Metaphors of Dissemination and Interaction of morphoCAs

Functional Analysis of the Graphematics of morphoCAs

Dr. phil Rudolf Kaehr

ISSN 2041-4358
(work in progress, v. 0.5.5, Dec., Oct. 2015)

Diagrams and Dissemination

Keywords

ECA, morphoCA, diagrams, reduction, minimization, flow charts, interaction, transaction, mediation, heterogeneous structures, poly-, dis-, intra-, trans-contexturality, contextuality, morphogrammatics, retrograde recursion, memristivity

Motivation

"Physical computing media are asymmetric.
Their symmetry is broken by irregularities, physical boundaries, external connections and so on. Such peculiarities, however, are highly variable and extraneous to the fundamental nature of the media. Thus, they are pruned *from theoretical models, such as* cellular automata*, and reliance on them is* frowned *upon in programming practice.*
However, computation, like many other highly organized activities, is incompatible with perfect symmetry. Some standard mechanisms must assure breaking the symmetry inherent in idealized computing models."
Leonid A. Levin, The Computer Journal Vol. 48, No. 6, 2005

Minimization and flow charts

It is not easy to explain how to understand the results of morphoCAs. It seems that there is a strong conflict between the millions of *visualizations*, *sonifications* and *structurations* managed by the approach of claviatures and the paradigmatic statement developed in the paper *"Asymmetric Palindromes"* for morphoCAs that *"What you see is not what it is".*

Instead of studying the multitude of the products of morphoCAs, another approach that is more focused on the *mechanism* of the production process of morphoCAs might help to uncover the deep-structural significance of *morphogrammatic* based cellular automata.

This paper offers some insights into the mechanism of production by the application of *reductions* (minimizations) of the functional interpretations of the morphoCA rules and by designing the *network* of the actions of the morphic automata by some *flow charts.*

There is not yet an algorthmic approach to reduce morphic CA functions accessible. But the distinction between reducible and non-reducible morphoCAs is well defined.

Hence, instead of considering the multi-millions of morphoCA productions, some specific *flow charts* of the mechanism of production is presented to continue the studies of morphoCAs. With that a kind of reflection a kind of a meta-theory of morphoCAs is introduced.

From this meta-theoretial point of view, morphoCAs might be involved into an introspection between Kaluzhnine-Graph-Schemata of recursivity and poly-contextural memristivity.

A further approach to study the deep-structure of the meaning of morphoCAs will be sketched in a further paper by an analysis of their underlying *poly-contextural logics.*

Contexturality vs. contextuality

The term *"polycontexturality"* occurs frequently in sociological studies. Often as a synonyme or replacement of '*polycentricity*' and linguistically, modal-logically or semiotically identified with '*contextuality*'.

Polycontexturality refers to a trans-classical paradigm of thinkind and writing that is not compatible with established concepts of science, while '*polycentricity*' and '*contextuality*' are parts of classical logic (say, modal logic), ontology and semiotics.

> *"Polycentricity is similar to the concept of polycontexturality in logic. Polycontexturality represents a many-system logic, in which the classical logic systems (called contextures) interplay with each other, resulting in a complexity that is structurally different from the sum of its components (Kaehr and Mahler 1996)."*
> (Rajendra Singh, Towards Information Polycentricity Theory: Investigation of a Hospital Revenue Cycle, 2011)

A similar approach, chosen out of the 'polycentricity or centextualist movement', is proposed e.g. by Lars Qvortrup:

> *"The implicit idea behind the first three theses is that we are on our way into a society, which is radically different from the so-called modern society. It has been described as* "functionally differentiated" *(Luhmann 1997), as* "polycontextural" *(Günther 1979) or as* "hypercomplex" *(Qvortrup 1998), emphasising that it does not offer one single point of observation, but a number of mutually competing observation points with each their own social context."*
> Lars Qvortrup, THE AESTHETICS OF INTERFERENCE: From anthropocentrism to polycentrism and the reflections of digital art
>
> http://www.hotelproforma.dk/Userfiles/File/artikler/lq.pdf

It might provoke some progress if the distinctions proposed in this paper would be applied to systems theory of intra-, inter- and trans-contexturally mediated complex dis-contextural constellations and dynamics.

http : // memristors.memristics.com / Morphospheres / Asymmetric %20 Palindromes.html

http : // scholarworks.gsu.edu / cgi / viewcontent.cgi ? article = 1003 & context = ceprin_diss

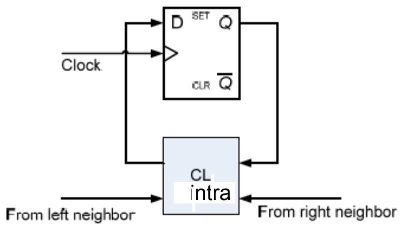
More entertainment with intra-, inter- and trans-disciplinarity of inter-, poly-, trans- and dis-contexturality at: *"Modular Bolognese, Paradoxes of postmodern education"* in: Short Studies 2008. Adventures in Diamond Strategies of Change(s)

http : // works.bepress.com / cgi / viewcontent.cgi ? article = 1007 & context = thinkartlab

Three kinds of morphoCA diagrams

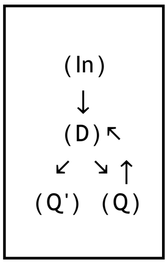
Three kinds of morphoCA diagrams have to be distinguished:

1. Mono-contextural diagrams (intra)



This ‘technical’ diagram has a *meta-mathematical* representation in the graph schemata calculus for recursion. With this connection, all the meta-theoretical results about computability are ready to be applied.

Graph scheme for mathematical recursion



$E(n, a) = (o, n, a),$

$\Gamma(i, n, \omega) \equiv n = i$

$A(i, n, \omega')$

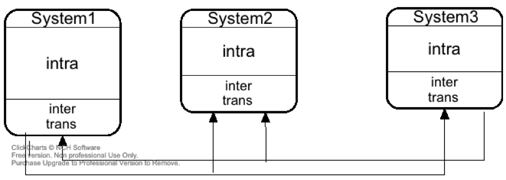
$\delta(i, n, \omega) = (i', n, \omega')$

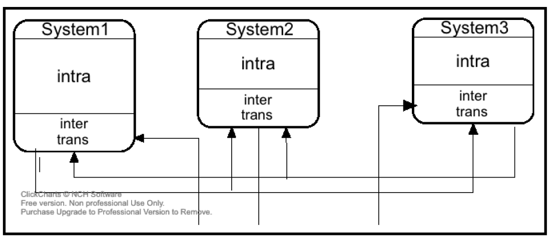
$m' = m + 1, \quad m \in \mathfrak{m}$

R. Peters, Dialectica 47/48, p. 375, 1958

The *first* kind is covered by the classical diagrams. These diagrams hold for classical ECAs as much as for mono-contextural morphoCAs of different topological complexity. Morphogrammatically, they are supported by the ‘classical’ morphograms of complexity 2.

2. Poly - contextural diagrams as interaction (inter, trans)



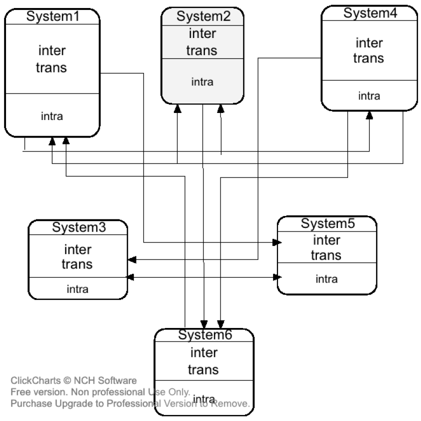


The *second* kind is based on a distribution of the diagram of at least 3 loci. This distribution is basic for the *interac-*

tions between otherwise autonomous automata. The internal structure of the memory/logic unit of the single automata is intrinsically changed toward a chiastic, i.e. memristive behavior of internal and external events.

The interactional activity of the second kind of diagrams is supported by the morphograms of complexity 3. In this field of interactional activity of complexity 3, two different modi might be distinguished:
• *inter-actional* with morphograms mg[5], mg[10] and mg[14], and head[{1,2,3}] -> i, i=1,2,3
• *trans-contextural* mg[11], mg[12] and mg[13] with head [{1,2}] -> 3.

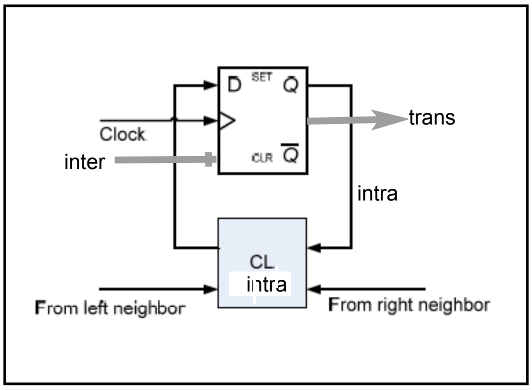
3. Poly - contextural diagrams as mediation



The *third* kind is based on the second kind but is involving the whole structural complexion of the distributed morpho-CAs. Only with this configuration the full graphematic character of morphoCAs enters the trans-classic game of computation, interaction, reflection and mediation.

Mediative actions are supported by morphograms of the minimal complexity 4, represented by the morphogram mg[15] with head[{1,2,3}]->4.

Poly - contextural basic component

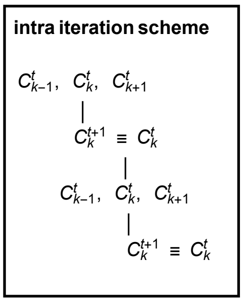


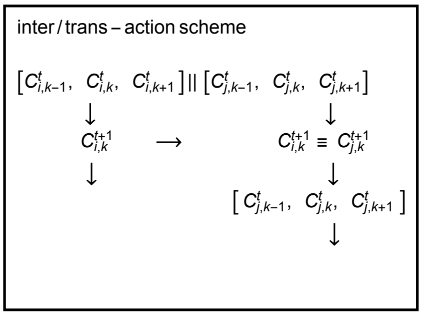
Examples

Functions

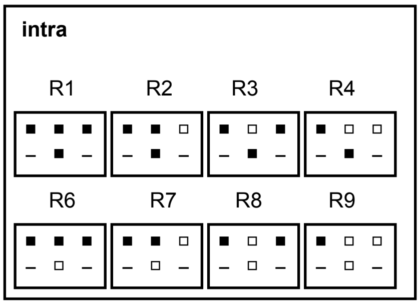
```
intra : {0, 0, 0} → 0 : sys1, 1, 1 → sys1, 1, 1
trans : {0, 0, 1} → 2 : sys1, 1, 1 → sys3 || sys1 || sys3
inter : {1, 2, 1} → 0 : sys2, 1, 2 → sys1 || sys3 || sys1
med   : {1, 0, 2} → 3 : sys1, 2, 3 → sys5 || sys4 || sys6
```

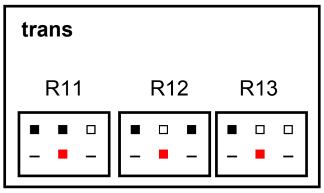
Action schemes

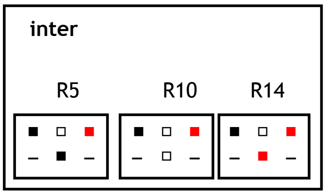


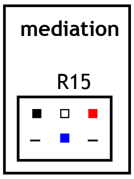


Morphograms

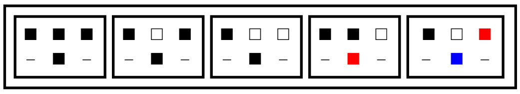








The compound morphogram **ruleDM[{1, 3, 4, 11, 15}]** inscribes the deep-structure of the *mediation* of *intra*- and *inter*-contextural actions.



Poly-contextural logic

Quite obviously, intra-contextural morphograms are representing the deep-structure of *junctional* mono- and poly-contextural operators.

As a first remark, inter- and trans-contextural morphograms are representing the deep-structures of *transjunctional* poly-contextural operators.

Morphogram mg[15] represents the full differentiations of the *interplay* of inter- and trans-contextural poly-contextural operators.

The proof-theoretical metaphor of polycontextural interplays is not anymore just a ‘tree’ but a *‘forrest of colored trees’.*

Example: ternary 3-contextural transjunctions of ruleDCM[{1, 2, 12, 13, 5}]

```
{0, 0, 0} → 0, {0, 0, 1} → 0 : dis - junctions in syst1,
{0, 0, 2} → 0 : junction in syst3,  "0∧(0∨2)≡ 0",
{2, 2, 0} → 2 : junction in syst3,  "2∧(2∨0)≡ 2",

{0, 1, 0} → 2, {1, 0, 0} → 2  : trans - junctions from syst1 to sys2 || sys3,

{1, 2, 1} → 0                 : trans - junctions  from syst2 to sys1 || sys3

{0, 2, 2} → 1, {2, 0, 0} → 1  : trans - junctions from syst3 to sys1 || sys2,

{0, 1, 2} → 0, {2, 1, 0} → 2  : trans - junctions from syst1, 3, 2 to sys1 || sys3
```

I. Mono-contextural diagrams: ECA and morphoCA$^{(m,2,n)}$

Diagram of the ECA scheme

K. Salman's paper *"Elementary Cellular Automata (ECA) Research platform"* gives an elaborated definition and explanation of the concept of ECAs.
For the purpose of an introduction of *morphoCAs* it suffice to connect to some of its terms and constructions.

> *"For ease of illustration we let the CA evolve according to one* uniform *neighborhood transition function and fixed radius which is a local function (rule)* $\mathcal{A}_0 : Q^{2r+1} \longrightarrow Q$ *where the CA evolves after a certain number of time steps T.*
> *In this case we have a total of* p^{2r+1} *distinct rules. It follows that a 1 - D CA is a linear lattice or register of* $\mathcal{K} \in \mathbb{N}$ *memory cells. Each cell is represented by* c_k^t *, where k = [1:* $\mathcal{K}$*],* $\mathcal{K} \in \mathbb{N}$ *and t = [1:* $\mathcal{T}$*],* $\mathcal{T} \in \mathbb{Z}$ *that describes the content of memory location at evolution time step* t.." (K. Salman)

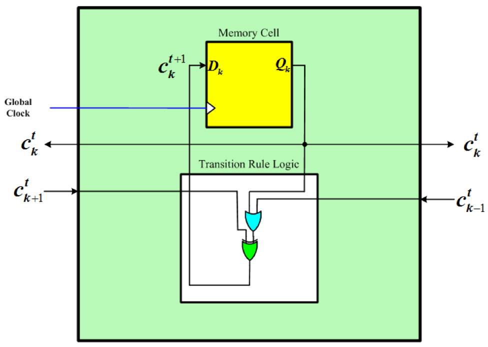
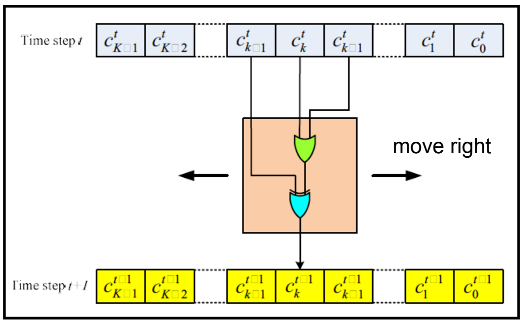


Figure 5, Detailed Structure of a typical Cellular Automaton Cell for rule 30.

http : // www.cyberjournals.com/Papers/Jun2013/02. pdf

Diagram of the CA rule in respect of input and output cells in time *t* to *t+1*



`http://www.slideshare.net/ijcsit/5413ijcsit03`

Description of the mechanism of the CA calculation

The object $\boldsymbol{D}_k$ of C_k^{t+1} of Fig. 5 is a result of the calculation of the logical unit U, i.e. *Transition Rule Logic*, in relation to its inputs C_{k+1}^t and C_{k-1}^t, but it is also at the same time the initial value, $\boldsymbol{Q}_k$, in the *Memory Cell*, of a new calculation of a next step of the CA.

This new calculation might happen *intra-contexturally* as a mapping from Q_k as Q^i to the logic unit U^i or *trans-contexturally* as a mapping from Q_k as Q^i to the new object D_k of D_k^{i+1} in $CA^{2.1}$where it becomes the new value of $Q_k^{i+1}$for a calculation in $CA^{2.2}$.

The presumption of the classical model of ECAs is certainly that all components are from the same contexture, and having the same clock.

Mono-contextural CAs are homogeneous structures.

Classical Cellular Automata. Homogeneous Structures
By V. Z. Aladjev

intra **iteration scheme**

$$\begin{array}{l}
[C_{k-1}^t,\ C_k^t,\ C_{k+1}^t \\
\qquad\quad \downarrow \\
\qquad C_k^{t+1} \equiv C_k^t \\
\qquad\qquad\quad \downarrow \\
\qquad C_{k-1}^t,\ C_k^t,\ C_{k+1}^t \\
\qquad\qquad\quad \downarrow \\
\qquad\qquad\quad C_k^{t+1} \equiv C_k^t
\end{array}$$

inter / trans interaction scheme

$$\begin{array}{ccc}
\left[C_{i,k-1}^t,\ C_{i,k}^t,\ C_{i,k+1}^t\right] & || & \left[C_{j,k-1}^t,\ C_{j,k}^t,\ C_{j,k+1}^t\right] \\
\downarrow & & \downarrow \\
C_{i,k}^{t+1} & \Longleftrightarrow_{\text{inter/trans}} & C_{j,k}^{t+1} \equiv C_{j,k}^t \\
 & & \downarrow \text{intra} \\
 & & \left[C_{j,k-1}^t,\ C_{j,k}^t,\ C_{j,k+1}^t\right]
\end{array}$$

inter / trans interaction network scheme

$$\begin{array}{ccccccc}
 & \left[C_{i,k-1}^t,\ C_{i,k}^t,\ C_{i,k+1}^t\right] & || & \left[C_{j,k-1}^t,\ C_{j,k}^t,\ C_{j,k+1}^t\right] & || & \left[C_{h,k-1}^t,\ C_{h,k}^t,\ C_{h,k+1}^t\right] & \\
 & \downarrow & & \downarrow & & \downarrow & \\
\Longleftrightarrow & C_{i,k}^{t+1} \equiv C_{i,k}^t & \Longleftrightarrow_{\text{inter/trans}} & C_{j,k}^{t+1} \equiv C_{j,k}^t & \Longleftrightarrow_{\text{inter/trans}} & C_{h,k}^{t+1} \equiv C_{h,k}^t & \Longleftrightarrow \\
 & \downarrow \text{intra} & & \downarrow & & \downarrow & \\
 & \left[C_{i,k-1}^t,\ C_{i,k}^t,\ C_{i,k+1}^t\right] & || & \left[C_{j,k-1}^t,\ C_{j,k}^t,\ C_{j,k+1}^t\right] & || & \left[C_{h,k-1}^t,\ C_{h,k}^t,\ C_{h,k+1}^t\right] &
\end{array}$$

Diagram of the sub-rule definition of ECAs

A sub-rule implementation of the ECA rules might augment its computational efficiency and reduce numeric complexity for programmable hybrid ECA compositions.

