

From Universe to Polyverses

The power of speculation by Schelling, Faraday, Gunther and Leon Chua

Rudolf Kaehr Dr. @

ThinkArt Lab Glasgow

Abstract

Some thoughts about the power of speculation behind important discoveries in mathematics, physics and computer science. The exercise shows that there is no need for a compulsory ultimate unifying universe. It is speculated that just this paradigm of a single ultimate universe is unmasking itself today as the main obstacle for further development in Western science and technology.

1. The Universe as the ultimate unity of possible worlds

1.1. Schelling, Faraday and Frege

Schelling proposed the unity of nature in his philosophy of nature. With that he was the first philosopher to develop a view on nature as a unity of all its existence. At this time fields of physical research had been separated and there was no idea or desire to unify them. As a direct consequence of Schelling's philosophical theory of the unity of nature, which was unifying highly unconnected research, a great unification of different principles and phenomena of nature and its studies has taken place.

Today it is not well known that phenomena like galvanic electricity and the interaction between electricity and chemical processes had been studied in the past separately without any idea of a unification.

"eine Einheit aller Kräfte in der Natur..." (Oerstedt)
Schelling, Ideen zu einer Philosophie der Natur..., 1797

Marie-Luise Heuser-Keller, Die Produktivität der Natur, Duncker&Humblot /Berlin 1986

Today a unity of nature or even of the world is generally accepted by all scientists.

One big support for this idea of a single and unified nature is found in the idea and practice of mathematics and mathematical logic.

Also things are highly complex on all levels of thematization and formalization, producing antinomies and paradoxes, and by no way intuitive, the singular unity of the rationality and reality is taken as granted. It might be called the *"One World, One Logic"* paradigm.

Schelling(1775-1855)

Schelling's *'community of causation'* as a unifying principle:

"To begin with, it should be noted that Schelling's Naturphilosophie has had a tremendous influence on science. Oersted's work on chemistry and electro-magnetism was strongly influenced by Schelling. The principle of 'conservation of energy' was formulated by scientists who had been influenced by Schelling. Schelling had a significant influence on the work of

Hermann Grassmann, the brilliant nineteenth century German mathematician (who strongly influenced Alfred North Whitehead)." (Arran Gare)

<http://www.cosmosandhistory.org/index.php/journal/article/viewFile/109/217>

All kind of unification attempt occurred, especially to find a unified theory of quantum physics and relativity. Today, billions of dollars are spent to find the reason, a wee piece of matter, for a final unification. Chua's missing *memristor* was finally found in the labs of HP, as it is called. But Peter Higgs "*Missing Piece*" is well hidden in Geneva while he is still waiting in an Edinburgh pub for a final result, which would, for sure, result in a Nobel Prize, and he is enjoying, again, the Fringe Festival.

One of the latest proposal to a unified theory is given by Samson Abramsky and Bob Coecke (Oxford, Edinburgh) with their theory of monoidal categories for quantum physics and everything else. A theory for all kind of processes, from cooking to doing quantum physics, and writing poems.

But speculation didn't stop only because of the immense and nearly total hegemony of the paradigm of a unique unity of the world got into trouble.

Frege did his best to develop a : "*Begriffsschrift*" for the description and calculation of such a unified rational world-view, applicable to all facts. The fact that he failed and produced "system shattering" antinomies (B. Russell), which cracked the principle of unital harmony, hasn't stopped his followers to continue this path of unification, albeit with some restrictions concerning the disillusionment in the strictness of the program.

What happened in philosophy, logic and mathematics, continued for computer and information science.

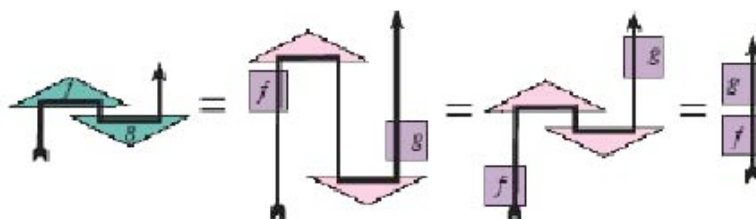
"This paper attempts to provide a common basis for physical and computational ways of thinking. . . . If this approach should turn out to be a small, but definite step towards the remote (perhaps illusory) goal of founding technology and natural sciences on a theory of information flow, the author would feel rewarded beyond merit." (Carl-Adam Petri)

From Abramsky: Petri, C.-A., State-Transition Structures in Physics and in Computation. International Journal of Theoretical Physics, 21(12), (1982) 979-993.

