	1	2	*a*	*b*	*c*
1	1	2	2	3	3
2	2	1	3	1	4

$$\mathrm{kmul}\left(\begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array},\ \begin{array}{|c|}\hline 1\\\hline 2\\\hline\end{array}\right) = \left\{\begin{array}{|c|c|}\hline 1&2\\\hline 2&1\\\hline\end{array},\ \begin{array}{|c|c|}\hline 1&3\\\hline 2&1\\\hline\end{array},\ \begin{array}{|c|c|}\hline 1&2\\\hline 2&3\\\hline\end{array},\ \begin{array}{|c|c|}\hline 1&3\\\hline 2&4\\\hline\end{array}\right\}$$

1. Identity: [1] x [1,2] = [1,2]: head- and body-iteration, $head_1 = head_2$, $body_1 = body_2$
2. Diversity: [2]x[1,2] =
   [2,1]: iterative component,
   [3,1] :head-accretion, body-iteration,
   [2,3]: head-iteration, body-accretion,
   [3,4]: head- and body-accretion,
   all accepting CRM: $head_1 \mathrel{!=} head_2$, $body_1 \mathrel{!=} body_2$

Multiplication table for kmul([1, 2, 2], [1, 2, 1])

kmul	1	2	*a*	*b*	*c*	1
1	1	2	2	3	3	1
2	2	1	3	1	4	2
2	2	1	3	1	4	2

$$\text{kmul}\left(\begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}1\\2\\1\end{bmatrix}\right) = \left\{\begin{bmatrix}1&2&1\\2&1&2\\2&1&2\end{bmatrix}, \begin{bmatrix}1&2&1\\2&3&2\\2&3&2\end{bmatrix}, \begin{bmatrix}1&3&1\\2&1&2\\2&1&2\end{bmatrix}, \begin{bmatrix}1&3&1\\2&4&2\\2&4&2\end{bmatrix}\right\}$$

Multiplication table for kmul([1, 2, 2], [1, 2, 3, 1])

kmul	1	2	3	1
1	**1**	2 3333 2222 3333 3333 3	3 2244 3434 2452 24545 5	**1**
2	**2**	1 1111 3333 4444 4444 4	4 3435 114 5 1113 53351 6	**2**
2	**2**	1 1111 3333 4444 4444 4	4 3435 114 5 1113 53351 6	**2**

$$\text{kmul}\left(\begin{bmatrix}1\\2\\2\end{bmatrix}, \begin{bmatrix}1\\2\\3\\1\end{bmatrix}\right) = \left\{\begin{bmatrix}1&2&3&1\\2&1&4&2\\2&1&4&2\end{bmatrix}, \begin{bmatrix}1&3&2&1\\2&1&3&2\\2&1&3&2\end{bmatrix}, \begin{bmatrix}1&3&2&1\\2&1&4&2\\2&1&4&2\end{bmatrix}, \ldots, \begin{bmatrix}1&3&5&1\\2&4&6&2\\2&4&6&2\end{bmatrix}\right\}$$

Exclusion table

kmul	1	2 ≠ CRM	3 ≠ CRM	1
1	**1**	1 1 3	1 1 1 1 3 4 5	1
2	**2**	2 3 2	2 3 4 5 2 2 2	2
2	**2**	2 3 2	2 3 4 5 2 2 2	2

Inclusion table

$$\begin{bmatrix}2&3&4&5&2&2&2&3&4&4&5&5\\1&1&1&1&3&4&5&4&3&5&3&6\\1&1&1&1&3&4&5&4&3&5&3&6\end{bmatrix} \in \text{CRM}$$

Decomposition

$$\textbf{kmul}\left([1,2,2],\ [1,2,3,1]\right) \Longrightarrow \textbf{kmul}\left(\left([1],[2,2]\right),\ \left([1],[2],[3],[1]\right)\right)$$

$$MG_1 = [1,2,2] \Longrightarrow [mg_1,\ mg_2]$$

$$MG_2 = [1,2,3,\ 1] \Longrightarrow [mg_1,\ mg_2,\ mg_3,\ mg_1]$$

kmul	$mg_{2.1}$	$mg_{2.2}$	$mg_{2.3}$	$mg_{2.1}$
$mg_{1.1}$	$mg_{1.1}$	$\{\overline{mg_{1.1}}\}$	$\{\overline{\overline{mg_{1.1}}}\}$	$mg_{1.1}$
$mg_{1.2}$	$mg_{1.2}$	$\{\overline{mg_{1.2}}\}$	$\{\overline{\overline{mg_{1.2}}}\}$	$mg_{1.2}$