As it is well known, CAs are understood as parallel computing concepts and devices.

There is no doubt that the sketched sub-rule appoach can be concretized and implemented as a '*hybrid*' ECA on a hardware board like Spartan-6 FPGA Connectivity Kit or similar.
(http://www.xilinx.com/products/boards-and-kits.html)

A further step in augmenting the granularity of CAs might be achieved with the sub-rule approach for ECA rules. Each ECA rule is defined in a sub-rule oriented approach as a composition of sub-rules. Thus all compatible sub-rules can be applied in parallel to realize a single ECA rule.

Also the sub-rule approach is defining the ECA rules is not yet showing the flow chart of the interactions of the sub-rules to build the ECA rule.

ECA-rule = [eca1, eca2, ..., eca8]

Example: ECA rule 210

$$\boxed{\begin{matrix} C_{k-1}^{t}, & C_{k}^{t}, & C_{k+1}^{t} \\ & C_{k}^{t+1} & \end{matrix}} \; : \text{intra}$$

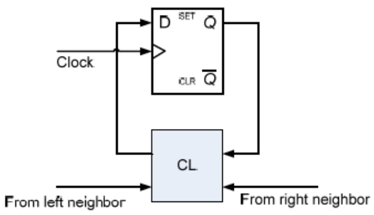
ruleECA[{6, 7, 3, 9, 10, 12, 13, 15}]

```
{{0, 0, 0} → 1,
 {0, 0, 1} → 1,
 {0, 1, 0} → 0,
 {0, 1, 1} → 1,
 {1, 0, 0} → 0,
 {1, 0, 1} → 0,
 {1, 1, 0} → 1,
 {1, 1, 1} → 0}
```

```
FromDigits[{1, 1, 0, 1, 0, 0, 1, 0}, 2]
```

```
210
```

Hence the ECA rule 210 is represented by the tuple (6,7,3,9,10,12,13,15) of ECA sub-rules.



Flow chart of the parallel realization of the 8 sub - rules of an ECA.

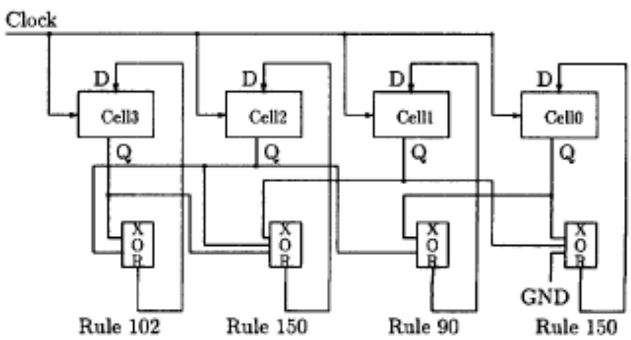


Fig. 1. A hybrid CA.

"If in a CA the same rule applies to all cells, then the CA is called a uniform *CA; otherwise the CA is called a* hybrid *CA (Fig. 1)."*

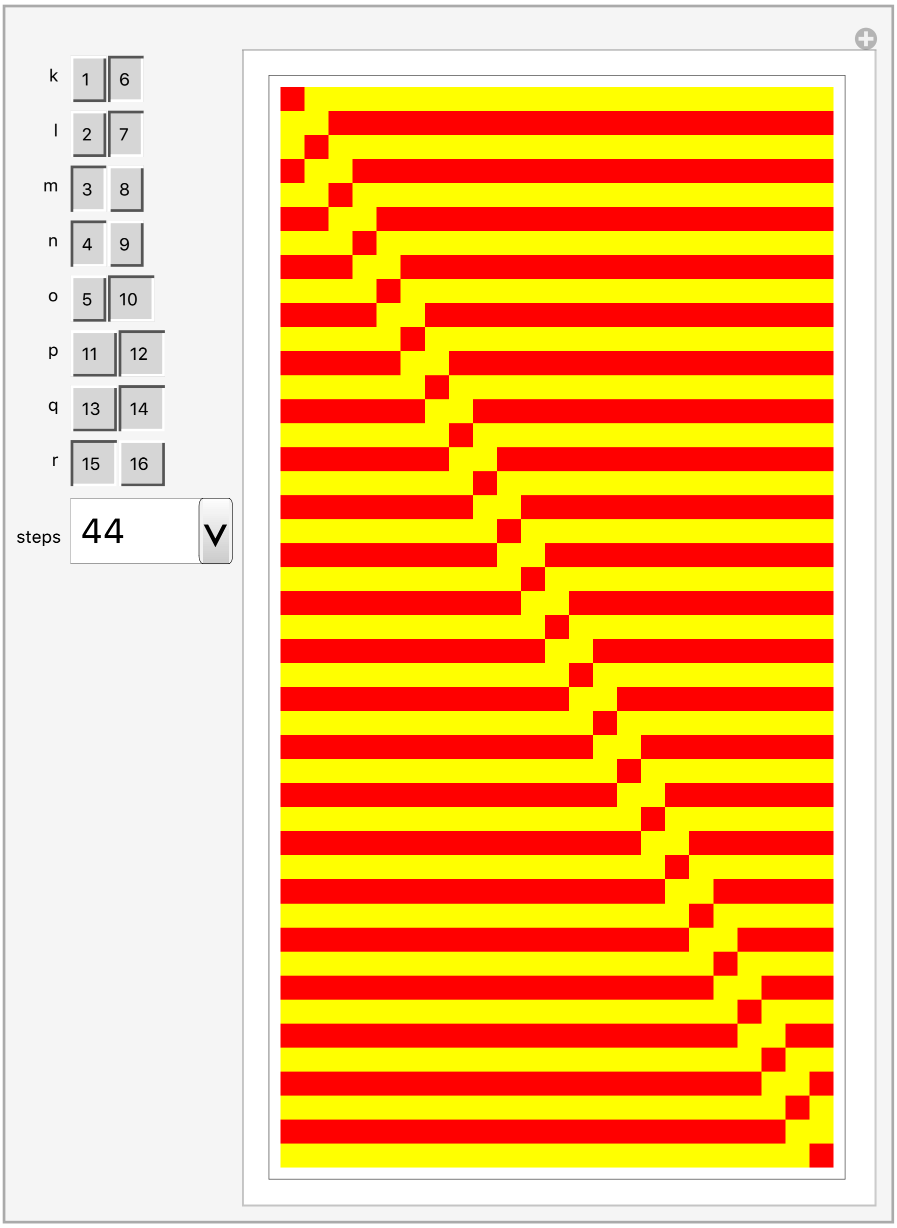
Theory and Applications of Cellular Automata in Cryptography
S.Nandi, B.K.Kar and P.Pal Chaudhuri

ECA sub-rule manipulators

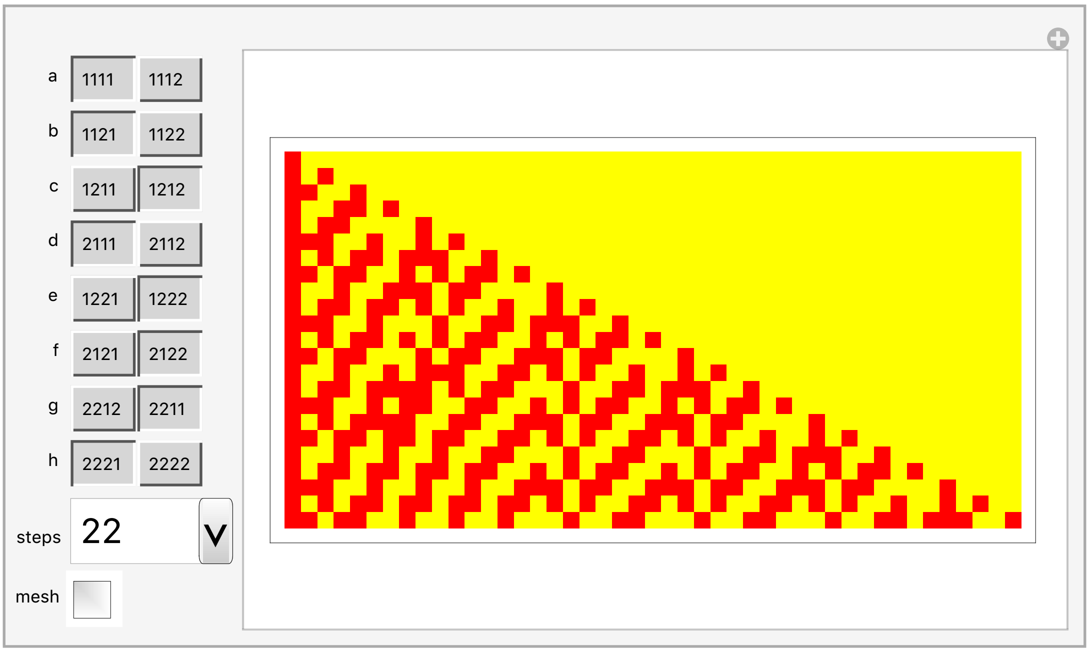
The method of sub-rules for ECAs is an *abstraction* and *parametrization* of the components of the rule schemes that allows a micro-anlysis of the ECAs. The ECA sub-rule manipulator manages all ECA rules of a 1D ECA. The sub-rule manipulators enables a micro-analysis of the behavior of all 2^8 ECA rules.

Each 1-D ECA rule number has a sub-rule number representation. There are just 8 disjunct pairs of sub-rules to define a 1D ECA rule.

The results are visualized below. The combination of the 8 sub-rules covers all the 256 well known ECA rules.

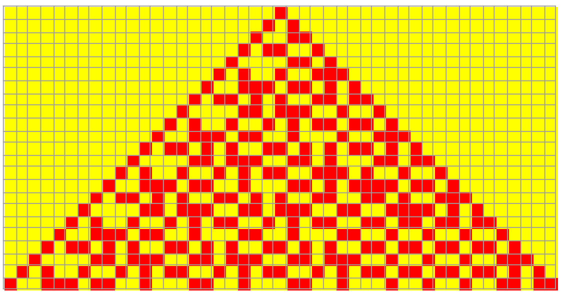


Mono-contextural ruleDCKF$^{(5,2,3)}$



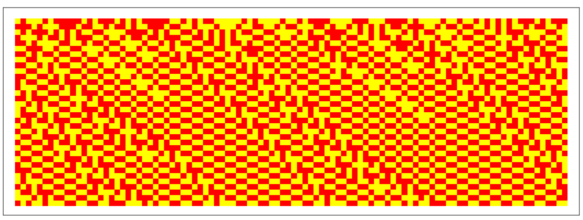
Reduction (trivial)

ruleDCKF[{**1111, 1122, 1211, 2112, 1221, 2121, 2211, 2222**}]

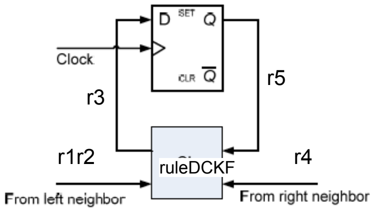


```
intra
{1, 1, 1, 1} → 1, {0, 0, 0, 0} → 0,  : Sys1
{1, 1, 1, 0} → 0, {0, 0, 0, 1} → 1,
{1, 1, 0, 1} → 1, {0, 0, 1, 0} → 0,
{1, 0, 1, 1} → 0, {0, 1, 0, 0} → 1,
{1, 1, 0, 0} → 1, {0, 0, 1, 1} → 0,
{1, 0, 1, 0} → 1, {0, 1, 0, 1} → 0,
{1, 0, 0, 1} → 1, {0, 1, 1, 0} → 0,
{1, 0, 0, 0} → 0, {0, 1, 1, 1} → 1
```

Random



Scheme: (r1, r2, r3, r4) $\Longrightarrow$ r5, r_i = {0, 1}



2. Poly-contextural diagrams with interactions

General approach

Internal structure of the memory unit of the second kind

Following for example K. Salman's classical modelling approach in *"Elementary Cellular Automata (ECA) Research platform"* a more explicit modelling of the mechanism of morphoCAs might be achieved.

A first crucial difference to the classical concept is the fact that the memory unit is not just passively receiving (D) and sending data (Q) but is also actively *deciding* to which system of its computational environment they belong and if the data remain in its domain or not. If not, the activity of the memory unit is deciding where that data belong and sends it to the evoked computational unit of the complexion.

In terms of *actors*, the memory unit is *receiving, sending* and *deciding* about the *contextures* of data. Classical memory actions are strictly intra-contextural. This holds in the same sense for multi-processor systems too. They are acting strictly intra-contexturally, keeping their distributed data together.

Hence, the logic devices in the modified diagram, Fig. 5b, have two function towards its memory units:
 1. a decision operation over the logical operations, i.e. to decide if an operation stays inside the contexture or if it leaves trans-contexturally the contexture for another contexture on another layer of the complex poly-layered morphoCA system.
 2. the intra-contextural function of producing the junctional values for the corresponding intra-contextural memory in the sense of ordinary logical functions, like NAND or NOR.

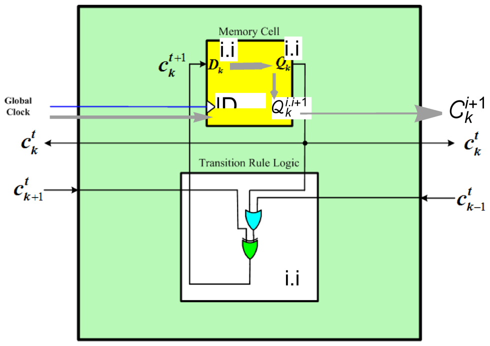
The object D_k of the CA diagram receives a value from the logic unit and it delivers it to Q as $Q_k$for the new calculation with c_k of the logical unit in time $t + 1$.
Secondly, Q receives the value from D as a value, not for $Q^{1.1}$ in CA^1 but for $Q^{2.1}$of the neighbor layer CA^2. This new value is memorized in the neighbor CA^2 as the new positive value for calculation in CA^2, hence it is placed in $CA^{2.1}$and not as a genuine value of CA^2 as $CA^{2.2}$.

The result of the application of the rule in all 3 sub-systems is delivered with the multi-layered system as a whole, i.e. with morpho$CA^{(3,3)}$ and its rules $\mathcal{R}^{(3,3)}$.

Obviously, the whole automaton with its different layers has to be designed in the epistemological mode of the '*as-abstraction'*, i.e. as *"A as B is C"* and not in the mode of identity with *"A is A"*.

> The modified diagram is introducing an *environment* to the original mono-contextural CA diagram that implies the possibility of interactions. The environment of a CA system is the primary condition for a possible self-reflection of the complex system of different and interacting CAs.
>
> The logic behind this construction was first introduced by Gotthard Gunther's *"Cognition and Volition"* (1970) which gives a profound explanation of the new concept of the '*proemial relation*'.

Modified diagram Fig. 5b



Memristive properties of the memory/logic unit

Why and how is the behavior of the memory units of morphoCAs of second-order and memristive and not just defined as first-order actions of storage and transformations? The main strategy of the whole maneuver is to avoid ‘information processing’. Interaction is prior to information exchange.

It could be said: morphoCAs without memristivity are reducible without loss to classical CAs.

internal :	external :
$D_k^{1.1} \Longrightarrow \begin{pmatrix} Q_k^{1.1} & \mathcal{D}_k^{2.2} \\ \downuparrows & \downuparrows \\ Q_k^{1.2} \Longrightarrow & Q_k^{2.1} \end{pmatrix}$	$U^{1.1} \to D_k^{1.1} \Longrightarrow \begin{pmatrix} Q_k^{1.1} & \mathcal{D}_k^{2.2} \longrightarrow Q_k^{2.2} : C_k^{2.1} \to U^{2.2} \\ \Updownarrow & \downuparrows \\ Q_k^{1.2} \Longrightarrow & Q_k^{2.1} \end{pmatrix}$

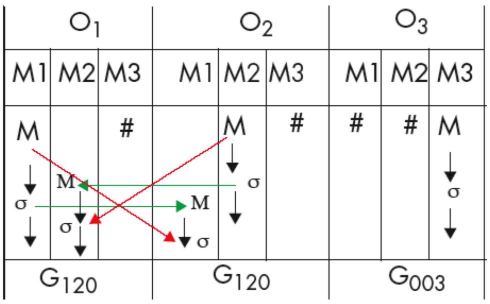
The diagram below, Fig. 1, shows again the *chiastic* interaction between operators (M) and operands ‘σ’ distributed over different loci of the kenomic matrix.

“M as σ” is obviously not the so called self-reference of “M is σ”.

$$X(M, \ \sigma) = \begin{pmatrix} M_{1.1} \Longrightarrow \sigma_{2.1} & M_{2.2} \Longrightarrow \sigma_{1.2} \\ & x \\ \sigma_{2.2} \Longrightarrow M_{1.2} & \sigma_{1.1} \Longrightarrow M_{2.1} \end{pmatrix}$$

```
http : //
 memristors.memristics.com / semi - Thue / Notes %20 on %20 semi - Thue %20 systems.html
```

Fig . 1 Chiasm (M, σ)



Explanation of Fig. 1

"The wording here is not only
" *types becomes terms and terms becomes types* " but
“*a type as a term becomes a term* " and, at the same time,
"*a type as type remains a type*".

Thus, "*a type as a term becomes a term and as a type it remains a type*".
And the same round for terms.

Full wording for a chiasm between terms and types over two loci

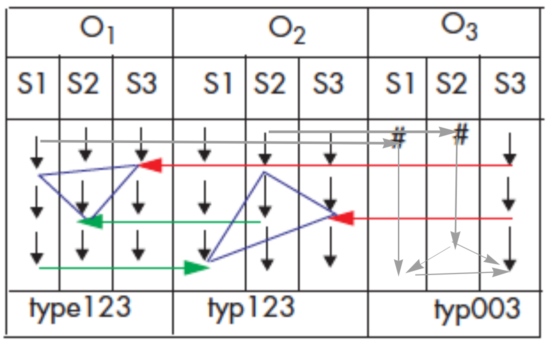
Explicitly, first the green round,
"A type $\boldsymbol{\sigma}_{1.1}$ as a term $M_{2.1}$ becomes a term $M_{2.1}$

and as a type $\boldsymbol{\sigma}_{1.1}$ it remains a type $\boldsymbol{\sigma}_{1.1}$ for a term $M_{1.1}$".
And,
"A type $\boldsymbol{\sigma}_{2.2}$ as a term $M_{1.2}$ becomes a term $M_{1.2}$
and as a type $\boldsymbol{\sigma}_{2.2}$ it remains a type $\boldsymbol{\sigma}_{2.2}$ for a term $M_{2.2}$".