"Note in particular the interesting phenomenon of "apparent reversal of the causal order". While on the left, physically, we first prepare the state labeled g and then apply the costate labeled f , the global effect is as if we first applied f itself first, and only then g . This corresponds to the apparent reversal of flow of computations in the token game on Petri nets achieved with deficits and cancellations." Abramsky, Festschrift

Antidromicity as "apparent reversal of the causal order" (Petri)

Abramsky/Coecke's approach of modeling "*antidromicity*" in the framework of monoidal categories.



This "*apparent reversal of the causal order*" or "*reversal of flow of computations*" is modeled as an equation between the left and the right part of the bipartite phenomenon of "*quantum information flow in (bipartite) entangled quantum systems*".

The strategy is to decompose composition into bras and kets and recombine them back to

composition.

Certainly, this modeling is showing intriguing results.

But the “*One World-One Logic*” presumptions are producing *amphicranial* headache.

From a polycontextural point of view the process of parallel and reversal order is not explicitly modeled by the proposed operators but by two different but equal “interpretations” or “decompositions” of the primary formula over f and g . The simultaneity proposed of the two aspects, presented as the two parts of an equation or equivalence is not realized in the notation as such but has to be understood, i.e. mentalized by the reader. Hence, the phenomenon of parallel and reverse order is modeled not explicitly in the formula but as a mental representation by the designer of the formula.

A further step of formalization would have to implement antidromicity directly into the operators as it happened with the polycontextural approach to diamond category theory.

If the reversal order would be conceived at once as “left” and “right” movement, the construction would automatically produce logical contradictions (in whatever logic).

But all that seems not to work under the general hypothesis of a single universe of unified physical and mathematical theories without producing logical and other problems, disturbance and paradoxes.

The epistemological status of “*as if*” (Abramsky) of both parts of the reversal order is not concretized by a formal definition and seems to refer therefore to the subjective understanding of the reader. Metaphysically, the whole approach unmasks itself as a further attempt of idealism in physical studies, especially the metaphysics of *as-if* fictionalism (Hans Vaihinger, *Philosophie des Als Ob*).

1.2. Power of speculation by Leon Chua

It was part of a speculative desire to harmonize the laws of electronics which lead Leon Chua to his construction of the memristor “*as the fourth element of electronics*”.

The physical memristor as a possible device wasn’t found by experiments but by an interpretation of results of experiments by an accidental discovery of Chua’s speculative formulas and diagrams (pinched-hysteresis loop) and the application of Chua’s speculated behaviors of his stipulated memristor and the pinched-hysteresis loops to such not yet properly interpreted experimental data. Nobody was searching for the “missed fourth element” because nearly nobody did know Chua’s paper about his memristor. What happened, as far as it is reported, was an accidental coincidence of research and a possible interpretation of that research by Chua’s speculative paper from 1971.

Further steps to unify the theory according to speculative harmony and unity of electronics and its nanoelectronic memristor, lead to the generalization of the discoveries towards memristive systems with mem-capacitor, mem-inductor, and other mem-devices. Those constructions are supporting a new level of harmonic symmetry but are also radically deconstructing this desire for unification. The just found fourth element, which closed the gap and re-installed harmony, is now embedded in a chain of further mem-elements. Hence, the memristor as a history- and time-dependent device, is playing a double role as the fourth element of electronics, closing the system, and as the “first” element of a new system of nanoelectronics.

The closure of electronics and the advent of nano-electronics inaugurated by Chua, is therefore a unification of both spheres, the micro- and the nano-electronics, into a unified physical world. All this formulated in a general theory of complex systems, which is, without doubt, based on Grothendieck’s universe.

The project started with Schelling, motivated Faraday's efforts to unify magnetism and electricity, and got a new climax with Chua's memristive systems. Nevertheless, Schelling transcended his approach of "One World-One Logic (in God)" subversively by the speculation of a trans-unitarian world of permanent interplay of discontextual spheres.