$$\begin{pmatrix} mg_{1.1}, & \{\overline{mg_{1.1}}\}, & \{\overline{\overline{mg_{1.1}}}\}, & mg_{1.1} \\ mg_{1.2}, & \{\overline{mg_{1.2}}\}, & \{\overline{\overline{mg_{1.2}}}\}, & mg_{1.2} \end{pmatrix} \in \text{CRM}$$

3.7.2. Properties of evolutive multiplication

1. Unit element:

$$\mathbf{kmul}\left(\mathbf{mg}\,[1]\right) = [\mathbf{mg}] = \mathbf{kmul}\left([1]\,\mathbf{mg}\right)$$

2. non – commutativity for $MG_1 \neq MG_2$:

$$\text{kmul}\left(\begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}, \begin{array}{|c|}\hline a\\\hline a\\\hline a\\\hline\end{array}\right) \neq \text{kmul}\left(\begin{array}{|c|}\hline a\\\hline a\\\hline a\\\hline\end{array}, \begin{array}{|c|}\hline a\\\hline b\\\hline\end{array}\right)$$

$$\text{kmul}\left(\begin{array}{|c|}\hline a\\ b\\\hline\end{array}, \begin{array}{|c|}\hline a\\\hline a\\\hline a\\\hline\end{array}\right) = \begin{array}{|c|}\hline a\\ b\\\hline a\\ b\\\hline a\\ b\\\hline\end{array} = \begin{array}{|c|c|c|}\hline a & a & a\\ b & b & b\\\hline\end{array}$$

$$\text{kmul}\left(\begin{array}{|c|}\hline a\\\hline a\\\hline a\\\hline\end{array}, \begin{array}{|c|}\hline a\\ b\\\hline\end{array}\right) = \begin{array}{|c|}\hline a\\ a\\ a\\\hline b\\ b\\ b\\\hline\end{array} = \begin{array}{|c|c|}\hline a & b\\ a & b\\ a & b\\\hline\end{array}.$$

Numeric notation

$[1, 2, 1, 2, 1, 2] \neq [1, 1, 1, 2, 2, 2]$

$[1^1, 2^1, 1^1, 2^1, 1^1, 2^1] \neq [1^3, 2^3]$.

Cardinality

$\left|\text{Kmult}_i\left([1, 2], [1, 2]\right)\right| = \left|[1, 2]\right| \times \left|[1, 2]\right|$

$\left|\text{Kmult}\left([1, 2], [1, 2]\right)\right| < \left|\text{Kadd}\left([1, 2], [1, 2]\right)\right|$

3.7.3. Integrative multiplication

$$\text{kmul}_{\text{integr}}\left(MG_1, MG_2\right) = \left(mg_{1_1}\ x\ [mg_{2_1}, mg_{2_2}], \left\langle mg_{1_2}\ x\ [mg_{2_1}, mg_{2_2}]\right\rangle, mg_{1_2}\ x\ [mg_{2_1}, mg_{2_2}\right.$$

Example

$$\begin{aligned}[1]\text{x}[1, 2] + [2]\text{x}[1, 2] &= [1, 2] + [2, 1] = [1, 2, 2, 1] \Longrightarrow [1, 2, (2, 1), 2, 1]\\ &= [1, 2] + [3, 1] = [1, 2, 3, 1] \Longrightarrow [1, 2, (3, 1), 3, 1]\\ &= [1, 2] + [2, 3] = [1, 2, 2, 3] \Longrightarrow [1, 2, (2, 3), 2, 3]\\ &= [1, 2] + [3, 4] = [1, 2, 3, 4] \Longrightarrow [1, 2, (3, 4), 3, 4].\end{aligned}$$

3.7.4. Reconfigurative multiplication

$$\text{kmul}_{recon}(MG_1, MG_2) = (mg_{1_1}\ x\ \langle [mg_{2_1}, mg_{2_2}] \rangle,\ mg_{1_1}\ x\ [mg_{2_1}, mg_{2_2}],\ mg_{1_2}\ x\ [mg_{2_1}, mg_{2_2}$$