And simultaneously, mediated,
the second round in red, the same for *terms*:
"A term $M_{1.1}$ as a type $\boldsymbol{\sigma}_{2.1}$ becomes a type $\boldsymbol{\sigma}_{2.1}$
and as a term $M_{1.1}$ it remains a term $M_{1.1}$ for a type $\boldsymbol{\sigma}_{1.1}$".
And,
"A term $M_{2.2}$ as a type $\boldsymbol{\sigma}_{1.2}$ becomes a type $\boldsymbol{\sigma}_{1.2}$
and as a term $M_{2.2}$ it remains a term $M_{2.2}$ for a type $\boldsymbol{\sigma}_{2.2}$".

And finally, between terms $M_{1.1}$ and $M_{2.2}$ and types $\boldsymbol{\sigma}_{1.1}$ and $\boldsymbol{\sigma}_{2.2}$,
a categorial *coincidence* is realized.
While between terms and types a *morphism* (order relation) exists.

Fig . 2 Complete interactional scheme



interaction scheme

$$
\begin{array}{ccccc}
\left[C^t_{i,k-1},\ C^t_{i,k},\ C^t_{i,k+1}\right] & \parallel & \left[C^t_{j,k-1},\ C^t_{j,k},\ C^t_{j,k+1}\right] & & \\
\downarrow & & \downarrow & & \\
C^{t+1}_{i,k} & \longrightarrow & C^{t+1}_{j,k} \equiv C^t_{j,k} & & \\
 & & \downarrow & & \\
 & & \left[C^t_{j,k-1},\ C^t_{j,k},\ C^t_{j,k+1}\right] & &
\end{array}
$$

Hence, this kind of memory is a complexion of '*memory*' and '*logic*' as it is supposed for *memristive* behavior.

There are four basic components plus the clock in the interaction paradigm of morphoCAs.

Calculation :
send / receive,
accept / reject
in generalized time

In contrast to the classical CA with its send/receive properties, there are four basic components plus the clock in the paradigm of morphoCAs. The *sens/receive* or read/write mechanism is augmented in morphoCAs by a *decision-making* (trans-logical) component of *accept/reject* in regard of the sub-system property.

The contrast to Konrad Zuse's conception of calculation is obvious :

"Rechnen heisst : Aus gegebenen Angaben nach einer Vorschrift neue Angaben bilden." (Konrad Zuse)

The discontexturaliy of morphoCAs is certainly also not in hamony with Karl Hewitt's monolithic actor approach to computation.

A systematic deconstruction has obviously to deconstruct all 4+1 components of the diagram.

The very first *deconstruction* happens by parametrizing the inputs. Each input/output, i.e. send/receive action might belong to a different contexture. Hence, the very first task of the automaton is to handle such profound diversity. This job is obsolete for classical CAs because all data are from/in the same contexture.

This contextural embodiment of the fourth term, c_k^{t+1}, explains why the term is not just an extensional result of a mapping but is structurally depending on the conceptual '*history*' of the 3 previous actions.

This understanding of the morphoCA rules relates back to the concept of the *ε/ν-structure* of morphic objects and actions within the concept of the proposed memristive automata.

`http://www.thinkartlab.com/pkl/media/SKIZZE-0.9 .5-medium.pdf`

`http://works.bepress.com/thinkartlab/20/`

`http://transhumanism.memristics.com/Diagrammatik.ppt.htm`

From memristive flip-flop to memristive interactions

Finite state machines and morphoCAs

"A Cellular Automaton (CA) is an infinite, regular lattice of simple finite state machines *that change their states synchronously, according to a local update rule that specifies the new state of each cell based on the old states of its neighbors."* (Kari)

http://users.utu.fi/jkari/ca/CAintro.pdf

> *"Furthermore, since the ECA is actually a finite state machine then the present state of the neighborhood C^t_{k-1}, C^t_k, C^t_{k+1} of cell C^t_k at time step t and the next state C^{t+1}_k at time step t + 1, can be analyzed by the* state transition table *and the* state diagram *depicted in figure 4."* (K. Salman)

Present State c^t_k	Neighborhood $c^t_{k+1} - c^t_{k-1}$ 0-0	0-1	1-1	1-0
0	0	1	0	1
1	1	1	0	0

Next State

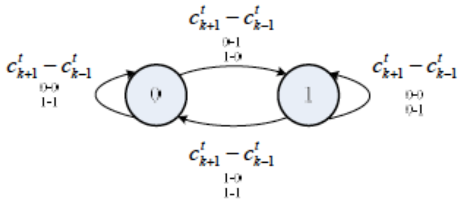


Figure 4, state machine analysis of Rule 30

`http://www.slideshare.net/ijcsit/5413 ijcsit03`

ECA Rule 30

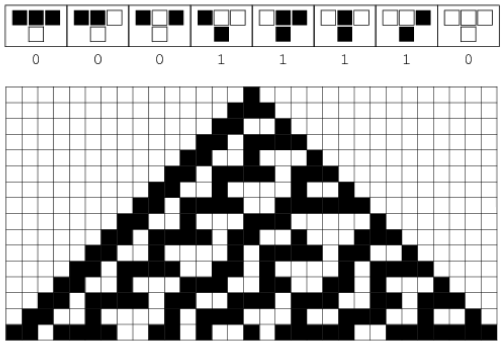
```
FromDigits[{0, 0, 0, 1, 1, 1, 1, 0}, 2]
```

```
30
```

```
FromDigits[kAryFromRuleTable[
  ruleECA[{1, 2, 3, 9, 5, 11, 13, 15}]], 2]
```

```
30
```

rule 30

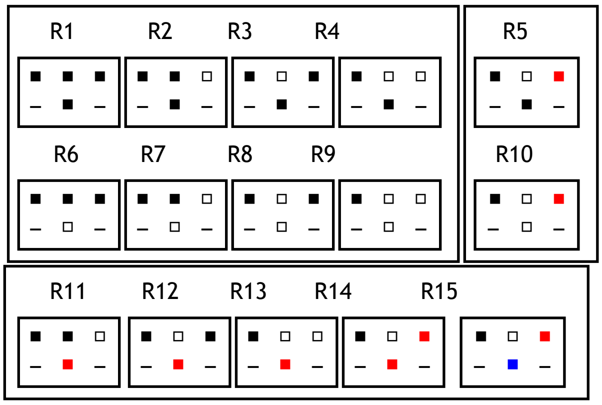


ruleECA[{1, 2, 3, 9, 5, 11, 13, 15}] = rule30

rule30	**111**	**110**	**101**	**100**	**011**	**010**	**001**	**000**
1	–	–	–	(5) : 1	(9) : 1	(3) : 1	(2) : 1	–
0	(15) : 0	(13) : 0	(11) : 0	–	–	–	–	(1) : 0

`http : // memristors.memristics.com / MorphoFSM / Finite %20 State %20 Machines %20 and %20 Morphogrammatics.html`

System of elementary kenomic cellular automata rules in trito-difference form



$$\begin{pmatrix} e1 & e2 & e4 \\ - & e3 & e5 \\ \mathbf{R1} & - & e6 \end{pmatrix} \begin{pmatrix} e1 & v2 & e4 \\ - & v3 & e5 \\ \mathbf{R2} & - & v6 \end{pmatrix} \begin{pmatrix} v1 & e2 & e4 \\ - & v3 & v5 \\ \mathbf{R3} & - & e6 \end{pmatrix} \begin{pmatrix} v1 & v2 & e4 \\ - & e3 & v5 \\ \mathbf{R4} & - & v6 \end{pmatrix} \quad \begin{pmatrix} v1 & v2 & v4 \\ - & e3 & v5 \\ \mathbf{R5} & - & v6 \end{pmatrix}$$

$$\begin{pmatrix} e1 & e2 & v4 \\ - & e3 & v5 \\ \mathbf{R6} & - & v6 \end{pmatrix} \begin{pmatrix} e1 & v2 & v4 \\ - & v3 & v5 \\ \mathbf{R7} & - & e6 \end{pmatrix} \begin{pmatrix} v1 & v2 & v4 \\ - & e3 & e5 \\ \mathbf{R8} & - & v6 \end{pmatrix} \begin{pmatrix} v1 & v2 & v4 \\ - & e3 & e5 \\ \mathbf{R9} & - & e6 \end{pmatrix} \quad \begin{pmatrix} v1 & v2 & v4 \\ - & v3 & v5 \\ \mathbf{R10} & - & e6 \end{pmatrix}$$

$$\begin{pmatrix} e1 & v2 & v4 \\ - & v3 & v5 \\ \mathbf{R11} & - & v6 \end{pmatrix} \begin{pmatrix} v1 & e2 & v4 \\ - & v3 & v5 \\ \mathbf{R12} & - & v6 \end{pmatrix} \begin{pmatrix} v1 & v2 & e4 \\ - & v3 & v5 \\ \mathbf{R13} & - & v6 \end{pmatrix} \begin{pmatrix} v1 & v2 & v4 \\ - & v3 & e5 \\ \mathbf{R14} & - & v6 \end{pmatrix} \quad \begin{pmatrix} v1 & v2 & v4 \\ - & v3 & v5 \\ \mathbf{R15} & - & v6 \end{pmatrix}$$

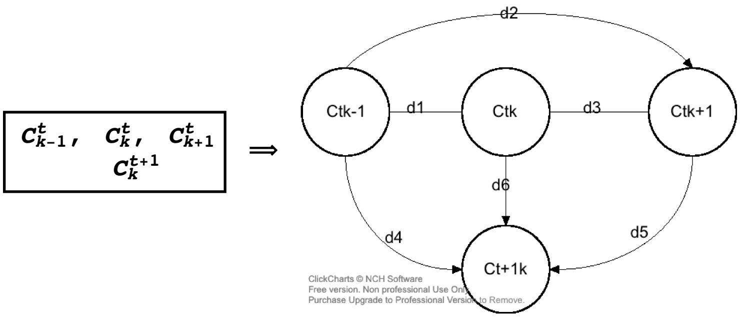
Interpretation

Difference scheme

The difference scheme is a *scheme of differences*, and not just a relational mapping from $C^3$to C.

Also an evolution from $[C^t_{k-1}, \; C^t_k, \; C^t_{k+1}]$ to C^{t+1}_k is defined by all previous elements of time *t* of the specified CA rule there is no concrete differentiation between the new state of C^{t+1}_k and the previous states defined.

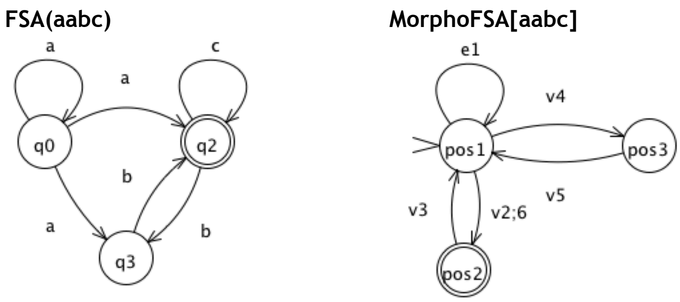
Hence, the new state C^{t+1}_k of a classical CA might incorporate any arbitrary value from a pre-given set of values and is not retro-recursive characterized by the differences of the previous constellation it depends.

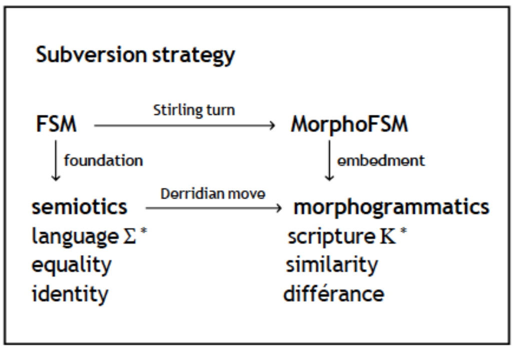


$$\boxed{\begin{matrix} C_{k-1}^{t}, & C_{k}^{t}, & C_{k+1}^{t} \\ & C_{k}^{t+1} & \end{matrix}} \Longrightarrow \begin{array}{c|c|c} d1 & d2 & d4 \\ \hline - & d3 & d5 \\ \hline - & - & d6 \end{array}, \text{ with } d = \{\epsilon, v\},\ \epsilon = \text{equal},\ v = \text{non-equal}$$

$d1 = \text{diff}\,(C_{k-1}^{t},\ C_{k}^{t}),$
$d2 = \text{diff}\,(C_{k-1}^{t},\ C_{k+1}^{t}),$
$d3 = \text{diff}\,(C_{k}^{t},\ C_{k+1}^{t}),$
$d4 = \text{diff}\,(C_{k-1}^{t},\ C_{k}^{t+1}),$
$d5 = \text{diff}\,(C_{k+1}^{t},\ C_{k}^{t+1}),$
$d6 = \text{diff}\,(C_{k}^{t},\ C_{k}^{t+1}).$

FSA Example





`http://memristors.memristics.com/CA-Overview/Short-%20Overview%20of%20Cellular%20Automata.pdf`

Monomorphic prolongation

First aspect: iteration

Given a morphogram MG, which is always a localized pattern in a kenomic matrix, a *prolongation* (successor, evolution) of the morphogram is achieved with the successor operator s_i. To each prolongation a further prolongation is defined by the iterated application of the operator s_i.

The morphogrammatic succession $\left(\text{MG} \xrightarrow{s_i} \text{MG}\right)$ is founded by its model $\left(\text{gm} \xrightarrow{h_i} \text{gm}\right)$ and the morphism *f*, guaranteeing the commutativity of the construction.

As a third rule, the iterability of the successor operation is *arbitrary,* which is characterised by the commutativity of the diagram. Hence, the conditions for a (retrograde) recursive formalisation are given.

Second aspect: anti-dromicity
Each prolongation is realized simultaneously by an iterative *progression* and an *antidromic retro-gression*. That is, the operation of prolongation of a morphogram is defined retro-grade by the possibilities given by the encountered morphogram. A concrete prolongation is selecting out of those possibilities its specific successions. All successions are to be considered as being realized at once.

Third aspect: simultaneity and interchangeability
This simultaneity of different successions defines the range of the prolongation. This definition of morphogrammatic prolongation is not requiring an alphabet and a selection of a sign out of the alphabet. Hence, the concept of morphogrammatic prolongation is defined by the two aspects of iteration and antidromic retro-gradeness of the successor operation. The simultaneity of the prolongations is modeled by the interchangeability of its actions.

Fourth aspect: diamond characterization of antidromicity
Both aspects together, repeatability and antidromicity with its simultaneous and interchangeable realizations, are covered by the diamond-theoretic concept of combination of operations and morphisms, i.e. composition and saltisition, between morphogramatic prolongations.

The philosophical status of morphoCAs has yet to be determined. *"What's after digitalism?"* might give a hint.

```
https : // www.academia.edu / 1 873 531 / Digital_Philosophy._Formal
  _Ontology _and _Knowledge _Representation _in _Cellular _Automata
```

Internal structure of the morphogrammatic transition rule

Recall definitions: classical transition rule

"*Rigid computations have another node parameter:* location *or* cell*. Combined with time, it designates the event uniquely. Locations have structure or proximity edges between them. They (or their short chains) indicate all neighbors of a node to which pointers may be directed.*

"CA are a parallel rigid model. Its sequential restriction is the Turing Machine (TM). The configuration of CA is a (possibly multi-dimensional) grid with a fixed (independent of the grid size) number of states to label the events. The states include, among other values, pointers to the grid neighbors. At each step of the computation, the state of each cell can change as prescribed by a transition function of the previous states of the cell and its pointed-to neighbors. The initial state of the cells is the input for the CA. All subsequent states are determined by the transition function (also called program)." Leonid A. Levin. Fundamentals of Computing.

http://www.cs.bu.edu/fac/lnd/toc/z/z.html

Morphogrammatic transition rule

```
http : // memristors.memristics.com / Memristive %20
  Cellular %20 Automata / Memristive %20 Cellular %20 Automata.html
```

General scheme	Example
rule set, start string ↓ string pos = (Nr., 1) ↓ReLabel ReLabel (string) ↙ ↓ ↘ NextGen NextGen (ReLabel (string)) ↘ ↓ ↙ ∈ rule - set ? ⟨yes; no⟩ ↓ apply rule result	rule set = {1, 7, 8, 4}, start string [bcb] : string at pos (Nr., 1) ↓ReLabel [aba] ↙ ↓ ↘ NextGen [abaa] [abab] [abac] ↘ ↓ ↙ [abaa] ∈ rule - set ? ⟨yes; no⟩ ↓ apply : [abaa] rule7 result = [a]

NextGen is in this morphoCA context a *retrograde recursive action* and not to be confued by a classical recursion.

What makes the difference?

