SEMIOTIC ABSTRACTIONS
IN THE THEORIES OF GOTTHARD GÜNTHER AND GEORGE SPENCER BROWN

By Rudolf Matzka, Munich, May 1993

Abstract

Three semiotic innovations, consisting in different abstractions from the standard notion of "string", are extracted from the works of the logicians Gotthard Günther and George Spencer Brown. These semiotic innovations are formally reconstructed in the context of classical string theory.

INTRODUCTION

Gotthard Günther's theory of "Polycontextural Logic"[1]) and George Spencer Brown's "Laws of Form"[2]) are both fairly revolutionary deviations from classical logic, and they are both very little acknowledged by mainstream logicians. That seems to be all they have in common: Gotthard Günther attempts to reorganize the whole of logic as a distributed network of logical systems, and George Spencer Brown attempts to generate the whole of logic as an unfoldment of the single concept of distinction.

By closer inspection one can find that the two theories have at least one more feature in common: they depart from classical logic not just on the level of logical rules, and not just on the level of grammatical rules, but on the level of semiotic rules. That is, neither "Polycontextural Logic" nor "Laws of Form" can be adequately represented within the constraints of the classical concept of "string". As far as I know, there are no other approaches to non-standard-logics for which a deviation from standard semiotics would play an essential part. This semiotic dimension of innovation appears to be what makes these two theories so revolutionary, and it may well be a reason why they are so hard to accept for the mainstream.

In Laws of Form, there is a special semiotic atom, called the "Cross", which can be combined with other terms in two modes: by concatenation or by enclosure. This is an obvious deviation from standard semiotics, where concatenation is the only mode of combination. Combination by enclosure is also the basis for the "reentrant forms", another semiotic innovation. A third deviation from classical semiotics is less obvious: the commutativity of the concatenation operation. For any two terms "a" and "b" the terms "ab" and "ba" are identical. That this is indeed a semiotic identity (and not just a logical equality) has been stressed by Varga[3]). In this paper we focus on this last de-

---

[1]) Gotthard Günther, Beiträge zur Grundlegung einer operationsfähigen Dialektik, Vol. 1-3, Hamburg 1976,1979,1980

[2]) G. Spencer Brown, Laws of Form (London 1969)

[3]) Varga suggested the term "Topologically Invariant Notation" for this property of Brownian semiotics. Varga von Kibed: Wittgenstein und Spencer Brown; Philosophy of the Natural Sciences, Proc. 13th Int. Wittgenstein Symposium 1988, Wien 1989

viation from standard semiotics, because it bears a close relationship to some of Gotthard Günther's work.

In Gotthard Günther's texts, the semiotic innovations come under the heading "Kenogrammatik". Upon analyzing the value sequences of the two-place operations of propositional calculus, Günther discovered a way to abstract from the identity of the logical values while retaining the structural patterns of the value sequences. He termed these abstract 4-place patterns "morphograms". Later he generalized them to be of arbitrary length and termed them "kenograms". The kenograms are then utilized by Günther in various ways, on which we cannot elaborate in this paper.[4]) Essentially, the kenograms are (claimed to be) a basis for the possibility to transcend Leibniz's "principle of identity". Günther considers the principle of identity to be the very core of classical logic - more than the axioms of forbidden contradiction and of the excluded third. And he considers the principle of identity to be incompatible with a truly distributed logic, which his "Polycontextural Logic" is supposed to be.

Although Günther himself did not use the term "Semiotics" in this context, and did not explicitly relate his concept of "kenogram" to the concept of "string", we shall see below that the whole issue becomes much clearer if we seperate it from the logical context in which it was discovered and reconsider it in the more fundamental context of string theory. By doing this, we will also find a systematic connection between the "Kenogrammatik" and the commutative concatenation found in George Spencer Browns "Laws of Form".

STANDARD SEMIOTICS: STRINGS OF ATOMS

As a framework for the analysis to follow we refer to the axiomatic theory of strings, which has been founded by Tarski and Hermes[5]). The latter suggested to give the name "Semiotics" to this theory. This theory is parametrized by a natural number n, representing the cardinality of the given alphabet. For each given n the theory is categoric. The axiomatic theory of strings can be interpreted in several ways[6]), and in the

---

[4]) An overview of Günther's work can be found in Joseph Ditterich, Rudolf Kaehr: Einübung in eine andere Lektüre. Diagramm einer Rekonstruktion der Güntherschen Theorie der Negativsprachen. Philosophisches Jahrbuch, Karl Alber Verlag Freiburg/München, 1979

[5]) Alfred Tarski: The Concept of tuth in formalized languages (1934), Logic, semantics and metamathematics, Oxford 1956; Hans Hermes: Semiotik, Eine Theorie der Zeichengestalten als Grundlage für Untersuchungen von formalizierten Sprachen, Forschungen zur Logik und zur Grundlage der exakten Wissenschaften, n.s. no. 5, Leipzig 1938. A metatheoretic analysis of these two axiom systems can be found in Corcoran, Frank and Maloney: String Theory; J. Symbolic Logic, 39/4, Dec.1974