Example

$$[1]\ x[1, 2] + [2]\ x[1, 2] = [1, 2] + [2, 1] \Longrightarrow [(2, 1), 1, 2, 2, 1] = [1, 2, 2, 1, 1, 2]$$
$$= [1, 2] + [3, 1] \Longrightarrow [(3, 1), 1, 2, 3, 1] = [1, 2, 2, 3, 1, 2]$$
$$= [1, 2] + [2, 3] \Longrightarrow [(2, 3), 1, 2, 2, 3] = [1, 2, 3, 1, 1, 2]$$
$$= [1, 2] + [3, 4] \Longrightarrow [(3, 4), 1, 2, 3, 4] = [1, 2, 3, 4, 1, 2].$$

New beginnings for kenogrammatics?

In arithmetic we distinguish between a zero – and a unit – element. This distinctions are necessary to run recursive functions.
Identity elements : "zero" and "one".
In addition :
$a+0=0+a=a$
In multiplication :
$a\,x1 = 1\,x\,a = a$
$a\,x0 = 0\,x\,a = 0$

```
[] a = [[]]
b [[] ] = [[]]
a [1] = [a]
[1] b = [b]
```

Iterative and accretive units

A kenomic unit (identity element in algebra) as an identity or as a difference.

(This distinction is connecting to the complementarity of objects in diamond category theory.)

$$a[1]_{iter} = [a_{iter}] = [a]$$

$$a\,[1]_{acc} = [a_{acc}] = [a']$$

$$a\,x[1] = \begin{pmatrix} [a_{iter}] \\ \updownarrow \\ [a_{acc}] \end{pmatrix}$$

Chiastic complementarity of beginnings

$$\begin{pmatrix} [a_{iter}] \\ \updownarrow \\ [a_{acc}] \end{pmatrix} \longleftrightarrow \begin{pmatrix} [a_{iter}] \\ \updownarrow \\ [a_{acc}] \end{pmatrix} : \text{tabular iteration / accretion}$$

$$\begin{pmatrix} [a_{iter}] & \longrightarrow & [a_{acc}] \\ \updownarrow & \square & \updownarrow \\ [a_{acc}] & \longleftarrow & [a_{iter}] \end{pmatrix} : \text{chiasm of (iteration / accretion and internal / external)}$$

Considering this observation, kenogrammatics has to start at the very beginning with the inscription of the difference of iteration/accretion. This observation is corresponding to the concept of *"bi-objects"* in diamond category theory.

At such a start of kenogrammatics there is no need for a dualism of iterativity and accretion but a *chiasm* is involved of the terms *internal/external* and *accretion/iteration*, producing the double determination of situations by the wording of iterative iterativity, accretive iterativity and iterative accretion, accretive accretion. Hence, basic terms in kenogrammatics are reflectional and second-order figures. In other terms, proemiality is opening up the beginnings of kenogrammatics.

To choose a beginning with a mark is putting a difference into the possibility of a choice for another mark of beginning. Such a difference in the notion of representation of a beginning by a mark shall be inscribed

as a *double* beginning. The question is: As which representation is a kenogram inscribed? If it is inscribed as “a” then it is differentiated from another possible inscription, say “b”. If it is inscribed as “b” then it is differentiated from another possibility, say “a”. A further differentiation, say into “c” , would be redundant and irrelevant for the characterisation of a kenomic beginning.
On the other hand, philosophically, with double beginnings, the necessity of a unique and ultimate *“coincidentia oppositorum”* (Cusanus, Hegel, Gunther) is differentiated and dissolved.

A beginning for kenogrammatics is not in the mode of an is-abstraction, i.e. a = a, but in the mode of an as-abstraction, thus, producing beginning as a complementarity of the monads “a” and “b”.

This, obviously, is not the same as an *isomorphism* between different sign systems which differ in their alphabet.

With that, the notational decision for a representation is represented as the choice for an iterative or an accretive notation of a kenogram. Both interpretations are, in *isolation* and without their chiastic interaction, isomorphic. The present *"Outline of morphogrammatics”* is not yet reflecting this situations of disremptive - iterative/accretive - identifications of kenoms by the as-abstraction.