„Seine [Schelling, rk] These, es gäbe weder die 'eine Wahrheit' noch die 'eine Wirklichkeit', sondern das Universum sei vielmehr als ein 'bewegliches Gewebe' aufeinander nicht zurückführbarer Einzelwelten zu denken, formulierte die entscheidende Aufgabe der Philosophie der Zukunft: eine Theorie bereitzustellen, die es gestattet, die Strukturgesetze des organischen Zusammenwirkens der je für sich organisierten Teilwelten aufzudecken.“
Gotthard Günther, Nachlass „GG“, 15. Juni 1980

Duration: Memory and Anticipation

"Bergson argued that time as duration is different than the spatialized time as measured by physicists. As durational it cannot be conceived as a point moving along a line but involves memory and anticipation." (Arran Gare)

This traditional Western unificational approach is behind the advent of a new epoch of hardware development in computational technology. On the other hand, it is exactly this desire for harmony and unification which will be discovered as the main obstacle for the invention and discovery of more lively devices and 'machines'.

Today it might be speculated that the difference of monocontextual *multi-layered crossbar* systems based on a clear distinction of the roles of memristors as *memory* and as *computational* devices, and in discontextual contrast, the polycontextual *poly-layered "crossbar"* constructions, based on the interplay of the memristive roles, will become a decisive challenge for a harmony-based technology. This challenge will force a radical transition to an overcoming of the single harmony-based paradigm of thinking and research.

2. Towards a theory of poly-verses

2.1. Deconstruction of the unity of the universe by Gotthard Gunther

Gotthard Gunther's theory of polycontextuality is paradoxically both at once, unifying natural science and humanities, i.e. Natur- und Geisteswissenschaften, but at the same time destroying the hegemony of unity of the world in favor of a multiplicity of distributed and mediated contextures.

"A great epoch of scientific tradition is about to end. It has lasted almost two-and-a-half millennia and philosophers and scientists begin to call it the classical period of science. However, there is not yet a clear conception of what basically characterizes the past scientific tradition and what distinguishes it from the era we are about to enter and which might rightly be called the age of trans-classical science. We shall start our reflections with a short analysis of the fundamental difference between the two. It is possible to trace the distinction between the classical and the trans-classical back to deeply hidden metaphysical assumptions about the nature of this Universe.

"We are now ready to see the deep ontological assumption which lies behind the epistemology of Aristotle. It can be formulated as follows: the Universe is, logically speaking, "mono-contextual". Everything there is belongs to the universal contexture of objective Being. And what does not belong to it is just Nothingness.

From all this follows that every logical operation we can perform is confined to the contextuality in which it originates. It is trivial to add that no logical operation can start in Nothingness or continue there. But also, if we count numbers this process of counting, i.e., the sequence of numbers, is confined to the contextuality in which it originates. You cannot cross the borderline between Being and Nothingness and still continue your process of counting.

"Such arguments are obvious. However, what is by no means self-evident is that we have to consider Nihilism or Nothingness also as an "ontological" contexture. The difficulty is that, if we insist on describing Nothingness as a contexture, we have to borrow our terms from Being, and doing so we discover we have only repeated our description of the contextuality of Being.

"On the other hand, if we speak about the Universe as a whole, the very term uni-verse suggest that all contextualities somehow form a unit, the unit of contextual existence and co-existence. We shall call such a unit a compound-contextuality.

"On the other hand, the turn from classic to trans-classic thinking means that the mono-contextual concept of Reality is abandoned and replaced by a poly-contextual theory of Existence which makes room for the phenomenon of Life within this Universe. In a poly-contextual Universe we do not have to consider Life as an element totally alien to inanimate matter, because matter in itself already contains the seeds of Life in its dialectical contraposition of Being and Nihilism." (Gotthard Gunther)

in: H. Fahrenbach (Hrsg.), Wirklichkeit und Reflexion, Festschrift für Walter Schulz, Pfullingen 1973