[6]) These interpretations can be found in Schröter, Ein allgemeiner Kalkülbegriff, Forschungen zur Logik und zur Grundlegung der exakten Wissenschaften, NF6, Hildesheim 1970

analysis to follow, we will use two of those interpretations, instead of the axiom system itself:

- the interpretation by means of a set theoretic model
- the "naive" interpretation by means of our everyday experience of using strings

The set theoretic model essentially consists of a set A and a set A^*, which are connected by the relation

$$A^* = \bigcup_{n=1}^{\infty} A^n$$

In the naive interpretation, the set A corresponds to the collection of all types of atoms of a given alphabet, and the set A^* corresponds to the collection of all types of strings which can be built from these atoms.

For a discussion of the structure of the token-type-relationship, the naive interpretation is more adequate. In the context of the naive interpretation, we can ask, for instance, under which circumstances we consider two given tokens x,y of strings as equal (i.e., as of the same type). The answer to this question, obviously, is the following algorithm:

(A) If the two given tokens of strings have different lengths, then they are different. If they have equal lengths, then go to (B).

(B) For each position i from 1 to the common length, check whether the atom at the i-th position of x equals the atom at the i-th position of y. If this is true for all positions i, then the given tokens are equal, otherwise they are different.

In this algorithm, as well as in the rest of this paper, we presuppose that it is known how to compare given tokens of atoms. The algorithm then determines our common notion of "string-type". Below we will modify part (B) of this algorithm, and will thereby generate non-standard variants of the notion of string-type.

ABSTRACTION FROM THE ORDER OF ATOMS: HEAPS OF ATOMS

As we noticed in the introduction, in the context of "Laws of Form" for any two terms "a" and "b" the concatenation results "ab" and "ba" are semiotically identical. If we start from our standard notion of string, we can introduce this property by building equivalence classes in the set A^* of all strings. We call two elements $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n) \in A^*$ equivalent, in short notation

(1) $x \approx_1 y$

if and only if there is a permutatation $f : \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$ such that

(2) $y_i = x_{f(i)}, \;\; i = 1, \ldots, n$

We could now say that a "string" in the Brownian sense is an equivalence class of A^* with respect to $\approx_1$, i.e., an element of the quotient set $A^* / \approx_1$.

But there is a more natural way to introduce the same property. What does it mean to say that "ab" and "ba" are semiotically identical? It means that they are but two tokens for the same type of string. This can only be true if, in the context of Laws of Form, a modified version of the token-type-relationship is valid. We make this modification explicit and substitute part (B) of the algorithm for comparing string-tokens by the following

(B') Check whether each atom appears equally often in both string-tokens. If this is the case, then they are equal, otherwise they are different.

Here we have enlarged the abstractive distance between string-token and string-type, by including the abstraction from the order of the atoms into the abstraction from token to type. Thereby we generate a non-standard notion of string, without having to refer to the standard notion of string. Since within these "strings" the order of occurence of the atoms is irrelevant, we may as well call them "heaps of atoms", to distinguish them from ordinary strings.

ABSTRACTION FROM THE IDENTITY OF ATOMS: STRINGS OF KENOMS

Let (T,F,F,F) and (F,T,T,T) be the value sequences of, say, the "and" and the "not and" operator, respectively. Both sequences have a common pattern, for which Günther introduced a notation like "(*,+,+,+)". The intention behind this notation was that the new signs "*" and "+" do not design values, but only indicate - in the context of the sequence - whether or not they are equal to each other.

Obviously, this procedure can be applied to strings in general, and it can be formally reconstructed by building equivalence classes on the set A^* of all strings. The equivalence relation needed for this reconstruction would be defined for $x = (x_1, ..., x_n)$, $y = (y_1, ..., y_n) \in A^*$ by saying that

(3) $x \approx_2 y$

if and only if

(4) $x_i = x_k \Leftrightarrow y_i = y_k,\ i,k = 1,...,n,\ i < k$

We could now say that a "kenogram" is an equivalence class of A^* with respect to $\approx_2$, i.e. an element of the quotient set $A^* / \approx_2$.

Again, there is a more natural way to introduce the kenograms, by including the equivalence relation into the abstraction from string-token to string-type. This time, we substitute part (B) or (B') of our algorithm for string-token comparison by a procedure which incorporates condition (4):

(B'') For each pair i,k, i<k, of positions, check whether within x there is equality between position i and k, and check whether wihin y there is equality between position i and k. If within both x and y there is equality, or if within both x and y there is inequality, then state equality for this pair of positions, otherwise state inequality for this pair of positions. If for each pair of positions there is equality, then x and y are equal. Otherwise they are not.