Hence, is it necessary to accept the unit element as a sole interpretation of a monadic multiplication?
As an example: [1] x [1, 2] = [1 x [1], 1 x [2]] = [1, 2],

Multiplication is a repetition. The multiplication “1 x [1]" is a single repetition of [1] .
But repetitions in kenomic systems might happen as *iterative* and as *accretive* repetitions .

Hence, 1 x [1] = [1] for *iteration*, the same for 1 x [2] = [2], and different for *accretion*: 1 x [1] = [2], 1 x [2] = [3].

Iterative and accretive multiplication
Because [1] and [2] are kenomically equal as isolated monads, $[1] =_{kenom} [2]$, the kenomic difference disappears. And with it the semiotic difference too.
But in a contextual environment of a morphogram the monadic difference is playing its part of differentiation. Hence, there is a possibility to distinguish between *iterative and accretive multiplication* as it is possible to distinguish iterative and accretive succession and coalition . This is not multiplicative accretion inherited by addition but a genuine multiplicative accretion. That is, a repetition of a kenogram with itself is not necessarily equal the repeated kenogram. The same monad is equal as an iteration and different as an accretion . Thus, $kmul_{iter} \neq kmul_{acc}$.
Similar ideas are presented at: “Lambda Calculi in polycontextural Situations”.[5]

Multiplication tables for kmul([1], [1, 2]) and kmul([1, 2], [1])

Iterative multiplication

kmul	1	2
1	1	2
∅	∅	∅

kmul	1
1	1
2	2

$$\text{kmul}_{\text{iter}}\left(\boxed{1}, \boxed{\begin{matrix}1\\2\end{matrix}}\right) = \boxed{\begin{matrix}1\\2\end{matrix}} = \text{kmul}_{\text{iter}}\left(\boxed{\begin{matrix}1\\2\end{matrix}}, \boxed{1}\right).$$

Accretive multiplication

kmul	1	2
1	2	3
∅	∅	∅

kmul	1
1	2
2	3

$$\left.\begin{matrix}\text{kmul}_{\text{accr}}\left(\boxed{1}, \boxed{\begin{matrix}1\\2\end{matrix}}\right) = \boxed{\begin{matrix}2\\3\end{matrix}}\\ \text{kmul}_{\text{iter}}\left(\boxed{\begin{matrix}1\\2\end{matrix}}, \boxed{1}\right) = \boxed{\begin{matrix}1\\2\end{matrix}}\end{matrix}\right) = \boxed{\begin{matrix}2\\3\end{matrix}} =_{\text{kenom}} \boxed{\begin{matrix}1\\2\end{matrix}}.$$

$$\Longrightarrow \text{kmul}_{\text{iter}}\left(\boxed{\begin{matrix}1\\2\end{matrix}}, \boxed{1}\right) =_{\text{kenom}} \text{kmul}_{\text{accr}}\left(\boxed{1}, \boxed{\begin{matrix}1\\2\end{matrix}}\right).$$

Mixed situations for kmul([1], [1, 1])

$$1 x_{\text{iter}} [1] + 1 x_{\text{iter}} [1] = [1, 1]$$
$$1 x_{\text{iter}} [1] + 1 x_{\text{acc}} [1] = [1, 2]$$
$$1 x_{\text{acc}} [1] + 1 x_{\text{iter}} [1] = [2, 1]$$
$$1 x_{\text{acc}} [1] + 1 x_{\text{acc}} [1] = [2, 2].$$

$$\text{kmul}([1], [1, 1]) = \begin{pmatrix}[1, 1]\\ [1, 2\end{pmatrix} \Longrightarrow [1, 1] \neq_{\text{kenom}} [1, 2].$$

Mixed situations for kmul([1, 2, 2], [1, 2, 3, 1])

kmul	1	2	3	1
1 2 2	**1** **2** **2**	□	□	**2** **3** **3**

Iteration of [1, 2, 2] at the first and

accretion of [1, 2, 2] at 4. column of [1, 2, 3, 1] by

" kmul([1, 2, 2], [1]) = {[1, 2, 2], [2, 3, 3]} "

kmul	1	2	3	1
1 2 2	**1** **2** **2**	3 3 3 3 3 3 3 3 1 4 4 4 4 1 4 4 4 4	3 4 5 4 5 4 1 4 5 6 4 1 4 5 6	**2** **3** **3**

End of " New Beginnings ".