"The opposition between Being and Nothingness is the most elementary case of discontextuality. If it would be the only one describing our universe then Hegel's logic would be completely superfluous and it would be impossible for ever to go beyond the classical tradition of thinking and reflecting. Our reality, however, is interwoven by lots of further discontextualities separating an infinite number of contextures. So the range of all bona fide objects represents one contexture and another one is represented by the psychical sphere of a conscious subject perceiving these objects. A further example of discontextuality is given by the radical separation between the sphere of consciousness of an I and the psychical sphere of a Thou. All our efforts to experience the conscious processes within another I will never be successful, because all psychical experiences bound to different I-centers belong to different contextures and are related discontextually to each other." Gunther, Hegel-Jahrbuch 1970, 34-61

3. From monoidal categories to polycontextuality

Polycontextual category theory differs from classical category by the fact that the latter is based on a single and unique universe (of objects, morphisms, composition and juxtaposition) while the first risks the option of a multitudes of universes, leading to a poly-verse of different discontextual categories, which are disseminated, i.e. distributed and mediated.

The fact of the universe of object is not well studied in ordinary category theory; it is supposed as a triviality, and if studied, then in meta-theoretical reflections. Grothendieck did crucial seminal work. Here, a universe is a set of all sets, i.e. a conglomeration, without considering possible implicit contradictions. universes are ruling the hierarchy of sets, classes and conglomerates.

The objects of category theory belong to these collections. Obviously, categorical objects are not simply sets but, e.g., categories of categories, hence surpassing all reasonable, i.e., contradiction-free notions of set theory.

Hierarchy of the Collections of the universe $U = [\text{sets, classes, conglomerates}]$.

Conglomerates

Classes = subcollections of the universe U

Sets = small classes = elements of U

3.1. Grothendieck's Universe

3.1.1. "*One universe as a foundation of category theory*", Mac Lane, 1969

http://modular.fas.harvard.edu/sga/sga/4-1/4-1t_185.html

<http://www.grothendieckcircle.org/>

- 185 -

S G A 4

E X P O S E I

II. Appendice : Univers (par N. BOURBAKI (*))1. Définition et premières propriétés des univers

DEFINITION 1. Un ensemble U est appelé un univers s'il satisfait aux conditions :

(U.I) si $x \in U$ et si $y \in x$, alors $y \in U$;

(U.II) si $x, y \in U$, alors $\{x, y\} \in U$;

(U.III) si $x \in U$, alors $P(x) \in U$;

(U.IV) si $(x_\alpha)_{\alpha \in I}$ est une famille d'éléments de U, et si $I \in U$, alors la réunion $\bigcup_{\alpha \in I} x_\alpha$ appartient à U ;

(U.?) si $x, y \in U$, alors le couple $(x, y) \in U$.

N.B. Comme il a été je crois décidé pour les prochaines éditions, on définit le couple à la Kuratowski par $(x, y) = \{\{x, y\}, \{x\}\}$, la condition (U.?) est inutile car elle résulte de (U.II).

Exemples.

- 1) L'ensemble vide est un univers noté U_0 .
- 2) Considérons les mots finis non vides formés avec les quatre symboles " { ", " }", " , " et " $\emptyset$ " (cf. Alg. I). Définissons, par

(*) Nous reproduisons ici, avec son accord, des papiers secrets de N. BOURBAKI. Les références de ce texte se rapportent à son savant ouvrage.

"A Grothendieck universe is a set U with the following properties:

1. If x is an element of U and if y is an element of x , then y is also an element of U . (U is a transitive set.)

2. If x and y are both elements of U , then $\{x,y\}$ is an element of U .
3. If x is an element of U , then $P(x)$, the power set of x , is also an element of U .
4. If $\{x_\alpha\}_{\alpha \in I}$ is a family of elements of U , and if I is an element of U , then the union $\bigcup_{\alpha \in I} x_\alpha$ is an element of U .

A Grothendieck universe is meant to provide a set in which all of mathematics can be performed." (Wiki)

Proposition1

If $x \in U$ and $y \subseteq x$, then $y \in U$.

Category

A consists of of a class of *objects*: $\text{ob}(C)$

A class of *morphisms*: $\text{hom}(C)$

A binary operation $\circ$, called *composition* of morphisms, with associativity and identity.

Because a category is considered as holding for all objects, the object of such a class belongs to a universe in the sense of Grothendieck.