1. retro - grade recursivity
2. irreducible heterogeneity
3. interactivity and reflectionality

Morphogrammatic example

evol $_i$ **(MG)** :

$\forall\, i \in (\mathrm{mg}\,(\mathrm{MG})),\ 1 \le i \le \mathrm{mg}\,(\mathrm{MG}) + 1$

$$\text{second} \xleftarrow{\text{retrogression}} \text{first}$$

$\downarrow$ *selection* $\qquad\qquad$ $\uparrow$ *choice*

$$([\mathrm{mg}_1 \ldots \mathrm{mg}_i \ldots \mathrm{mg}_n] \longrightarrow [\mathrm{mg}_1 \ldots \mathrm{mg}_i \,..\, \mathrm{mg}_n]) \longrightarrow [\mathrm{mg}_1 \ldots \mathrm{mg}_i \ldots \mathrm{mg}_n\ \mathrm{mg}_{n+1}]$$

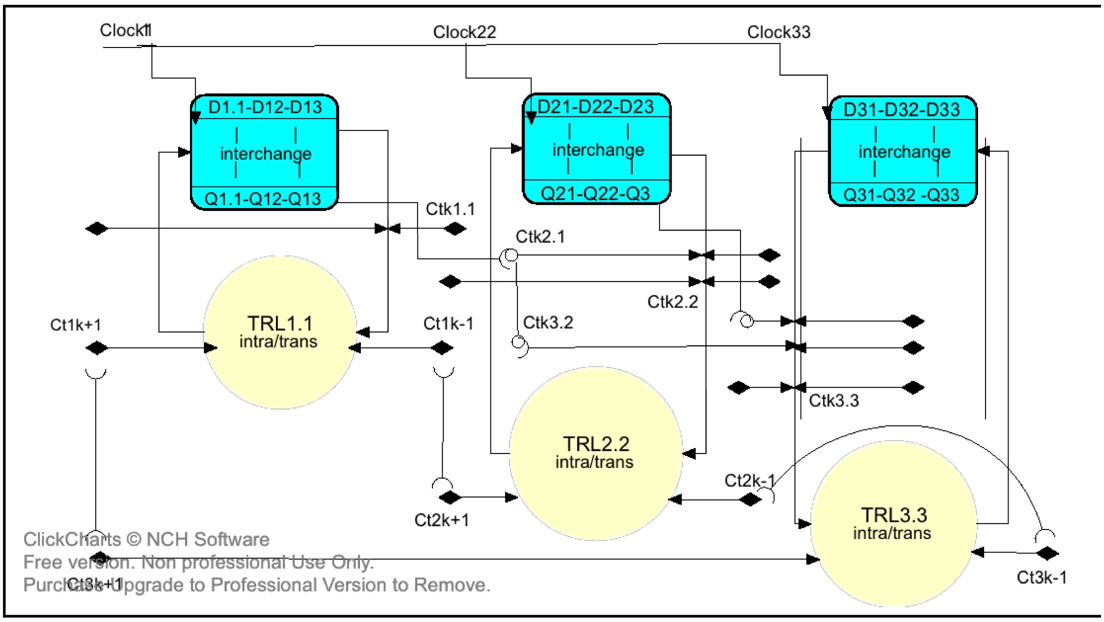
$\downarrow$ *choice* $\qquad\qquad$ $\uparrow$ *selection*

$$\text{first} \quad \underset{\text{progression}}{\longrightarrow} \quad \text{second}$$

`http : // memristors.memristics.com / MorphoReflection / Morphogrammatics %20 of %20 Reflection.html`

Flow charts for morphoCAs

Full mediation of input



Basic scheme : Explanation for morphoCA $^{(3,3)}$

$\mathbf{Clock}^{(3,3)} = \mathrm{synch}\left(\mathrm{Clock}^{1.1},\ \mathrm{Clock}^{2.2},\ \mathrm{Clock}^{3.3}\right)$

$\mathbf{Calculation}^{(3,3)} = \mathrm{mediation}\left(\mathrm{TRL1}\,.1,\ \mathrm{TRL2}\,.2,\ \mathrm{TRL3}\,.3\right)$:

$$\left(\mathrm{TRL}^1 \amalg_{1.2,0} \mathrm{TRL}^2\right) \amalg_{1.2,3} \mathrm{TRL}^3 = \begin{pmatrix} \mathrm{TRL}^1 & - \\ - & \mathrm{TRL}^3 \\ \mathrm{TRL}^2 & - \end{pmatrix}$$

$$\mathbf{intra}^{(3,3)} = (\,\mathrm{TRL1}\,.1\,(\mathrm{Ct1k},\ \mathrm{Ct1k}+1,\ \mathrm{Ct1k}-1) \\ \amalg \\ \mathrm{TRL2}\,.2\,(\mathrm{Ct2k},\ \mathrm{Ct2k}+1,\ \mathrm{Ct2k}-1)) \\ \amalg$$

$$\mathrm{TRL3.3}\,((\mathrm{Ct3k},\ \mathrm{Ct3k}+1,\ \mathrm{Ct3k}-1)$$

$$\mathbf{ENV}^{(3,3)} = [\,\mathrm{ENV}^1 \amalg \mathrm{ENV}^2\,] \amalg \mathrm{ENV}^3 \ :$$

$$\begin{array}{lcl} \mathrm{Ct1k}+1 & \amalg & \mathrm{Ct3k}+1 \\ \mathrm{Ct2k}-1 & \amalg & \mathrm{Ct2k}+1 \\ \mathrm{Ct3k}-1 & \amalg & \mathrm{Ct3k}-1 \end{array}$$

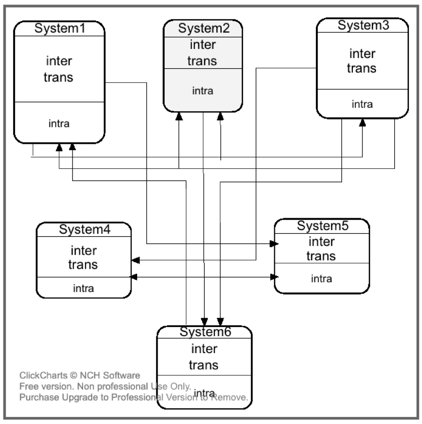
$$\left(\mathrm{ENV}^1 \amalg_{1.2,0} \mathrm{ENV}^2\right) \amalg_{1.2,3} \mathrm{ENV}^3 = \begin{pmatrix} \mathrm{ENV}^1 & - \\ - & \mathrm{ENV}^3 \\ \mathrm{ENV}^2 & - \end{pmatrix}$$

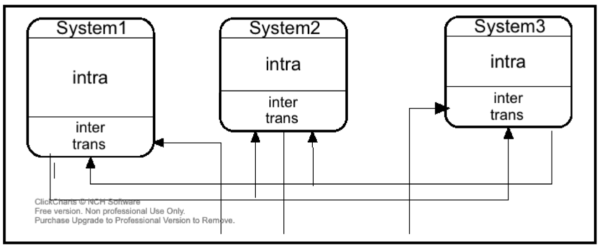
Arrows

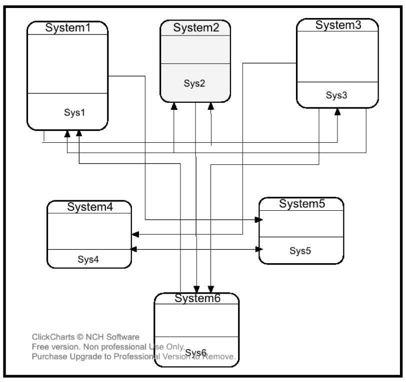
directed arrows : input / output arrows,
open headed arrows : inter - and trans - action, mediation arrows

Explanation for morphoCA$^{(4,4)}$

$$\mathrm{morphoCA}^{(4,4)} = \boxed{\begin{array}{ccc} \mathrm{CA1} & \square & \square \\ - & \mathrm{CA3} & \square \\ \mathrm{CA2} & \square & \mathrm{CA6} \\ \square & \mathrm{CA5} & \square \\ \mathrm{CA4} & \square & \square \end{array}} \ ::$$







Full interaction and mediation table for morphoCA$^{(3,3)}$

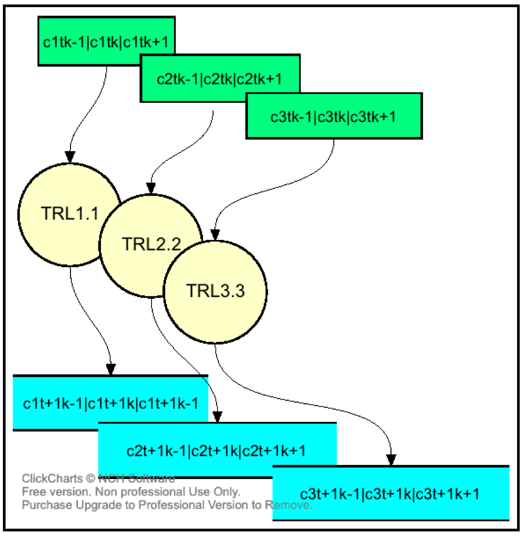
S11	S11	S21, 31
S22	S22	S12, 32
S33	S33	S13, 23

S1	D11 – Q11	Q12 – D21, Q13 – D31
S2	D22 – Q22	Q21 – D12, Q23 – D32
S3	D33 – Q33	Q13 – D31, Q23 – D32

$\text{morphoCA}^{(3,3)}$:

CA	1	2	3
1	$\mathbf{CA^{1.1}}$	$\mathbf{CA^{2.1}}$	$CA^{3.1}$
2	$\mathbf{CA^{1.2}}$	$CA^{2.2}$	$CA^{3.2}$
3	$CA^{1.3}$	$CA^{2.3}$	$\mathbf{CA^{3.3}}$

Discontexturality of distributed CAs



Poly-layered grid structure

...	C1tk + 1	C1tk	C1tk – 1	...
...	C2tk + 1	C2tk	C2tk – 1	...
...	C3tk + 1	C3tk	C3tk – 1	...

⟹

...	C1t + 1 k + 1	**C1t + 1 k**	C1t + 1 k – 1	...
...	C2t + 1 k + 1	**C2t + 1 k**	C2t + 1 k – 1	...
...	C3t + 1 k + 1	**C3t + 1 k**	C3t + 1 k – 1	...

An interpretation of the discontexturality diagram shows that the grid structure of distributed CAs of the morphoCAs are in fact not 1-D CAs but disseminated 1-D CAs. It also shows that disseminated CAs are not necessarily 2- or 3-dimensional or higher. What we see as a linear 1-D grid by the visualization of morphoCA actions is in fact a *composition* of different parallel 1-D grids projected onto an 1-D grid of an uninterpteted output.

Hence, in functional terms, there is no mapping from $\{0, 1, 2, 3\}^3$ -> {0,1,2,3} but a composition of partial sub-maps.

Nevertheless, poly-layered grids are not multi-layered, because the layers of a multi-layered system are unified under the umbrella of First-Order Logic with Modal logic and General Ontology (Upper Ontology). While dis-contexturality implies an *interplay* of a multitude of irreducibly different logics, each containing their inter- and trans-logical operators, additionally to the full set of intra-logicial operators too.

Multi-layerd systems are logically defined by the basic intra-logical operations only. Poly-layerd systems are involved in an interplay of dis-contextural operations of inter- and trans-contextural actions.

This discontextural approach obviously is in strict conflict with *Proposition1* of category theory and its unique universe U:

If $x \in U$ and $y \subseteq x$, then $y \in U$.

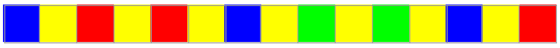
As usual in such fundamental situation, the proposition is circular. It presumes the uniqueness of its logical universe to work for a definition of its unique category-theoretical universe which is taken as the base for the definition of First-Order Logic and its unique universe of terms.

> *“Polycontexturality alone is not enough to realize the interwoven dynamics a new world-view is desperate for. Gotthard Gunther introduced his proemial relationship to dynamize his contextures, albeit still restricted to a uni-directional movement.The concept of metamorphosis as part of the diamond strategies, based on polycontexturality and disseminated over the kenomic matrix, is a further step to realize a radical paradigm change in our way of thinking and designing futures.”*

http : // memristors.memristics.com/Polyverses/Polyverses.html

The projection marks the difference of the deep-structure and the surface structure of the productions of morphoCAs.

It makes it clear, again, that *“What you see is not what it is”.* Hence, any ontologizing will fail.



yellow	C1t + 1 k	–	C3tk – 1	–
red	–	–	–	C4t + 1 k
blue	–	C2t + 1 k	–	–
green	–	–	–	–

projection $\Longrightarrow$ | C1t + 1 k | C2t + 1 k | C3tk – 1 | C4t + 1 k |

The difference between *multi-layered* and *poly-layered* systems got a conceptual sketch with the paper:

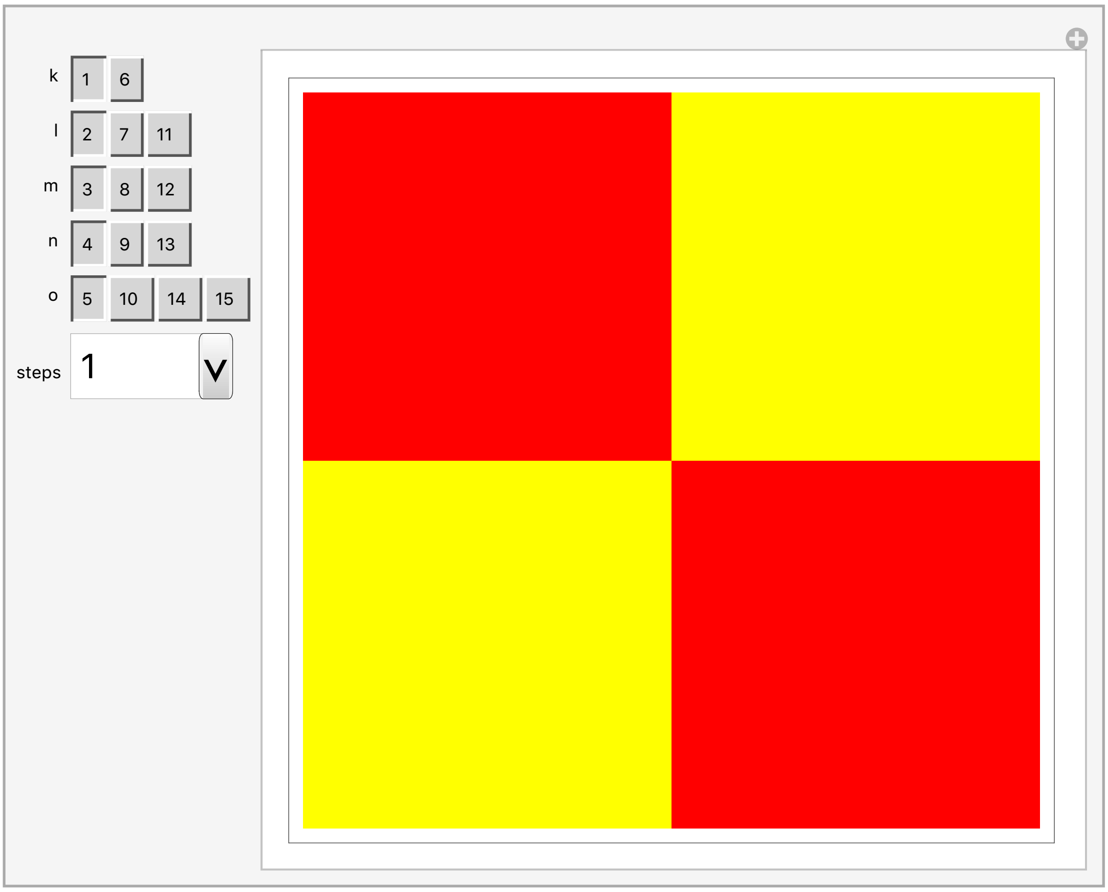
Memristics: Dynamics of Crossbar Systems
Strategies for simplified polycontextural crossbar constructions for memristive computation

“Interchangeability is part of a new axiomatics of poly-categorical diamond systems still to be developed. Interchangeability is defined intra-contextural for composition and yuxtaposition, and trans-contextural for interactions, like mediation, replication, iteration and transposition.”

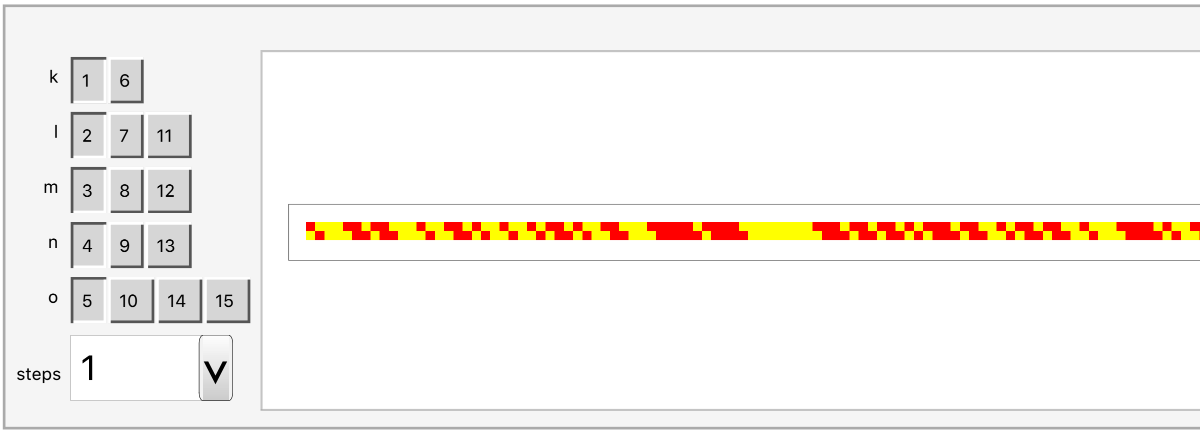
http://www.thinkartlab.com/Memristics/Poly-Crossbars/Poly-Crossbars.pdf

Claviatures for morphoCAs

Claviature: ruleDM



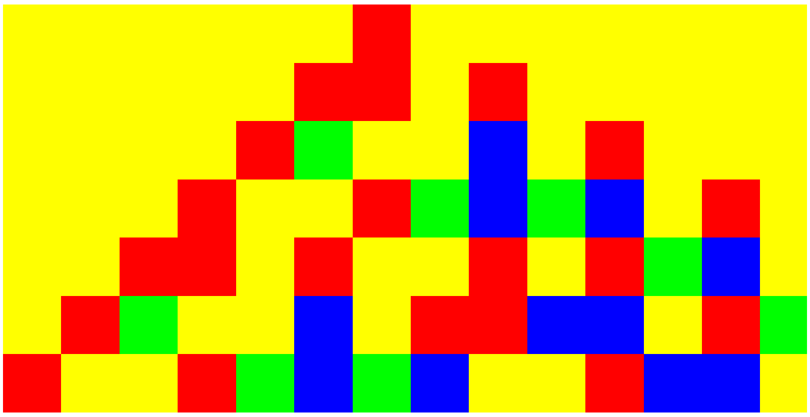
Claviature: Random ruleDM



Claviature: ruleDCKV

a	1111	1112		
b	1121	1122	1123	
c	1211	1212	1213	
d	1221	1222	1223	
e	2121	2122	2123	
f	2211	2212	2213	
g	2221	2222	2223	
h	2111	2112	2113	
i	2131	2132	2133	2130
j	1231	1232	1233	1230
k	2231	2232	2233	2230
l	2311	2312	2313	2310
m	2321	2322	2323	2320
n	2331	2332	2333	2330
o	2301	2302	2303	2300