3.8. Monomorphic subsitution

3.8.1. Substitution and context rules

Substitution might happen in the modi of the as-abstraction, which are contextualized by the context rules of substitution. Substitution is a main operation for the definition of many kinds of self-applicative functions. Self-application is the main construction for all kinds of proofs of fundamental properties of computability and its limitations.

Without doubt, kenomic function are showing fundamentally different behaviors and features. Conflicts or even antinomies or paradoxes producing substitutions are based on the very special case of substitution in the mode of identity, i.e. substitution ruled by the is-abstraction. Hence, a whole catalogue of non-paradox substitutions, framed by the context rules, are enabled by the as-abstractions of kenomic substitutions.
With this approach of multiple substitutions, polycontextural considerations of as-substitutions, as developped in the 70s, might be clearly separated and moved to further thematizations.

Kenomic substitution

Substitution too is depending on contexts. Not everything substituted for a monomorphy in a morphogram is supporting the equality between two equal morphograms. In contrast to kenomic substitution, semiotic substitution is context-free. That is, to two equal sign sequences any substitution of equal parts restores the equality between the two new sequences.

Semiotic Substitution:

$\forall h,\ k_1 \in H_1,\ k_2 \in H_2,\ k_1 =_{sem} k_2:$

$$\mathbf{H_1 =_{sem} H_2 \Longleftrightarrow Subst_{h/k}(H_1) =_{sem} Subst_{h/k}(H_2)}$$

$k_1 =_{sem} k_2 \text{ iff } length(k_1) = length(k_2) \bigwedge \forall i, j \in k_1, k_2 : loc_i(atom) = loc_j(atom)$

Example

$H_1 = H_2 = (aabbcde),$

$h = (bb),\ k_1 = k_2 = (\overline{lkmbc}):$

$H_1 =_{sem} H_2 \Longleftrightarrow Subst_{bb/\overline{klmbc}}(aabbcde) = Subst_{bb/\overline{klmbc}}(aabbcde)$

$(aabbcde) =_{sem} (aabbcde) \Longleftrightarrow (aa\,\overline{lkmbc}\,bbcde) = (aa\,\overline{lkmbc}\,bbcde).$

Monomorphic substitution

Context rules for substitution CRS
$\forall h, m_1 \in H_1, m_2 \in H_2, m_1 =_{MG} m_2,$
$m_1 \neq_{sem} m_2, h \neq_{sem} m_1, m_2,$
$\text{length}(m_1) = \text{length}(m_2),$
$\text{kenom}(m_1) \bigcap \text{kenom}(H_1) = \emptyset,$
$\text{kenom}(m_2) \bigcap \text{kenom}(H_2) = \emptyset :$
$\mathbf{H_1 =_{MG} H_2 \Longrightarrow Subst_{h/m_1}(H_1) =_{MG} Subst_{h/m_2}(H_2)}$ **; modulo CRS**

Example
$H_1 = [\text{aabbacc}], H_2 = [\text{aaccabb}],$
$H_1 =_{MG} H_2, H_1 \neq_{sem} H_2$
$\text{Dec}(H_1) = ([\text{aa}], [\text{bb}], [a], [\text{cc}]),$
$\text{Dec}(H_2) = ([\text{aa}], [\text{cc}], [a], [\text{bb}]),$
$h = [\text{aa}], m_1 = [\text{ddd}], m_2 = [\text{eee}],$

$\text{length}(m_1) = \text{length}(m_2),$
$m_1 \neq_{sem} m_2, h \neq_{sem} m_1, m_2,$
$\text{sem}(m_i) \bigcap \text{sem}(H_i) = \emptyset, i = 1, 2$

$\text{Dec}(H_1) = ([\text{aa}], [\text{bb}], [a], [\text{cc}])$
$\text{Subst}(H_1)_{[\text{aa}]/[\text{ddd}]}([\text{aa}], [\text{bb}], [a], [\text{cc}]) = ([\text{ddd}], [\text{bb}], [a], [\text{cc}])$