It is therefore quite nonsensical or "*non-scientific*" to reclaim the possibility of a *poly-verse* of poly-objects for poly-contextural category theory. Nevertheless, there is also some strangeness in the introduction of a Grothendieck universe, it seems to be circular and its mono-contextuality is not justified at all.

All kinds of mathematical constructions for multitudes of domains, sorts, sets, categories are based on the unique Grothendieck universe.

Hence, multi-sorted set theory, within multi-sorted predicate logic,

indexed categories,

multi-valued set theories,

context theories,

monoidal categories,

n -categories,

are all introducing multitude as a *secondary* concept, related to the unique first-order concept of a Grothendieck universe.

<http://www.thinkartlab.com/pkl/lola/Elements/Elements.html>

3.1.2. Complementarity of set and category

Sets as categories

"Any set S is a category whose objects are the elements of S and the only arrows are the identity arrows. A category in which the only arrows are the identity arrows is a *discrete category*." (Awodejy, Bauer, 2003)

For *small* categories, objects are sets, otherwise they are classes.

According to the hierarchy of conglomerates, classes and sets, generally, categories are not sets but classes, and able to construct categories of categories without automatically producing paradoxes.

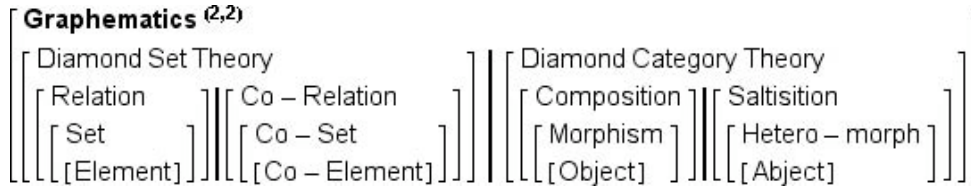
Also mathematicians are denying their ontological heritage, the classification simply still is: one and only one *fundamental* ontology, many sorts of *regional* ontologies.

Now, after category theory got enough academic recognition and applause even from computer scientists, it is time to speculate about the complementarity of both approaches. Following the commitment to singular unification there is a hierarchy between set theory and category theory. Depending on the school, category theory is sublime, for the others, set-theory is the ultimate endeavour.

In fact, observed from a more neutral position, not involved in propaganda, both achievements are thematizing *different* aspects of conceptualizations and formalizations of mathematico-

logical informational theories.

Hence, both are well placed in an interactional game of heterarchical thematizations, dissolving, again, the terror of compulsory hierarchical unification. The opposite of hierarchy is not *anarchy* as the propagandist of scientific unitarianism are preaching, it is heterarchy. It is an irony or at least riddle that Grothendieck, who did fundamental work for a unification of mathematics was politically and biographically an active *radical anarchist*.



3.1.3. Different logics

There could also be a chapter about *deviant*, *alternative*, *heterodox* or non-classical, non-Aristotelian, Hegelian, dialectical, complex logics, etc. The list was growing daily in the 70s. The exercise shows that they all depend on a *single ultimate universe*, and certainly also on classical semiotics as their sign base. There is not much to add. Some German philosophers called such formalism, even modal logic, not logics but “*logoide Formalismen*.”

I leave this exercise to the reader.

Certainly, the game has to involve a deconstruction of arithmetics too. (G. Gunther, A. Yessenin Volpin)

Kaehr, R.: Neue Tendenzen in der KI-Forschung-Metakritische Untersuchungen über den Stellenwert der

Logik in der Künstlichen-Intelligenz-Forschung, Stiftung Warentest, Berlin 1980.

http://www.vordenker.de/ggphilosophy/rk_meta.pdf

3.2. Monoidal categories

3.2.1. Bifunctoriality

Monoidal categories might be considered as another strategy to introduce *multitude* within the framework of a Grothendieck universe. A multitude over objects of a universe is established by the introduction of a new type of composition: *yuxtaposition*.