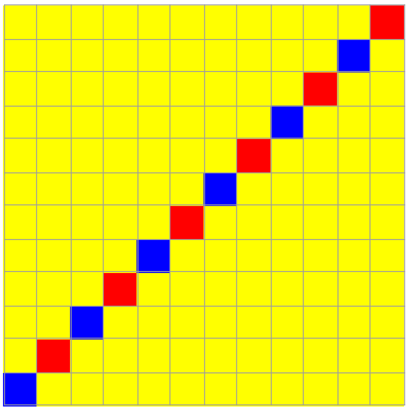
steps 6



Analysis of ruleDM[{1,11,3,9,x}]

```
ruleDM[{1, 11, 3, 9, 5}]
```

```
reduced ruleDM[{1, 11, 3, 9, x}]

          Subrules               Subsystems

ruleDM[1]  : {0, 0, 0} → 0, S1, 3 : yellow
ruleDM[3]  : {0, 1, 0} → 0, S1
ruleDM[9]  : {1, 0, 0} → 0, S1
ruleDM[3]  : {0, 2, 0} → 0, S3
ruleDM[9]  : {2, 0, 0} → 0, S3
ruleDM[11] : {0, 0, 1} → 2, S1 → S2, 3 : blue
ruleDM[11] : {0, 0, 2} → 1, S3 → S1, 2 : red
```

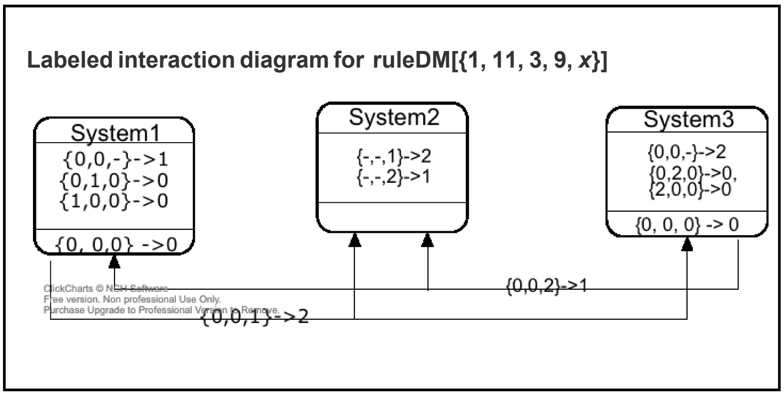


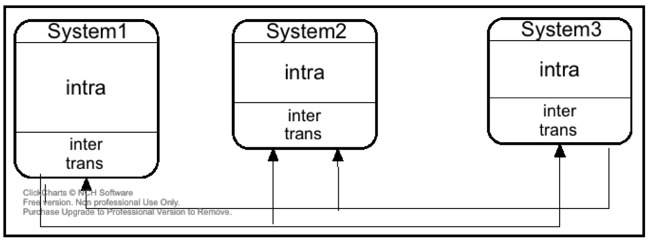
ruleDM[{1, 11, 3, 9, x}]					
x / yz	**00**	**10**	**20**	**01**	**02**
0	[1] : $0_{1,3}$	[8] : 0_{1}	[8] : 0_{3}	[11] : $2_{2,3}$	[11] : $1_{1,2}$
1	[9] : 0_{1}	–	–	–	–
2	[9] : 0_{3}	–	–	–	–

Distribution density: [5,1,1]

0 / 5	1 / 1	2 / 1
000	002	001
010		
100		
200		
020		

Flow chart for ruleDM[{1,11,3,9,x}]





Explicit transition system table for ruleDM[{1,11,3,9,x}]

TRL1 .1 || TRL3 .3
{0, 0, 0} -> 0
: D1 .1 → Q1 .1 → TRL1 .1 || D3 .3 → Q3 .3 → TRL3. 3

TRL1 .1
{0, 1, 0} -> 0,

```
{1, 0, 0} -> 0
: D1 .1 → Q1 .1 → TRL1 .1
```

TRL3 .3

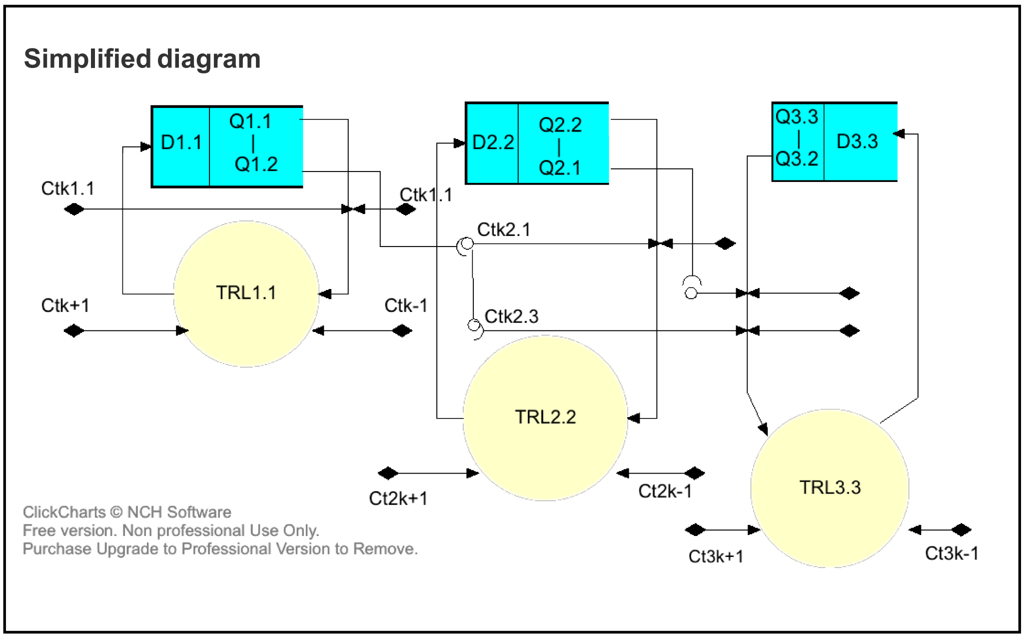
```
{0, 2, 0} → 0,
{2, 0, 0} → 0
: D3 .3 → Q3 .3 → TRL3 .3
```

TLR2 .2 || TLR3 .3

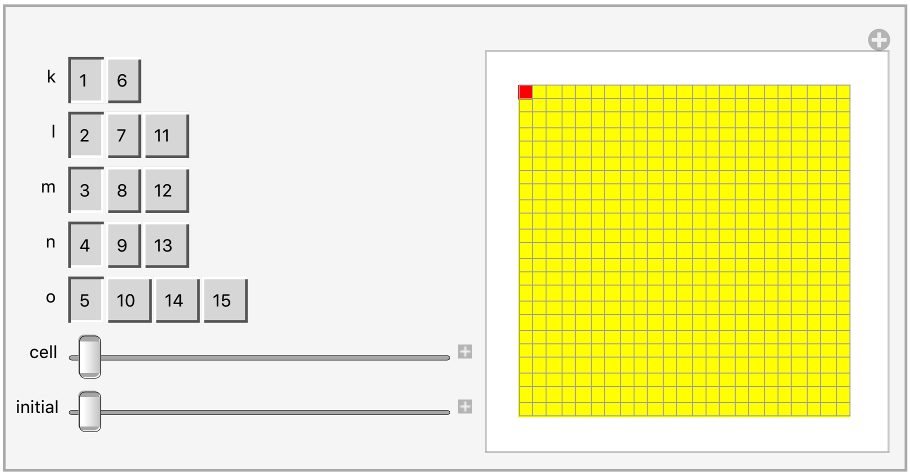
```
{0, 0, 1} → 2

: Q1 .2 → Q3 .2 → TLR3 .3 || Q1 .2 -> Q2 .1 → TLR22
```

TLR1 .1 || TLR2 .2

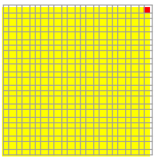
```
{0, 0, 2} → 1
: Q3 .1 → Q1 .3 → TLR1 .1 || Q3 .2 -> Q2 .1 → Q2 .2 → TLR22
```



Example: ruleDM[{1, 11, 3, 9, 15}] step-wise realization



start (init) : yellow, red

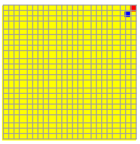
 ,

step 37 : {0, 0, 0} → 0, {0, 1, 0} → 0, {1, 0, 0} → 0 :

TRL1 .1 || TRL3 .3
{0, 0, 0} -> 0
: D1 .1 → Q1 .1 → TRL1 .1 || D3 .3 → Q3 .3 → TRL3 .3
TRL1 .1
{0, 1, 0} -> 0, : D1 .1 → Q1 .1 → TRL1 .1
{1, 0, 0} -> 0

Start of the morphoCA with init {{1}, 0} producing the entry “red” with an environment “yellow” with the properties defined at step 37 by the rules:
{0,0,0}→0 of sys1||sys3 and {0,1,0}→0, {1,0,0}→0 of sys1.

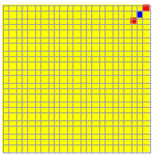
construction: blue, yellow, red, yellow



step 44 : **{0, 0, 1} → 2 :**

TLR2 .2 || TLR3 .3
{0, 0, 1} → 2
Q1 .2 → Q3 .2 → TLR3 .3 || Q1 .2 -> Q2 .1 → TLR22

At step 44, the memory decides that the received value “2” doesn’t belong to its range, i.e. the system 1, defined by the values {0,1}.
The value “2” of system 3 defines a new start at the system 3 with the properties of {0,0,2}→1, {0,2,0}→0, {2,0,0}→0.



step 66 : **{0, 0, 2} → 1 :**

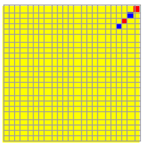
TLR2 .2 || TLR3 .3
{0, 0, 1} → 2
Q1 .2 → Q3 .2 → TLR3 .3 || Q1 .2 -> Q2 .1 → TLR22

TRL3 .3
{0, 2, 0} → 0, : D3 .3 → Q3 .3 → TRL3 .3
{2, 0, 0} → 0

Again, at the step 66, the decider of the memory unit of system 3 decides that the value “1” doesn’t belong to its range, i.e. the system 3, defined by the values {0,2}.
The value “1” of memory 3 defines a continuation in the system 1 with the background properties of {0,2,0}→0, {2,0,0}→0.

The background is symbolized numerically by 0, i.e. yellow. But “0” belongs to 2 different sub-systems defined by {0, 1} and {0, 3}.
What counts is not just the value in a system but its contextual relation or difference to other values. Hence the presupposed rule: {0,0,0} -> 0, holds in general but its significance depend on its context.

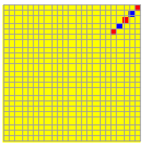
iteration of construction



```
step 88 : {0, 0, 1} → 2 :
```

```
TLR2 .2 || TLR3 .3
{0, 0, 1} → 2
Q1 .2 → Q3 .2 → TLR3 .3 || Q1 .2 -> Q2 .1 → TLR22
```

At step 88, the memory decides that the received value “2” doesn’t belong to its range, i.e. the system 1, defined by the values {0,1}.
The value “2” of system 3 defines a new start at the system 3 with the properties of {0,0,2}→1, {0,2,0}→0, {2,0,0}→0.



```
step 110 :  {0, 0, 2} → 1, {0, 2, 0} → 0, {2, 0, 0} → 0,
```

```
TLR1 .1 || TLR2 .2
{0, 0, 2} → 1
Q3 .1 → Q1 .3 → TLR1 .1 || Q3 .2 -> Q2 .1 → Q2 .2 → TLR22
```

```
TRL3 .3
{0, 2, 0} → 0, : D3 .3 → Q3 .3 → TRL3 .3
{2, 0, 0} → 0
```

Again, at the step 110, the decider of the memory unit of system 3 decides that the value “1” doesn’t belong to its range, i.e. the system 3, defined by the values {0,2}. The value “1” of memory 3 defines a continuation in the system 1 and in system 2 with the properties of {0,1,0}→0, {1,0,0}→0.
And so on.

Unfortunately it is necessary to go through these tedious phenomenological interpretations of the mechanism of morphoCAs because without this kind of modelling it isn’t possible to understand the nature of their outcome. Just to enjoy interesting pictures and listening to unheard sounds is not yet enough to understand the novelty of the morphogrammatic approach towards cellular automata and automata in general.

The switch from one automaton to the net of automata is not just ruled by the clock but also by the logic of the unit. If there is a transjunctional result of the logical unit, the calculations have to switch to another automaton. Different types of polycontextural transjunctions are ruling such interactions. Otherwise, without a switch, it stays inside the domain of the automaton for further intra-contextural calculations.

http://www.thinkartlab.com/pkl/media/Dynamic%20 Semantic %20 Web.pdf

http : // memristors.memristics.com/Notes %20 on %20 Polycontextural %20 Logics/Notes %20 on %20 Polycontextural %20 Logics.pdf

PCA, programmable CAs

“As the matter of fact, PCA are essentially a modified CA structure. It employs some control signals on a CA structure. By specifying certain values of control signals at run time, a PCA can implement various functions dynamically in terms of different rules.”

http://infonomics-society.ie/wp-content/uploads/ijicr/published-papers/volume-3-2012/Security-of-Telemedical-Applications-over-the-Internet-using-Programmable-Cellular-Automata.pdf

For morphoCAs, the range of reconfiguring processors is not limited to the range of classical CAs but spans over a wide range of trans-classical paradigms of morphoCAs also including classical CAs.

The specification of morphoCAs shows clearly the paradigmatical difference between morphoCAs, ECAs and PCAs.

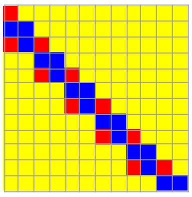
Concerning the sub-rule approach, morphoCAs might be seen as ‘*hybrid*’ CAs with transjunctional functions and mediation to be considered.

3. PCL diagrams for morphoCA$^{(3,3)}$ with interaction and

mediation

Analysis of minimized ruleDCM[{1, 2, 12, 13, 5}]

```
ArrayPlot[CellularAutomaton[
  {
   {0, 0, 0} → 0,

   {0, 0, 1} → 0, {0, 0, 2} → 0, {2, 2, 0} → 2,

   {1, 2, 1} → 0, {0, 1, 0} → 2,

   {0, 2, 2} → 1, {1, 0, 0} → 2, {2, 0, 0} → 1,

   {0, 1, 2} → 0, {2, 1, 0} → 2
  },
  {{1}, 0}, 11],
 ColorRules -> {1 -> Red, 0 -> Yellow, 2 → Blue, 3 → Green},
 Mesh → True, ImageSize → 100]
```



Analysis

ruleDM[{1, 11, 3, 9, x}]

x / yz	00	10	12	01	02	22
0	[1] : $0_{1,3}$	[12] : 2_1	[5] : 0	[11] : 0	[11] : $0_{1,2}$	[13] : 1_3
1	[13] : 2_1	–	–	–	–	–
2	[13] : 1_3	[5] : 2	–	–	–	–

Distribution density: [7,2,2]

The *distribution density* of a morphoCA constellation gives a simple measure for classification and comparison of morphoCAs.
It holds for reduced and non-reduced morphoCA constellations.

0 / 7	1 / 2	2 / 2
000	200	010
010	022	210
001		
002		
012		

S1	S1	S3
S3	S3	–
S2, 1, 2	–	S1, 3, 1
S1, 3, 2	–	S1, 1, 3

Analysis of the interaction patterns

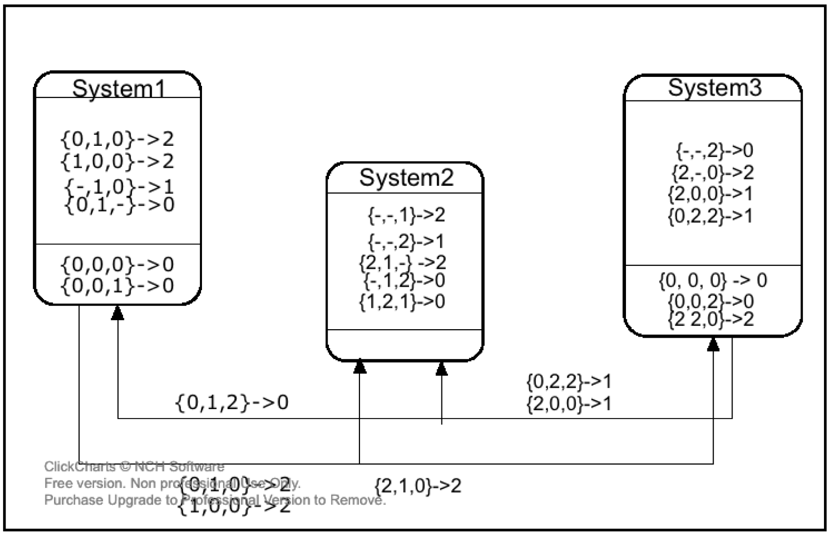
```
{0, 0, 0} → 0,   : sys1
{0, 0, 1} → 0,
```
: intra

```
{0, 0, 2} → 0,   : sys3
{2, 2, 0} → 2,
{0, 0, 0} → 0
```

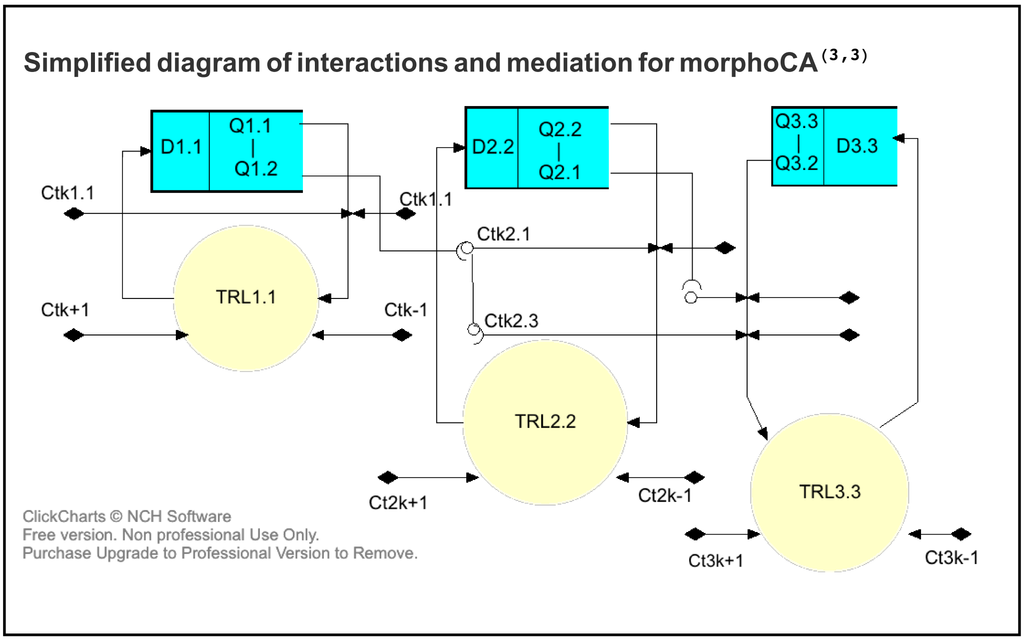
```
{1, 2, 1} → 0,  : sys2, 1, 2 → sys1 || sys3 || sys1
{0, 1, 0} → 2,  : sys1, 1, 1 → sys3 || sys2 || sys3
{1, 0, 0} → 2,  : sys1, 1, 1 → sys2 || sys3 || sys3   : inter
{0, 2, 2} → 1,  : sys3, 3, 3 → sys1 || sys2 || sys2
{2, 0, 0} → 1,  : sys3, 3, 3 → sys2 || sys1 || sys1
```

```
{0, 1, 2} → 0,  : sys1, 3, 2 → sys1 || sys3 || sys1   : trans
{2, 1, 0} → 2   : sys2, 3, 1 → sys3 || sys3 || sys2
```