$\text{Dec}(H_2) = ([\text{aa}], [\text{cc}], [a], [\text{bb}])$
$\text{Subst}(H_2)_{[\text{aa}]/[\text{eee}]}([\text{aa}], [\text{cc}], [a], [\text{bb}]) = ([\text{eee}], [\text{cc}], [a], [\text{bb}])$

$H_1 =_{MG} H_2 \Longrightarrow \text{Subst}(H_1)_{[\text{aa}]/[\text{ddd}]} =_{MG} \text{Subst}(H_2)_{[\text{aa}]/[\text{eee}]}$

$\mathbf{[aabbacc] =_{MG} [aaccabb] \Longrightarrow ([dddbbacc]) =_{MG} ([eeeccabb]}.$

Standard representation
$\mathbf{[aabbacc] =_{MG} [aabbacc] \Longrightarrow ([aaabbcdd]) =_{MG} ([aaabbcdd]}.$

Obviously, the case of semiotic equality of substituted monomorphies in morphograms is trivial.

4. Morphogrammatic reflection

4.1. Reflection of morphograms

Reflection or inversion of morphograms offers a convenient form of structural change without touching the complexity/complication features of a morphogram. Hence, reflection is realizing

morphogrammatic change without any change in the structure of the morphogram involved into reflection.
Nevertheless, such a simple transformation might produce *conceptual* changes which are not always easily to accept. Typical conceptual examples for dualities are: individualism/collectivism, matter/mind, true/false, virtual/rel; they all turn out to be morphogrammatically the same.
In logic, negation is fundamental. Dualities are ruled by negations. Morphogrammatics is conceived as a *negational-invariant* formalism. Nevertheless, laws of reflections, similar to the duality principle, can be studied in extenso.

In logic, reflection of a morphogram of logical operations (connectives) is interpreted as *dualization*.

Duality in logic

Duality of conjunction and disjunction: $(p \wedge q) = \neg(\neg p \vee \neg q)$

Morphogram of:

Negation: $\text{morph}(\neg p) = \text{morph}(p) = [ab]$

Conjunction: $\text{morph}(p \wedge q) = [abbb]$

Disjunction: $\text{morph}(p \vee q) = [aaab]$

Morphogrammatic reflection vs. logical duality:

$\text{refl}([abbb]) = [aaab]$,

$\text{refl}([aaab]) = [abbb]$,

$\text{compl}([aaab]) = \text{compl}([abbb]) =_{proto} \{1^1, 2^3\}$.

Duality in general:

$\text{dual}(\text{dual}(X)) = X$

$\text{dual}(X \oplus Y) = \text{dual}(Y) \oplus \text{dual}(X)$.

Morphogrammatic reflection

$\text{refl}(\text{refl}(MG)) = MG$

$\text{refl}(MG_1 \otimes MG_2) = \text{refl}(MG_2) \otimes \text{refl}(MG_1)$.

Example

$\text{refl}(MG_1^1 \otimes MG_2^1) = \text{refl}(MG_2^2) \otimes \text{refl}(MG_1^2)$ with

$MG_1^1 =_{MG} MG_1^2 \wedge MG_1^1 \neq_{sem} MG_1^2 \Longrightarrow [a] =_{KG} [b]$

$MG_2^1 =_{MG} MG_2^2 \wedge MG_2^1 \neq_{sem} MG_2^2 \Longrightarrow [abb] =_{KG} [bcc]$

$\text{refl}([a] + [abb]) = \text{refl}([bcc]) + \text{refl}([b])$

$(1): (1): \text{refl}([a] + [abb]) = \text{refl}(\{[aabb], [abaa], [abcc]\} = \{[bbaa], [aaba], [ccba]\}$

$= \{[aabb], [aaba], [aabc]\}$.

$(2): \text{refl}([bcc]) + \text{refl}([b]) = [ccb] + [b] = \{[ccbc], [ccbb], [ccbd]\}$

$= \{[aaba], [aabb], [aabc]\}$.

$\Longrightarrow (1) =_{MG} (2)$ •

Semiotic example

$\text{inv}([a] + [abb]) = \text{inv}([abb]) + \text{inv}([a])$

$(1): \text{inv}([a] + [abb]) = \text{inv}(aabb) = bbaa$

$(2): \text{inv}([abb]) + \text{refl}([a]) = bba + a = bbaa$

$\Longrightarrow (1) =_{sem} (2)$ •

4.2. Reflection of compound morphograms

Reflection happens to the whole morphogram or to its parts, i.e. to single or multiple monomorphies of the morphogram or to different morphograms of a compound of morphograms.
Again, this topic is best covered by “Morphogrammatik".