This juxtaposition, which is a parallel operation in contrast to the serial operation of composition, is considered as of the same level of abstraction as the fundamental operation of composition. This gets its reason with a change of strategy from an *abstract* mathematical to a more *concrete* physical modeling of operations and processes, i.e. *serial* for composition ($\circ$) and *parallel* for juxtaposition ($\otimes$). With the obvious condition of strictness that no composition becomes a juxtaposition and vice versa, no juxtaposition becomes a composition.

$$\text{CAT}_{\text{BIF}} = (\text{obj}, \circ, \otimes)$$

The great advantage of this subversive approach to category theory is the introduction of an inter-relation between composition and juxtaposition inscribed as bifunctoriality.

classical composition and juxtaposition

BIFUNCT $\begin{pmatrix} 2 \\ \end{pmatrix}$ $\begin{bmatrix} g_1 & g_2 \\ f_1 & f_2 \end{bmatrix} :$

$$\begin{pmatrix} f_1 \\ \otimes \\ f_2 \end{pmatrix} \circ \begin{pmatrix} g_1 \\ \otimes \\ g_2 \end{pmatrix} = \begin{pmatrix} (f_1 \circ g_1) \\ \otimes \\ (f_2 \circ g_2) \end{pmatrix}$$

Monoidal category

(C1) Rather than an underlying set of elements, as in the case of a group,

we have two sorts of things, one to which we referred as *systems*, and the other to which we referred as *processes*.

(C2) There is an operation $- \circ -$ on systems as well as an operation $- \otimes -$ on processes, with

respective units I and 1 . In addition to this operation, there is also an operation $- \circ -$ on processes,

but for two processes $A \xrightarrow{f} B$ and $C \xrightarrow{g} D$, their composite $g \circ f$ exists if and only if we have $B = C$.

(C3) The way in which $- \otimes -$ and $- \circ -$ interact with each other is given by the laws." (Coecke)

3.2.2. Back in Bob's Kindergarten

The classic mono-contextural wording for physical processes, parallel and serial, is given by Coecke's cooking example:

"That is, 'boiling the potato and then salting it, while, frying the carrot and then peppering it', is equal to 'boiling the potato while frying the carrot, and then, salting the potato while peppering the carrot'." (Coecke)

(cf. <http://memristics.com>, "Bob's Kitchen, refurbished" lost on the hddisk)

Cooking while cooking

"That is,

'boiling the potato (and then)¹ salting the potato,

(while)¹,

frying the carrot (and then)¹ peppering the carrot',

is (equal)¹ to

'boiling the potato (while)¹ frying the carrot,

(and then)¹,

salting the potato (while)¹ peppering the carrot'."

Category Cat1:

Objects: *potato, carrot*

Processes: *boiling, salting, peppering, frying*

Operations: *(and then), (while)*

$$\text{boil potato then fry carrot} = \text{fry carrot then boil potato}.$$

This law is only an instance of a more general law on recipes, namely

$$(\zeta \circ \xi) \otimes (\kappa \circ \omega) = (\zeta \otimes \kappa) \circ (\xi \otimes \omega),$$

which in the particular case of $\xi := f$, $\zeta := g$, $\kappa := k$ and $\omega := h$ reads as:

$$\begin{array}{c} \text{boil potato then salt potato, while, fry carrot then pepper carrot} \\ \parallel \\ \text{boil potato while fry carrot, then, salt potato while pepper carrot} \end{array}$$

boil potato *then* salt potato, *while*, fry carrot *then* pepper carrot
 Π
 boil potato *while* fry carrot, *then*, salt potato *while* pepper carrot.

Bob Coecke, Introducing categories to the practicing physicist

No-operation

"Our use of colours already indicated that states are themselves processes too:

$I \xrightarrow{\psi} A;$

where I stands for **unspecified** or **unknown**, i.e. we don't need to know from what **system** A has been produced, just that it is in **state** ψ and available for **processing**." (Coecke, p. 9)
<http://arxiv.org/pdf/0908.1787v1>

Cooking while thinking

"That is, in the 2-contextual modelling, *the distribution is:*

'boiling the potato (and then)^{1.0} salting the potato,
 (while)^{1.2},
 thinking the carrot (and then)^{0.2} evaluating the carrot',
 is (equal)^{1.2} to
 'boiling the potato (while)^{1.2} thinking the carrot,
 (and then)^{1.2},
 salting the potato (while)^{1.2} evaluating the carrot'."

(boiling *and then* salting)
while
 (thinking *and then* evaluating)
equal
 (boiling *while* thinking)
and then
 (salting *while* evaluating)

Because terms and actions in the polycontextural model of the following example are distributed over different loci, the meanings have to belong to different contextures. One contexture might contain the *physical data* of the cooking example. Another contexture might contain the *mental data* accompanying the cooking processes.