Diagram scheme for ruleDCM[{1, 2, 12, 13, 5}]



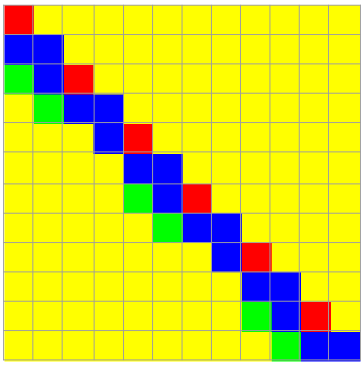
Simplified diagram of interactions and mediation for morphoCA$^{(3,3)}$



Analysis of ruleDM[{1, 2, 12, 13, 5}]

ruleDM[{1, 2, 12, 13, 5}]

```
ArrayPlot[CellularAutomaton[
  {
   {0, 0, 0} → 0,
   {0, 0, 1} → 0, {0, 0, 2} → 0, {0, 0, 3} → 0,
   {2, 2, 0} → 2,
   {1, 2, 1} → 0, {0, 1, 0} → 2,
   {0, 2, 2} → 3, {1, 0, 0} → 2, {2, 0, 0} → 1, {3, 2, 2} → 0,
   {0, 2, 1} → 0, {0, 3, 2} → 0, {2, 1, 0} → 2, {3, 2, 1} → 3
  },
  {{1}, 0}, 11],
 ColorRules -> {1 -> Red, 0 -> Yellow, 2 → Blue, 3 → Green},
 Mesh → True, ImageSize → 100]
```



Analysis

intra + inter + trans

ruleDM[{1, 2, 12, 13, 5}]

x / yz	00	01	02	03	10	20	21	22
0	0	0	0	0	2	□	0	3
1	2	□	□	□	□	□	0	□
2	1	□	□	□	2	2	□	□
3	□	□	□	□	□	□	3	0

Distribution density: [7,1,3,1]

0 / 7	1 / 1	2 / 3	3 / 1
000	200	100	321
001		010	
002		210	
003			
021			
121			
322			

Analysis of the interaction patterns

```
{0, 0, 0} → 0,   : sys1
{0, 0, 1} → 0,              : intra
```

```
{0, 0, 2} → 0,   : sys3
{2, 2, 0} → 2,
{0, 0, 0} → 0
```

```
{0, 0, 3} → 0,   : sys6
{0, 0, 0} → 0
```

```
{1, 2, 1} → 0,  : sys2, 2, 2 → sys1 || sys3 || sys1
{0, 1, 0} → 2,  : sys1, 1, 1 → sys3 || sys2 || sys3
{1, 0, 0} → 2,  : sys1, 1, 1 → sys2 || sys3 || sys3
{0, 2, 2} → 3,  : sys3, 3, 3 → sys1 || sys2 || sys2    : inter
{2, 0, 0} → 1,  : sys3, 3, 3 → sys2 || sys1 || sys1

{3, 2, 2} → 0,  : sys4, 4, 4 → sys6 || sys3 || sys3
```

```
{0, 2, 1} → 0, : sys3, 1, 2 → sys1 || sys3 || sys1
{2, 1, 0} → 2  : sys2, 3, 1 → sys3 || sys3 || sys2
                                                     : trans
{0, 3, 2} → 0, : sys6, 3, 4 → sys4 || sys6 || sys3
{3, 2, 1} → 3  : sys4, 5, 2 → sys3 || sys4 || sys5
```

```
{0, 1} = sys1, {1, 2} = sys2, {2, 3} = sys4
       {0, 2} = sys3, {1, 3} = sys5,
              {0, 3} = sys6
```

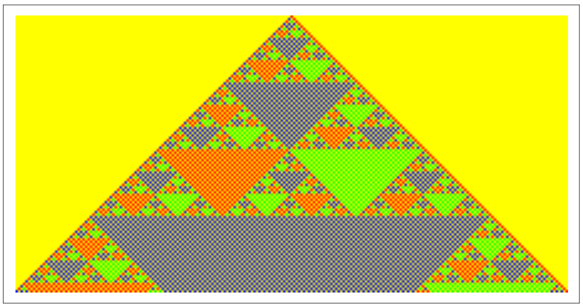
4. PCL diagrams with interactions and mediations: morphoCA$^{(4,3,3)}$

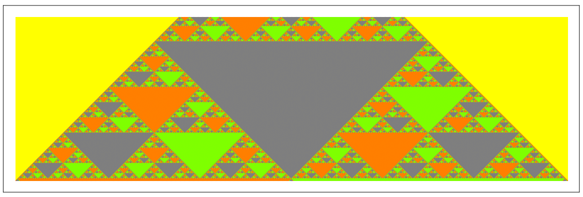
Analysis of ruleDM[{1,11,3,4,15}]

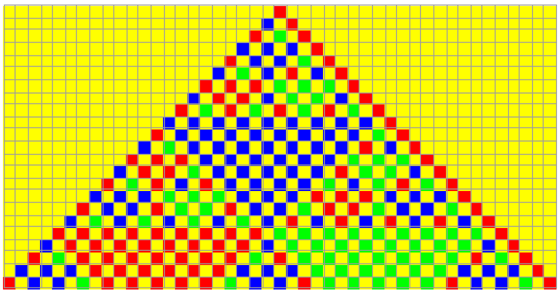
```
ruleDM[{1, 11, 3, 4, 15}]
```

```
{1}
{2, 0, 1}
{1, 0, 3, 0, 1}
{2, 0, 2, 0, 2, 0, 1}
{1, 0, 2, 0, 2, 0, 3, 0, 1}
{2, 0, 3, 0, 2, 0, 1, 0, 2, 0, 1}
{1, 0, 1, 0, 1, 0, 3, 0, 3, 0, 3, 0, 1}
{2, 0, 1, 0, 1, 0, 2, 0, 3, 0, 3, 0, 2, 0, 1}
{1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1, 0, 3, 0, 1}
{2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 1}
{1, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 3, 0, 1}
{2, 0, 3, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 2, 0, 1, 0, 2, 0, 1}
```

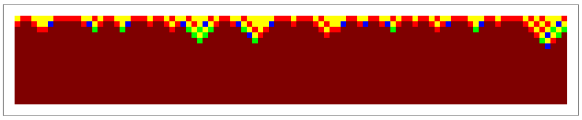
Reduced







Random (restricted by reduction)



Analysis

Analysis of ruleDM[{1,11,3,4,15}]

ruleDM[{1, 11, 3, 4, 15}]

x / yz	00	01	10	02	03	20	30
0	**0; sys1, 3**	2; sys2, 3	**0; sys1**	1; syss1, 3	–	0; sys2	0; sys3
1	1; sys1	**1; sys1**	–	3; sys2, 3	2; sys	–	–
2	–	3; sys	–	2; sys2	1; sys2, 4	–	–
3	–	2; sys	–	2; sys	3; sys3	–	–

Distribution density: [4,5,4,3]

ruleDM[{1, 11, 3, 4, 15}]

x / yz	00	01	10	02	03	20	30
0	**0**	2	**0**	1	–	**0**	**0**
1	**1**	**1**	–	3	2	–	–
2	–	3	–	**2**	1	–	–
3	–	2	–	2	**3**	–	–

0 / 4	1 / 5	2 / 4	3 / 3
000	101	001	303
010	100	202	102
020	002	103	201
030	203	301	
	302		

{1}
{2, 0, 1}
{1, 0, 3, 0, 1}
{2, 0, 2, 0, 2, 0, 1}

Analysis of the interaction patterns

Calculation: intra-contextural action

```
{0, 0, 0} → 0,  : sys1
{0, 1, 0} → 0,
{1, 0, 1} → 1,
{1, 0, 0} → 1,
```

```
{0, 2, 0} → 0,   : sys3
{2, 0, 2} → 2,
{0, 0, 0} → 0
```

```
{0, 3, 0} → 0,   : sys6
{3, 0, 3} → 3,
{0, 0, 0} → 0
```

Alternation: trans-contextural action from sys1 to sys2||sys3 and from sys3 to sys2||sys1

```
{0, 0, 1} → 2 : sys1 → sys3 || sys1 || sys3
{0, 0, 2} → 1 : sys3 → sys1 || sys2 || sys1
```

Mediation: poly-layered action

```
{1, 0, 2} → 3 : sys1, 2, 3 → sys5 || sys6 || sys4
{1, 0, 3} → 2 : sys1, 6, 5 → sys2 || sys3 || sys4
{2, 0, 3} → 1 : sys3, 6, 4 → sys2 || sys1 || sys5
{2, 0, 1} → 3 : sys3, 6, 4 → sys2 || sys1 || sys5
{3, 0, 1} → 2 : sys6, 5, 1 → sys4 || sys1 || sys3
{3, 0, 2} → 1 : sys6, 4, 3 → sys5 || sys2 || sys1
```

Interpretation of mediation

```
{1, 0, 2} → 3 : sys1, 2, 3 → sys5 || sys4 || sys6
{1, 0, 3} → 2 : sys1, 5, 6 → sys2 || sys4 || sys3
```

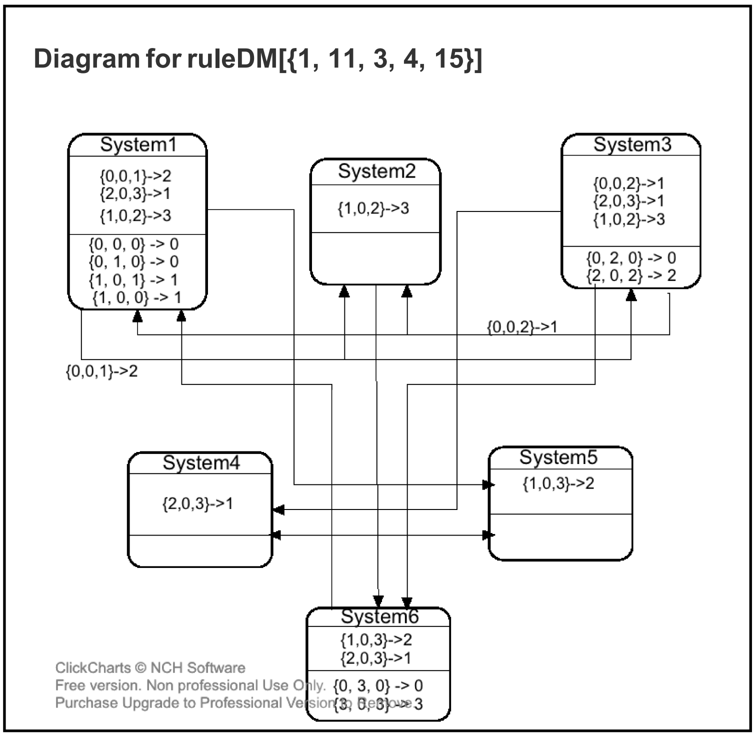
```
{0, 1} = sys1, {1, 2} = sys2, {2, 3} = sys4
       {0, 2} = sys3, {1, 3} = sys5,
              {0, 3} = sys6
```

Transition system table for ruleDM[{1,11,3,4,15}]

S1	S1	S2, 3
S2	S2	–
S3	S3	S1, 2

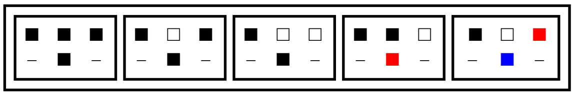
S1, 2, 3	S5, 6, 4
S1, 6, 5	S2, 3, 4
S3, 6, 4	S2, 1, 5

Diagram scheme for ruleDM[{1, 11, 3, 4, 15}]



The rules placed in the first half are the rules of intra-contextural actions. They don't refer to other contextures. The rules in the upper part represent the trans-contextural actions between different contextures depicted as directed arrows.

The compound morphogram of **ruleDM[{1, 3, 4, 11, 15}]** reflects the *mediation* of *intra*- and *inter*-contextural actions of the flow chart. It is the morphogram compound of the flow chart of the actions of the morphoCA **ruleDM[{1, 3, 4, 11, 15}]** .



Non-reducible examples

Non-reducible automata definitions might be used as *complete irreducible building-blocs* for complex morphoCAs.

For *complete irreducible building-blocs,* all entries of the transition table are occupied. In other terminology, all intra-, inter- and trans-contextural sections of the flow-chart scheme are occupied.

Irreducible rules are playing the same role for morphoCAs as the irreducible binary functions like NAND, XOR for binary reductions. With NAND or NOR, all other two-valued binary function are defined. Because they are not reducible they are used as elementary devies in electronic circuit consturctions.

Unfortunately, there is not yet an algorithmic procedure to minimize (reduce) the functional representation of morphoCA rules.

The question for morphic patterns arises: *How many irreducible patterns exist for morphoCA$^{(3,3)}$?*

In analogy:
“No logic simplification is possible for the above diagram. This sometimes happens. Neither the methods of Karnaugh *maps nor* Boolean algebra *can simplify this logic further. [..] Since it is not possible to simplify the Exclusive-OR logic and it is widely used, it is provided by manufacturers as a* basic integrated circuit *(7486).”*

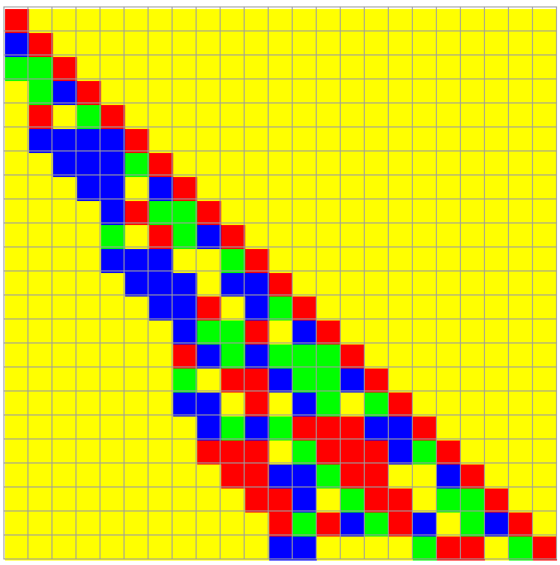
http://www.allaboutcircuits.com

http : // memristors.memristics.com // Reduction %20 and %20 Mediation/Reduction %20 and %20 Mediation.pdf

Example : ruleDM[{1, 2,12,4,15}]

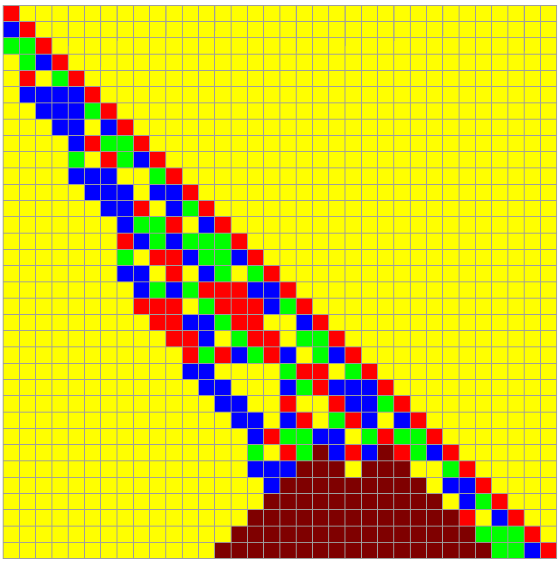
reducible to steps 22

```
ruleDM[{1, 2, 12, 4, 15}]
```



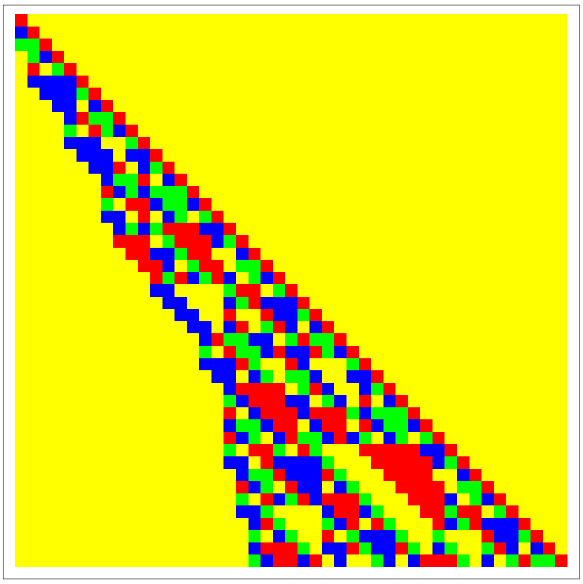
Reducts

```
{1, 1, 3} → 1, {3, 3, 0} → 3, {0, 2, 0} → 1,
{1, 0, 1} → 2, {3, 1, 3} → 2,
{2, 1, 1} → 2 {3, 0, 0} → 3, {3, 2, 2} → 3
```