4.2.1. Morphogrammatic compounds

Compounds of morphograms are build by the chaining composition for morphograms.

$$MG^{(3)} = \left(MG_1, MG_2, MG_3\right) \xrightarrow{\text{composition}} \left[MG_1, MG_2, MG_3\right]$$

4.2.2. Reflection of morphogrammatic compounds

Reflection[6] of morphograms in compounds happens as reflections of single or multiple composed morphograms.
The tables are showing the examples for a combination of 3 morphograms of length 4. Reflection happens to the single morphograms by the reflectors r_1, r_2 and r_3 and the composed reflectors $r_{1.2}$, $r_{1.3}$, $r_{2.3}$ and $r_{1.2.3}$.
The example shows the *scheme* of a morphogrammatic compound $Q^{(3)}$ independently of its morphogrammatic interpretation by concrete morphograms.
The composition conditions are given by the main diagonal of the composition where the morphograms are connected (chained) together:

$$\begin{array}{|l|} \hline S_1 \cap S_3 = (1,1) \\ S_1 \cap S_2 = (2,2) \\ S_2 \cap S3 = (3,3) \\ \hline \end{array} \quad \text{with subsystems} \quad \begin{array}{l} S_1 = (1, 2) \\ S_2 = (2, 3) \\ S_3 = (1, 3) \end{array}$$

$r_1(Q^3)$	1	2	3
1	●	●	○
2	●	●	○
3	○	○	○

$r_2(Q^3)$	1	2	3
1	○	○	○
2	○	●	●
3	○	●	●

$r_3(Q^3)$	1	2	3
1	●	○	●
2	○	○	○
3	●	○	●

$r_{1.2}(Q^3)$	1	2	3
1	●	●	○
2	●	●	●
3	○	●	●

$r_{1.3}(Q^3)$	1	2	3
1	●	●	●
2	●	●	○
3	●	○	●

$r_{2.3}(Q^3)$	1	2	3
1	●	○	●
2	○	●	●
3	●	●	●

$r_{1.2.3}(Q^3)$	1	2	3
1	●	●	●
2	●	●	●
3	●	●	●

5. Natural numbers in morphogrammatics

5.1. Natural numbers, de-mystified

Now, what happened to our natural numbers?
Without surprise, natural numbers[7], after having lost their innocence of naturality and their hegemony of being ultimate and their dignity of uniqueness, natural numbers are re-appearing again, multiplied, distributed, cloned [8][9] and mediated over multiple kenomic loci, not loosing anything of their grandeur of pragmatic numeric relevancy. They are now embedded into a grid that enables number systems to have neighbors, to interact with other number systems and to reflect on such actions. And they might intervene into each others axiomatics, changing the rules of the game while playing it.

5.2. Natural numbers, disseminated

The tabular successiveness structure of morphogrammatic systems of change are offering space enough to distribute full natural number systems over multiple locations of the kenomic grid.

5.2.1. Numeric evolution

The most obvious numeric interpretation of evolutionary morphogrammatics is given by the observation of purely iterative and purely accretive evolutions or repetitions.
Hence, for a morphogram $[1^1, 2^n, ..., m^n]$ an iterative system like $([1^1], evol_{iter})$ with $evol_{iter}[1^n] = [1^{n+1}]$ and an accretive system like $([m^1], evol_{acc})$ with $evol_{acc}[m^1] = [(m+1)^1]$ are both representing the successiveness structure of natural numbers. The same holds for all intermediary morphograms as beginnings.

1: $[1^1]$
2: $[1^2][1^1 2^1]$
3: $[1^3][1^2 2^1][1^1 2^1 1^1][1^1 2^1][1^1 2^1 3^1]$
4: $[1^4][1^3 2^1][1^2 2^1 1^1][1^1 2^1 1^2][1^1 2^3][1^2 2^2][1^1 2^1 1^1 2^1][1^1 2^2 1^1][1^2 2^1 1^1 3^1][1^2 2^1 3^1][1^1 2^1 1^1 3^1][1^1 2^1 3^1 1^1][1^1 2^1 3^1 2^1][1^1 2^1 3^2][1^1 2^1 3^1 4$

5.2.2. Numeric interactivity

Interaction between different distributed number systems is guided by the emanation rules between different morphograms of the same complexity.