The connection between the two equivalence relations $\approx_1$ and $\approx_2$ can be seen more clearly by observing that condition (4) is tantamount to the condition that there exists a permutation $p: A \to A$ such that

(5) $$y_i = p(x_i),\ i = 1,\ldots,n$$

Comparing conditions (2) and (5), we can see that there is some symmetry in the relationship between G. S. Brown's commutative strings and G. Günther's kenograms: The former are invariant w.r.t. permutations of the index set {1,...,n}, while the latter are invariant w.r.t. permutations of the alphabet A.

Obviously, a kenogram is composed of some sort of "atoms", in the sense of indivisible parts, but those "atoms" have no identity as types. In fact, if we ask how many "atoms" there are, and if we equate "atom" with "kenogram of length one", then the answer is that there is one and only one "atom". The concept of an alphabet, as a set of two or more types of atoms, becomes obsolete in the context of kenograms. Because of this very strange property of the kenogrammatic "atoms", we term them "kenoms", so that the kenograms can be called "strings of kenoms".

ABSTRACTION FROM THE ORDER AND IDENTITY OF ATOMS: HEAPS OF KENOMS

In his development of the theory of kenograms, Gotthard Günther introduced three layers of abstraction, and called them "Trito Structure", "Deutero Structure" and "Proto Structure". The Trito Structure coincides with what we have called "strings of kenoms". The Deutero Structure was derived from Trito Structure by abstracting from the order in which the kenoms occur. The Proto Structure was derived from Deutero Structure by excluding patterns in which more than one atom occur repeatedly.

We shall not go into a discussion of the Proto Structure, but the Deutero Structure is interesting since it shows that the idea of abstracting from the order of atoms was present in Günther's work as well as in Brown's Laws of Form. Günther saw the possibility to abstract from the order of atoms only after he had abstracted from their identity, and didn't see it (or wasn't interested in it) as a possible abstraction from ordinary strings.

We can reconstruct the combined abstraction from order and identity quite easily in set theoretic terms, by defining the equicalence relation "$\approx_3$" on A^*as the product of the relations "$\approx_1$" and "$\approx_2$". This means that two elements $x, y \in A^*$are equivalent in the sense of "$\approx_3$" if and only if there are a permutation f of the index set and a permutation p of the alphabet such that

(6) $$y_i = p(x_{f(i)}),\ i = 1,\ldots,n$$

Again we can include the equivalence relation into the rule for comparing string-tokens, by using the algorithm

(B''') Take an atom a from x, find out the number k of atoms in x equal to a, and check whether in y there is an atom which occurs exactly k times. If not, then x and y are unequal. If yes, then remove the atoms just considered from x and y. If nothing is left, x and y are equal. Otherwise apply B''' to the remaining string-tokens.

According to the terminology introduced earlier, this third type of non-standard-string can be called a "heap of kenoms".

CONCLUDING REMARKS

As a result of the above analysis, we have identified three non-standard variants of classical semiotics. Each of them is defined by implementing an additional abstraction into the token-type-relationship for strings, and each of them defines a new type of semiotic object. Abstraction from the order of atoms leads to heaps of atoms, abstraction from the identity of atoms leads to strings of kenoms, and abstraction from both order and identity of atoms leads to heaps of kenoms. These non-standard semiotic objects have been extracted from the works of G. Günther and G. S. Brown, but I believe that they should also be checked for usefulness independently of the theories of the two authors.

If we look at these new semiotic objects from a mathematical point of view, we find that

- heaps of atoms are stucturally very similar to multisets
- heaps of kenoms are structurally very similar to number theoretic partitions

while strings of kenoms do not - as far as I know - resemble any well-known mathematical structure.

From a semiotic point of view, these new semiotic objects have to be viewed as notational raw material, which can be used to build languages, calculi, theories, etc., just as ordinary strings are the raw material for all our classical languages, calculi, etc. Since the new semiotic material is structered differently to the standard semiotic material, those new languages etc. might be quite different from the classical ones. For the time being, we have very little intuitions about how this could be done, and for which purposes.[7])

Imagine for example we would actually possess a language based on kenoms instead of atoms. Then the process of writing in such a language would put the writer into a sequence of decision making situations, the dynamics of which would be very different from the classical process of writing. In clasical writing, providing "the next letter" involves a repetitive choice of one out of a given and constant number of atoms. In keno-writing, the number of kenoms to choose from as "the next letter" would be a dynamically changing variable: it equals the number of different kenoms used so far, plus one - because the next letter could always be a new one.

---

[7]) Gotthard Günther had the vision of a "Negative Language", although he did not intend to give a precise definition of such a language. Cf. Gotthard Günther, "Martin Heidegger und die Weltgeschichte des Nichts", Beiträge zur Grundlegung einer operationsfähigen Dialektik, Vol. 3, p. 260-296, Hamburg 1980