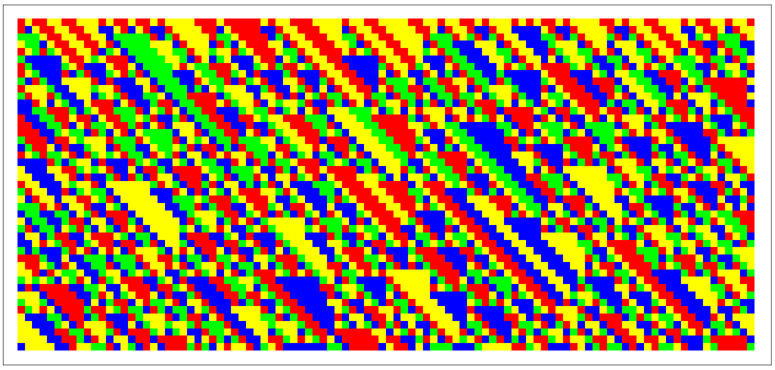


Not reduced

```
ruleDM[{1, 2, 12, 4, 15}]
```



Random



Analysis

Analysis of the interaction patterns

Computation: intra-contextural actions

```
{0, 0, 0} → 0,  : sys1
{0, 0, 1} → 0,
{0, 1, 1} → 0,
{1, 0, 0} → 1,
{1, 1, 0} → 1,
{1, 1, 1} → 1,
```

```
{1, 1, 1} → 1,  : Sys2
{1, 1, 2} → 1,
{1, 2, 2} → 1,
{2, 1, 1} → 2,
{2, 2, 1} → 2,
{2, 2, 2} → 2
```

```
{0, 0, 0} → 0,  : Sys3
{0, 0, 2} → 0,
{0, 2, 2} → 0,
{2, 2, 0} → 2,
{2, 0, 0} → 2,
{2, 2, 2} → 2
```

```
{2, 2, 2} → 2,  : Sys4
{2, 2, 3} → 2,
{2, 3, 3} → 2,
{3, 2, 2} → 3,
{3, 3, 2} → 3,
{3, 3, 3} → 3
```

```
{1, 1, 1} → 1,  : Sys5
{1, 1, 3} → 1,
{1, 3, 3} → 1,
{3, 1, 1} → 3,
{3, 3, 1} → 3,
{3, 3, 3} → 3
```

```
{0, 0, 0} → 0,  : Sys6
{0, 0, 3} → 0,
{0, 3, 3} → 0,
{3, 0, 0} → 3,
{3, 3, 0} → 3,
{3, 3, 3} → 3
```

Alternation: inter-contextural actions

```
{0, 1, 0} → 2,  : sys1 → sys2 || sys3
{1, 0, 1} → 2   :  sys1 → sys3 || sys2

{0, 0, 2} → 1 : sys3 → sys1 || sys2
{2, 2, 0} → 1,

{0, 2, 0} → 1,
{2, 0, 2} → 1,

{0, 0, 3} → 2,  : sys6 → sys3 || sys4
{3, 3, 0} → 2,

{0, 3, 0} → 2,
{3, 0, 3} → 1,

{1, 2, 1} → 0,
{2, 1, 2} → 0,  : sys2 → sys1 || sys3

{2, 2, 3} → 0,  : sys4 → sys3 || sys6
{3, 3, 2} → 1,  : sys4 → sys5 || sys2

{2, 3, 2} → 1,
{3, 2, 3} → 1,

{1, 1, 3} → 2,  : sys5 → sys2 || sys4
{3, 3, 1} → 0,  : sys5 → sys6 || sys1

{3, 1, 3} → 2,
{1, 3, 1} → 2,
```

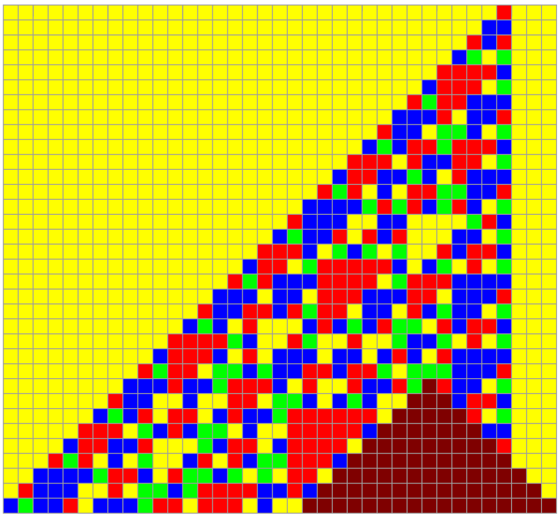
Mediation: poly-layered trans-contextural action

```
{0, 2, 1} → 3,
{0, 1, 2} → 3,
{1, 0, 2} → 3,
{1, 2, 0} → 3,
{2, 0, 1} → 3,
{2, 1, 0} → 3  : sys1, 2, 3 → sys5 || sys6 || sys4

{0, 3, 1} → 2,
{0, 1, 3} → 2,
{1, 0, 3} → 2,
{1, 3, 0} → 2,
{3, 1, 0} → 2,
{3, 0, 1} → 2  : sys1, 6, 5 → sys2 || sys3 || sys4

{0, 2, 3} → 1,
{0, 3, 2} → 1,
{2, 3, 0} → 1,
{2, 0, 3} → 1,
{3, 0, 2} → 1,
{3, 2, 0} → 1   : sys3, 6, 4 → sys2 || sys1 || sys5

{1, 2, 3} → 0,
{1, 3, 2} → 0,
{2, 1, 3} → 0,
{2, 3, 1} → 0,
{3, 1, 2} → 0,
{3, 2, 1} → 0 : sys4, 5, 2 → sys6 || sys3 || sys1
```

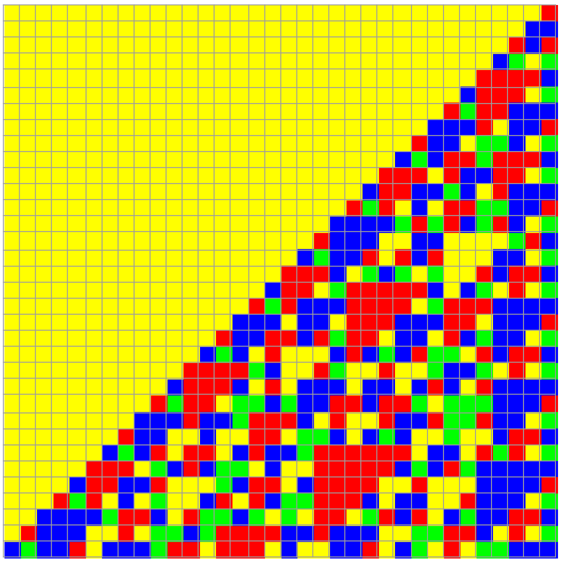
Example: ruleDM[{1, 11, 12, 9, 15}]

ruleDM[{1, 11, 12, 9, 15}] : reducible with {3,3,3}-> 3 for steps <33

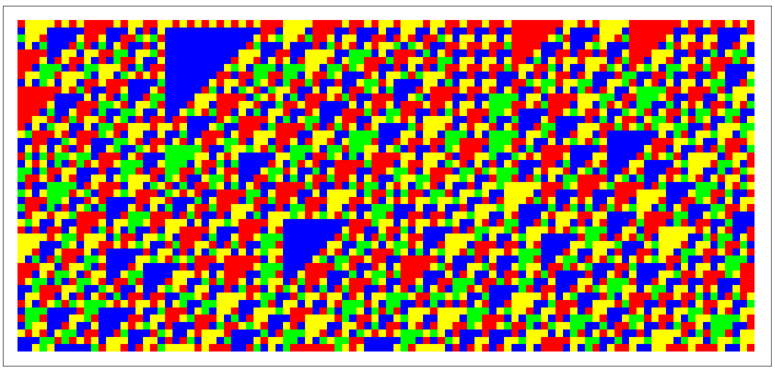
```
ruleDM[{1, 11, 12, 9, 15}]
```



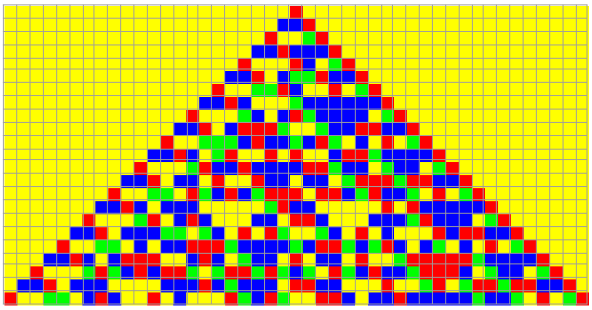
Non - reducible for steps >22



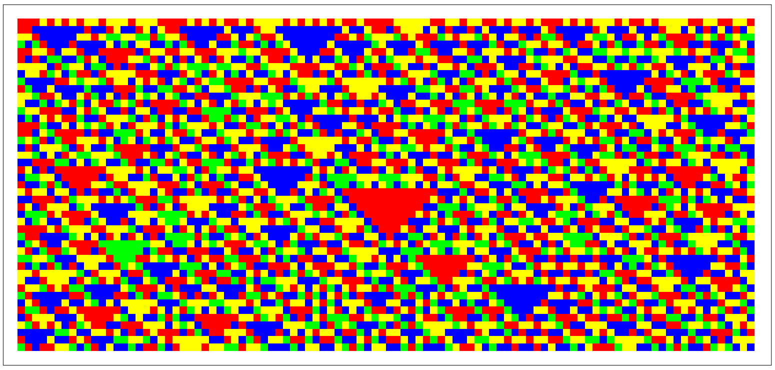
Random



Example : ruleDM[{1, 11, 12, 4, 15}]



Random



Analysis

ruleDM[{1, 11, 12, 4, 15}]

x / yz	00	01	02	03	10	20	30	11	12	13	21	22	23	31	32	33
0	0	2	1	2	2	1	2	0	3	2	3	0	1	2	1	0
1	1	2	3	2	2	3	2	1	1	2	0	1	0	2	0	1
2	2	3	1	1	3	1	1	2	0	0	0	2	0	0	1	2
3	3	2	1	1	2	1	2	3	0	1	0	3	1	2	1	3

Distribution density: [14,18,15,10]

0 / 14	1 / 18	2 / 15	3 / 10
000	100	200	300
011	002	001	201
212	202	101	102
312	302	003	210
213	203	103	120
121	303	010	311
221	020	130	012
321	220	211	021
022	320	013	322
123	230	113	333
223	111	222	
231	112	031	
132	313	131	
033	122	331	
	023	233	
	323		
	032		
	133		

DistrDense (ruleDM[{1, 11, 12, 4, 15}]) = (14, 18, 15, 10)

Analysis of the interaction patterns

Computation: intra-contextural actions

```
{0, 0, 0} → 0,  : sys1
{0, 1, 1} → 0
{1, 0, 0} → 1,
{1, 0, 0} → 1,
{1, 1, 1} → 1,
```

```
{1, 1, 1} → 1,  : Sys2
{1, 2, 1} → 2,
{1, 2, 2} → 1,
{2, 1, 1} → 2,
{2, 2, 2} → 2
```

```
{0, 0, 0} → 0,  : Sys3
{0, 2, 2} → 0,
{2, 0, 0} → 2,
{2, 2, 2} → 2
```

```
{2, 2, 2} → 2,  : Sys4
{2, 3, 3} → 2,
{3, 2, 2} → 3,
{3, 3, 3} → 3
```

```
{1, 1, 1} → 1,  : Sys5
{1, 3, 3} → 1,
{3, 1, 1} → 3,
{3, 3, 3} → 3
```

```
{0, 0, 0} → 0,  : Sys6
{0, 3, 3} → 0,
{3, 0, 0} → 3,
{3, 3, 3} → 3
```

Alternation: inter-contextural actions

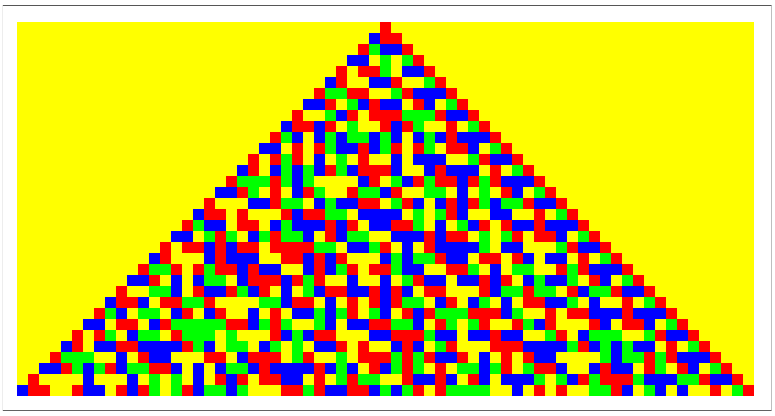
```
{0, 0, 1} → 2 : sys1 → sys2 || sys3
{1, 1, 0} → 2 :  sys1 → sys3 || sys2

{0, 1, 0} → 2,
{1, 0, 1} → 2,

{0, 0, 2} → 1 : sys3 → sys1 || sys2
{2, 2, 0} → 1,

{2, 0, 2} → 1,
{0, 2, 0} → 1,

{0, 0, 3} → 2,  :sys6 → sys3 || sys4
{3, 3, 0} → 2,

{0, 3, 0} → 2,
{3, 0, 3} → 1,

{1, 1, 2} → 0,  :sys2 → sys1 || sys3
{2, 2, 1} → 0,

{1, 2, 1} → 0,
{2, 1, 2} → 0,

{2, 2, 3} → 0,  :sys4 → sys3 || sys6
{3, 3, 2} → 1,  :sys4 → sys5 || sys2

{2, 3, 2} → 1,
{3, 2, 3} → 1,

{1, 1, 3} → 2,  :sys5 → sys2 || sys4
{3, 3, 1} → 0,  :sys5 → sys6 || sys1

{3, 1, 3} → 2,
{1, 3, 1} → 2,
```

Mediation: poly-layered trans-contextural action

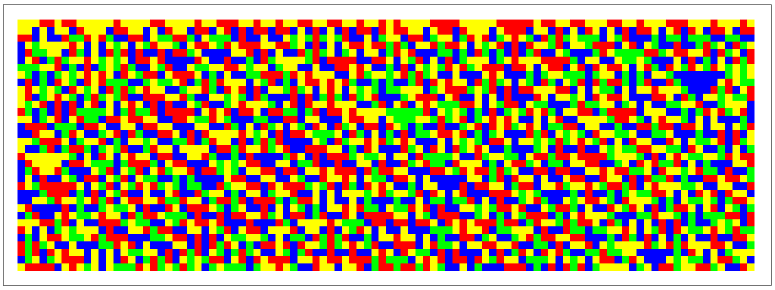
```
{0, 2, 1} → 3,
{0, 1, 2} → 3,
{1, 0, 2} → 3,
{1, 2, 0} → 3,
{2, 0, 1} → 3,
{2, 1, 0} → 3  : sys1, 2, 3 → sys5 || sys6 || sys4

{0, 3, 1} → 2,
{0, 1, 3} → 2,
{1, 0, 3} → 2,
{1, 3, 0} → 2,
{3, 1, 0} → 2,
{3, 0, 1} → 2  : sys1, 6, 5 → sys2 || sys3 || sys4

{0, 2, 3} → 1,
{0, 3, 2} → 1,
{2, 3, 0} → 1,
{2, 0, 3} → 1,
{3, 0, 2} → 1,
{3, 2, 0} → 1   : sys3, 6, 4 → sys2 || sys1 || sys5

{1, 2, 3} → 0,
{1, 3, 2} → 0,
{2, 1, 3} → 0,
{2, 3, 1} → 0,
{3, 1, 2} → 0,
{3, 2, 1} → 0 : sys4, 5, 2 → sys6 || sys3 || sys1
```

Example : ruleDM[{1, 11, 8, 4, 15}]

ruleDM[{**1**, **11**, **8**, **4**, **15**}]



Random



Analysis

```
ruleDM[{1, 11, 8, 4, 15}]
```

x / yz	00	01	02	03	10	20	30	11	12	13	21	22	23	31	32	33
0	0	2	1	2	1	2	3	0	3	2	3	0	1	2	1	0
1	1	0	3	2	2	3	2	1	1	2	2	1	0	3	0	1
2	2	3	2	1	3	1	1	2	1	0	0	2	0	0	3	2
3	3	2	1	0	2	1	2	3	0	1	0	3	2	0	1	3

Analysis of the interaction patterns

Computation: intra-contextural action

```
{0, 0, 0} → 0,  : sys1
{0, 1, 0} → 0,
{0, 1, 1} → 0
{1, 0, 0} → 1,
{1, 0, 1} → 1,
{1, 1, 1} → 1,
```

```
{1, 1, 1} → 1,  : Sys2
{1, 2, 1} → 2,
{1, 2, 2} → 1,
{2, 1, 2} → 1,
{2, 1, 1} → 2,
{2, 2, 2} → 2
```

```
{0, 0, 0} → 0,  : Sys3
{0, 2, 2} → 0,
{0, 2, 0} → 2,
{2, 0, 0} → 2,
{2, 0, 2} → 0,
{2, 2, 2} → 2
```

```
{2, 2, 2} → 2,  : Sys4
{2, 3, 2} → 3,
{2, 3, 3} → 2,
{3, 2, 2} → 3,
{3, 2, 3} → 2,
{3, 3, 3} → 3
```

```
{1, 1, 1} → 1,  : Sys5
{1, 3, 1} → 3,
{1, 3, 3} → 1,
{3, 1, 1} → 3,
{3, 1, 3} → 1
{3, 3, 3} → 3
```

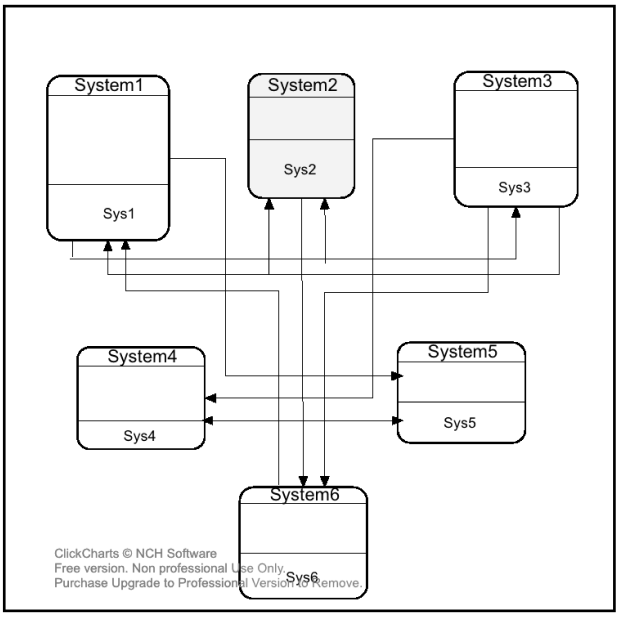
```
{0, 0, 0} → 0,  : Sys6
{0, 3, 0} → 3,
{0, 3, 3} → 0,
{3, 0, 0} → 3,
{3, 0, 3} → 0,
{3, 3, 3} → 3
```