$\boxed{3}: [1^3] \leftrightarrow [1^2 2^1] \leftrightarrow [1^1 2^1 1^1] \leftrightarrow [1^1 2^1] \leftrightarrow [1^1 2^1 3^1]$

Hence, there is a interactional mediation between the strict "*cardinal*" numbers $[1^m]$ and the strict "*ordinal*" numbers $[1^1, 2^1, ..., m^1]$ and also between intermediary number systems.

5.2.3. Morphogrammatics of arithmetic

There are many aspects under this rubric of *"Morphogrammatics of arithmetics"* to report and to study. At first, we can learn that morphogrammatics is dealing "arithmetically" not with arithmetical numbers but with arithmetical number *systems* as such. Hence, morphogrammatics offers a kind of an *"arithmetics of arithmetic" or* an arithmetics of natural number systems.
In contrast to existing alternative number systems, like fuzzy-number systems, number systems based on multiple-valued logic and set theory, etc., morphogrammatically disseminated number systems are not reducible, back to the conformity of classical natural number systems.
On the other hand, those alternative or deviant systems are easily disseminated over the kenomic grid keeping and mixing their deviancy to interesting interplays of interactions.

5.2.4. Diamond structure of the "arithmetics of arithmetic"

The diamond categorical approach to arithmetics is reflecting on the double character of any operation.

A first step towards a categorical construction might be sketched:

$$\left.\begin{array}{ccc} (8)_1 & \longrightarrow & (5+3)_1 \\ \Updownarrow & x & \Updownarrow \\ (8)_2 & \longleftarrow & (5+3)_2 \end{array}\right\} (8)_1 =_{Arith} (8)_2$$

Arithmetically, the relations $(5+3)_1 \longrightarrow (8)_1$ and $(5+3)_2 \longrightarrow (8)_2$ are well obvious because their relata are all belonging to the same arithmetical systems A_1 and A_2. The situation is getting slightly more intriguing if the relations are belonging to two different arithmetical systems, A_1 an A_2, with $A_1 \cap A_2 = \emptyset$. Hence the relations between $(5+3)_1 \longrightarrow (8)_2$ and $(5+3)_2 \longrightarrow (8)_1$ are of special interest.

The relations (or morphisms) $(5+3)_1 \longrightarrow (8)_2$ and $(5+3)_2 \longrightarrow (8)_1$ can be seen as *translational* morphisms between two *discontextural* arithmetical systems A_1 an A_2. Hence, a possibility of a *comparison* between $(8)_1$and $(8)_2$ is established, which is demanding its own third contexture to take place.
This little example is of interest independently of the numeric values used and the definition of their axiomatics.

Notes

1 § 1673
The identity with the other individual, the individual's universality, is thus as yet only internal or subjective; it therefore has the longing to posit this and to realise itself as a universal. But this urge of the genus can realise itself only by sublating the single individualities which are still particular relatively to one another.
In that first instance, in so far as it is these latter which, in themselves, universal, satisfy the tension of their longing and dissolve themselves into the universality of their genus, their realised end identity is the negative unity of the genus that is reflected into itself out of its **disremption**.
Hegel, Sience of Logic, Life
http://www.marxists.org/reference/archive/hegel/works/hl/hl764.htm

2 http://www.thinkartlab.com/pkl/pcl-lab.htm

3 www.thinkartlab.com/pkl/media/Web_Mobility/Web_Mobility.html

4 http://www.thinkartlab.com/pkl/tm/MG-Buch.pdf

5 http://www.thinkartlab.com/pkl/lola/poly-Lambda_Calculus.pdf

6 http://vordenker.de/ggphilosophy/gg_cyb_ontology.pdf

7 http://comet.lehman.cuny.edu/fitting/bookspapers/pdf/unpubbooks/NumbersBook.pdf

8 http://www.thinkartlab.com/pkl/media/DERRIDA/Cloning%20the%20Natural.html

9 www.thinkartlab.com/pkl/media/DERRIDA'S%20MACHINES.pdf