Alternation: inter-contextural actions

```
{0, 0, 1} → 2 : sys1 → sys2 || sys3
{1, 1, 0} → 2 :  sys1 → sys3 || sys2

{0, 0, 2} → 1 : sys3 → sys1 || sys2
{2, 2, 0} → 1,

{0, 0, 3} → 2,  : sys6 → sys3 || sys4
{3, 3, 0} → 2,

{1, 1, 2} → 0,  : sys2 → sys1 || sys3
{2, 2, 1} → 0,

{2, 2, 3} → 0,  : sys4 → sys3 || sys6
{3, 3, 2} → 1,  : sys4 → sys5 || sys2

{1, 1, 3} → 2,  : sys5 → sys2 || sys4
{3, 3, 1} → 0,  : sys5 → sys6 || sys1
```

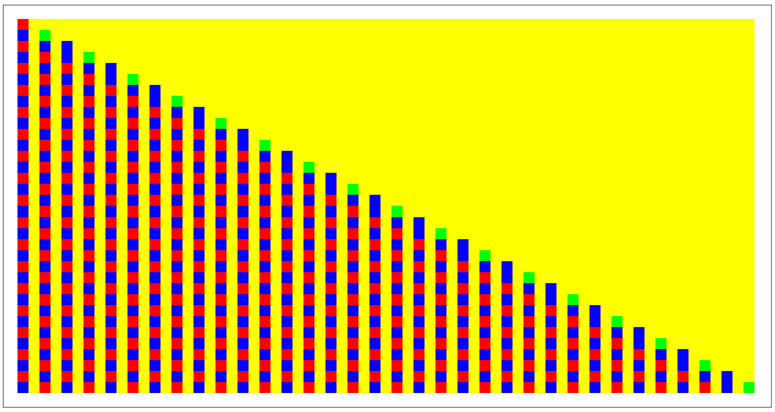
Mediation: poly-layered trans-contextural action

```
{0, 2, 1} → 3,
{0, 1, 2} → 3,
{1, 0, 2} → 3,
{1, 2, 0} → 3,
{2, 0, 1} → 3,
{2, 1, 0} → 3 : sys1, 2, 3 → sys4 || sys5 || sys6

{0, 3, 1} → 2,
{0, 1, 3} → 2,
{1, 0, 3} → 2,
{1, 3, 0} → 2,
{3, 1, 0} → 2,
{3, 0, 1} → 2,  : sys1, 6, 5 → sys4 || sys3 || sys2

{0, 2, 3} → 1,
{0, 3, 2} → 1,
{2, 3, 0} → 1,
{2, 0, 3} → 1,
{3, 0, 2} → 1,
{3, 2, 0} → 1 : sys3, 6, 4 → sys5 || sys2 || sys1

{1, 2, 3} → 0,
{1, 3, 2} → 0,
{2, 1, 3} → 0,
{2, 3, 1} → 0,
{3, 1, 2} → 0,
{3, 2, 1} → 0 : sys4, 5, 2 → sys6 || sys3 || sys1
```



ruleDCKV reduction examples

```
ruleDCKV5[{1111, 1121, 1213, 1223, 1231, 2111, 2121, 2131, 2211,
           2223, 2234, 2311, 2321, 2331, 2344}]
```



Analysis

ruleDCKV5[{ ...}]

x / yz	000	001	002	010	020	101	100	102	201	200	202	203	300
0	0	0	0	2	1	0	0	0	0	0	0	0	0
1	3	□	□	□	1	□	□	□	□	□	□	□	□
2	3	□	□	□	2	□	□	□	□	□	□	□	□
3	2	□	□	□	□	□	□	□	□	□	□	□	□

distribution density: [11,2,4,2]

0 / 11	1 / 2	2 / 4	3 / 2
0, 0, 0, 0	0, 0, 2, 0	2, 0, 2, 0	1, 0, 0, 0
0, 0, 0, 1	1, 0, 2, 0	0, 0, 1, 0	2, 0, 0, 0
0, 1, 0, 0		0, 0, 1, 0	
0, 1, 0, 1		3, 0, 0, 0	
0, 2, 0, 2			
0, 2, 0, 0			
0, 0, 0, 2			
0, 3, 0, 0			
0, 1, 0, 2			
0, 2, 0, 1			
0, 2, 0, 3			

```
DitrDense (ruleDCKV5[{1111, 1121, 1213, 1223, 1231, 2111, 2121, 2131, 2211,
           2223, 2234, 2311, 2321, 2331, 2344}]) = (11, 2, 4, 2)
```

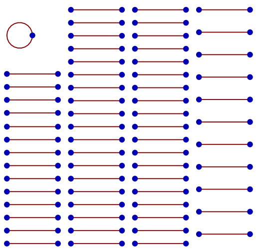
Analysis of the interaction patterns

```
{0, 0, 0, 0} → 0, Sys1
{0, 0, 0, 1} → 0,
{0, 1, 0, 0} → 0,
{0, 1, 0, 1} → 0,

{0, 2, 0, 2} → 0,  Sys2           : intra
{0, 2, 0, 0} → 0,
{0, 0, 0, 2} → 0,
{2, 0, 2, 0} → 2,

{0, 3, 0, 0} → 0 : Sys3
```

```
{0, 0, 1, 0} → 2,  : sys1 → sys2 || sys3
{0, 0, 2, 0} → 1   : sys3 → sys1 || sys2     : inter
```

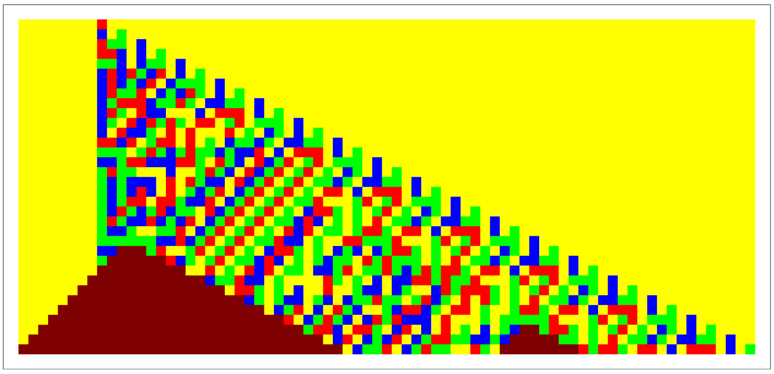
```
{1, 0, 0, 0} → 3,  : sys1 → sys5 || sys6
{2, 0, 0, 0} → 3,  : sys3 → sys4 || sys6   : trans
{3, 0, 0, 0} → 2   : sys6 → sys4 || sys3
```

```
{0, 1, 0, 2} → 0,
{0, 2, 0, 1} → 0,
{1, 0, 2, 0} → 1,
{2, 0, 1, 0} → 2,     : mediation

{0, 2, 0, 3} → 0,
{2, 0, 3, 0} → 2
```



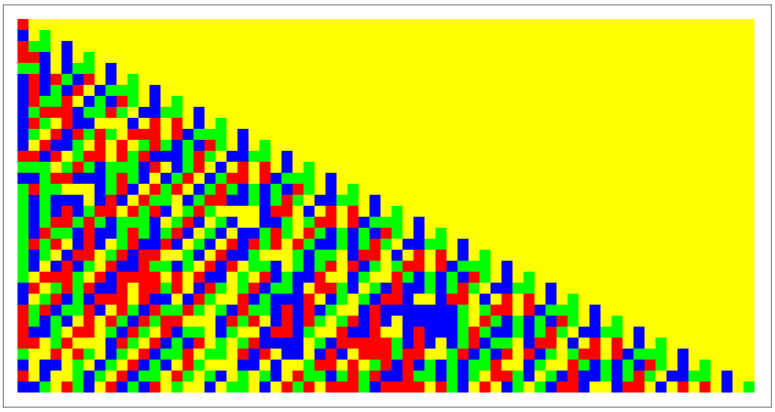
Further example: non-reducible

```
ruleDCKV5[{1111, 1121, 1213, 1223, 1232,
  2111, 2121, 2133, 2211, 2223, 2230, 2311, 2323, 2332, 2300}]
```

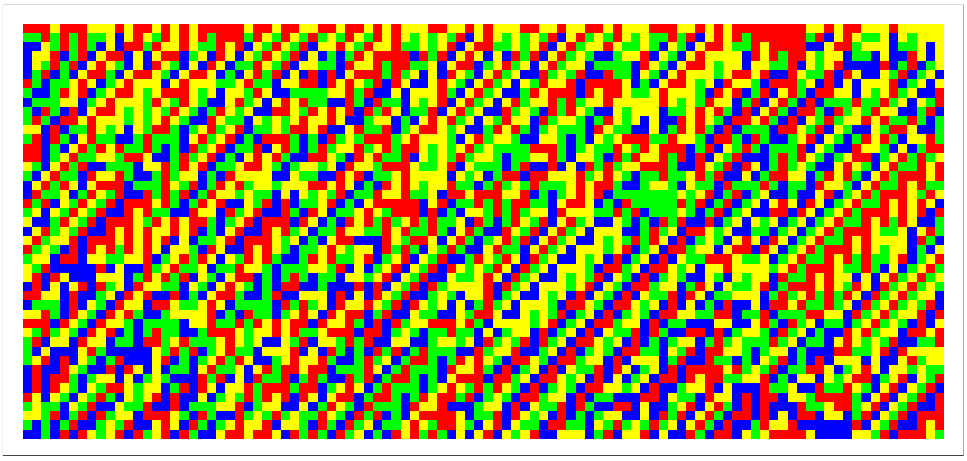


```
{1, 1, 1, 1} → 1, {2, 2, 2, 2} → 2, {3, 3, 3, 3} → 3,
```

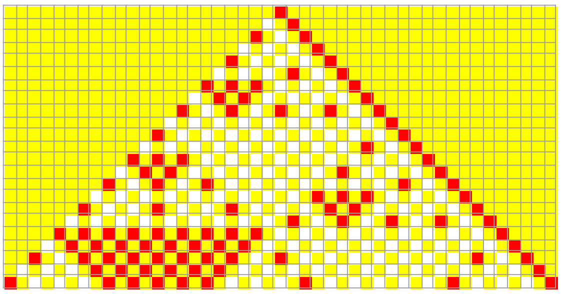
Not reduced

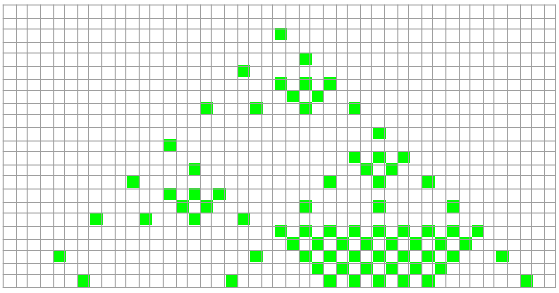


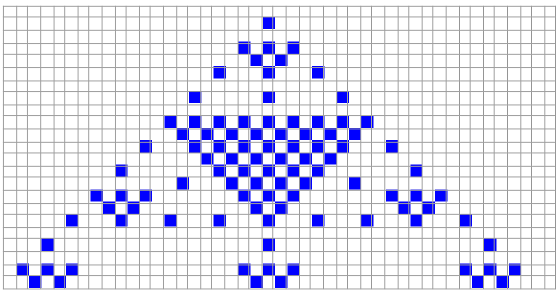
Random

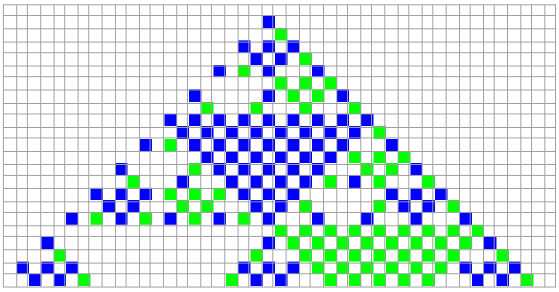


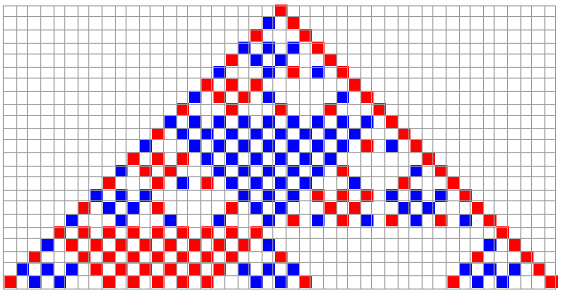
Layers of morphoCA sub-systems of ruleDM[{1,11,3,4,15}]

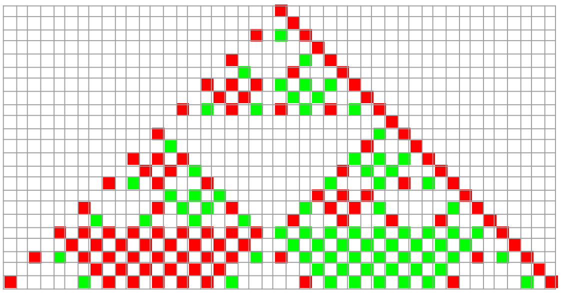




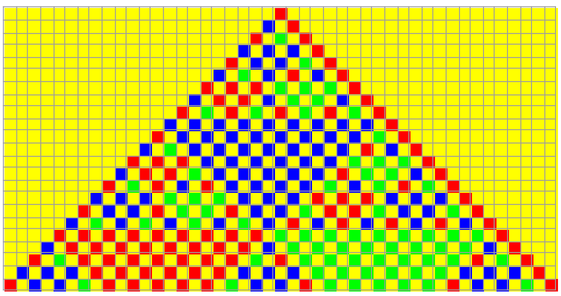




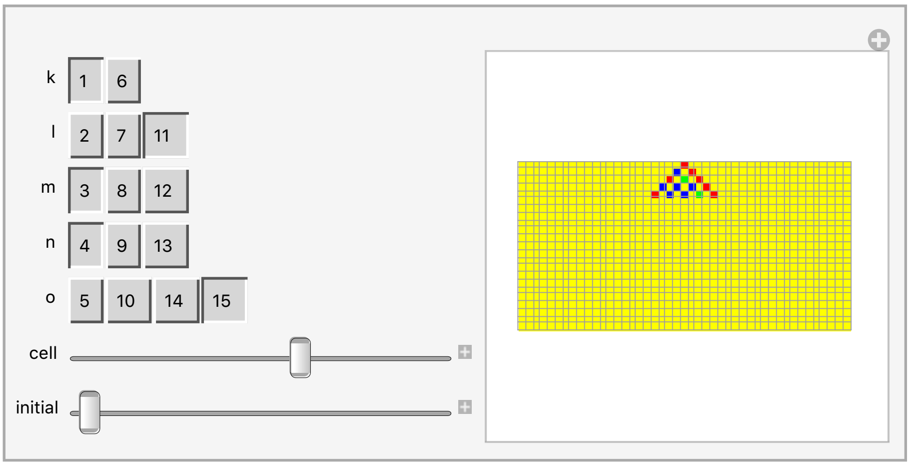




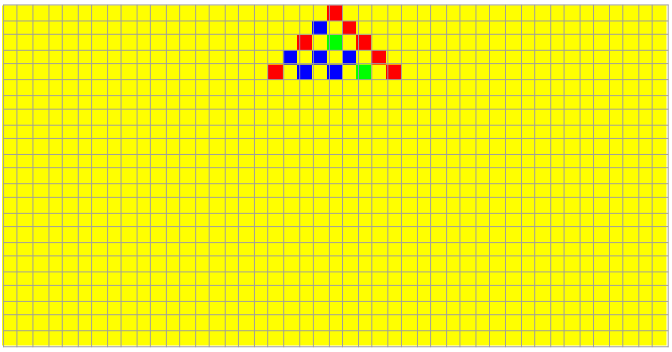
All together



Step - wise developments of ruleDM[{1,11,3,4,15}]



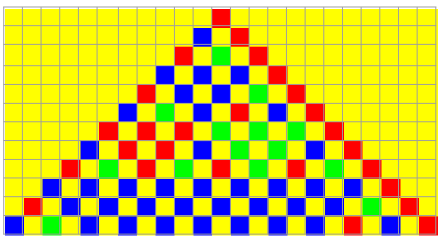

```
                       22 : red
                66 : blue   68 : red
         110 : red  112 : green 114 : red
     154 : blue  156 : blue   158 : blue 160 : red
198 : red  200 : blue 202 : blue 204 : green  206 : red
```



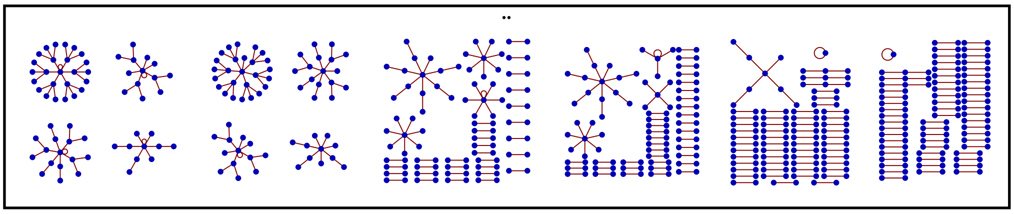
Transition graphs of reduced ruleDM[{1,11,3,4,15}]

Additionally to the difference of reduced and non-reduced morphoCA rule in respect to their seed structure, there is also an interesting difference between **ArrayPlot** visualizations and transition graph representations by the **GraphPlot** of reduced morphoCA rules to observe. All reductions are conserving the full visualization of the original, while the transition structure is significantly reduced.

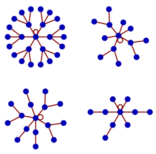
```
ruleDM[{1, 11, 3, 4, 15}]
```

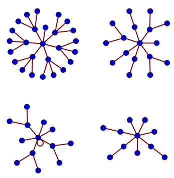


Reduction steps

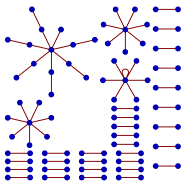


Full pattern

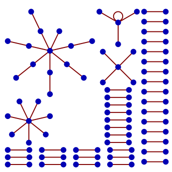


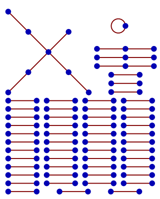


Without {1, 1, 1} -> 1, {2, 2, 2} -> 2, {3, 3, 3} -> 3,



```
Without:
   {0, 0, 3} → 2,
{1, 1, 0} → 2, {1, 1, 2} → 0, {1, 1, 3} → 2, {2, 2, 3} → 0,
{2, 2, 0} → 1, {2, 2, 1} → 0, {3, 3, 2} → 1, {3, 3, 0} → 2, {3, 3, 1} → 0,
```





{3, 2, 0} → 1, {3, 1, 2} → 0, {1, 2, 3} → 0

Fully reduced