Hence, "*boiling*" and "*salting*", "*carrot*" and "*potato*" belong to the physical contexture represented by the category Cat1.

While the accompanying mental processes "*evaluating*", and "*thinking*" belong to the category Cat2.

The objects "*carrot*" and "*potato*" are appearing as physical objects in Cat1 and as representations, i.e. signs, in Cat2.

Both categories, Cat1 and Cat2, are mediated and part of a 3-contextural category Cat⁽³⁾.

The interactivity of both thematizations, the physical and the mental, are represented by the bifunctionality of the distributed operations .

$$\left(f_1 \circ^{1.0} g_1 \right) \Pi^{1.2} \left(f_2 \circ^{0.2} g_2 \right) =^{1.2} \left(f_1 \Pi^{1.2} f_2 \right) \circ^{1.2} \left(g_1 \Pi^{1.2} g_2 \right)$$

(boiling *then* salting) *while* (thinking *then* evaluating) *equal* (boiling *while* thinking) *then* (salting *while* evaluating)

$$\left(\begin{array}{c} \left(\text{boiling} \circ_{1.0} \text{salting} \right) \\ \Pi_{1.2} \\ \left(\text{thinking} \circ_{0.2} \text{evaluating} \right) \end{array} \right) = \left(\begin{array}{c} \text{boiling} \\ \Pi_{1.2} \\ \text{thinking} \end{array} \right) \circ_{1.2} \left(\begin{array}{c} \text{salting} \\ \Pi_{1.2} \\ \text{evaluating} \end{array} \right)$$

$$\left(\left(\begin{array}{c} \left(f_1 \circ_{1.0} g_1 \right) \\ \Pi_{1.2} \\ \left(f_2 \circ_{0.2} g_2 \right) \end{array} \right) \right) = \left(\begin{array}{c} f_1 \\ \Pi_{1.2} \\ f_2 \end{array} \right) \circ_{1.2} \left(\begin{array}{c} g_1 \\ \Pi_{1.2} \\ g_2 \end{array} \right)$$

3.2.3. Reflectional monoidal categories

What is missing in the above examples of modeling mental and physical processes of cooking is an instance to reflect on both. Hence, the 2-categorical frame has to be augmented to 3-categorical frame containing a new operator, reflecting on the mental and physical processes. This is again a kind of a parallel operator but not on processes but on the contextures containing cognitive and informatic processes.

The weakest modeling might be realized by the mediating contexture between the first and the second contexture in a 3-contextural category.

Category Cat3 = (Cat1, Cat2):

Objects: *potato*, *carrot*

Processes: *boiling*, *salting*

Operations: (*and then*), (*while*)

Term-Objects: "*potato*", "*carrot*"

Processes: *thinking*, *evaluating*

Operations: (*and then*), (*while*)

Semiotic modeling	
(boiling and then salting)	$\begin{pmatrix} f_1 \circ 1.0.0 & g_1 \end{pmatrix}$
while	$\Pi_{1.2.0}$
(thinking and then evaluating)	$\begin{pmatrix} f_2 \circ 0.2.0 & g_2 \end{pmatrix}$
and reflecting on	$\blacklozenge_{1.2.3}$
$\left[\left(\text{boiling then salting} \right) \text{ while } \left(\text{thinking then evaluating} \right) \right]$	$\begin{pmatrix} f_3 \circ 0.0.3 & g_3 \end{pmatrix}$
equal	$= 1 = 2 = 3$
(boiling while thinking) and reflecting on (boiling while thinking)	$\begin{pmatrix} f_1 \\ \Pi_{1.2.0} \\ f_2 \\ \blacklozenge_{1.2.3} \\ f_3 \end{pmatrix}$
and then and while	$\circ_1 \circ_2 \circ_3$
(salting while evaluating) and reflecting on (salting while evaluating)	$\begin{pmatrix} g_1 \\ \Pi_{1.2.0} \\ g_2 \\ \blacklozenge_{1.2.3} \\ g_3 \end{pmatrix}$

$$\begin{pmatrix} \begin{pmatrix} f_1 \circ 1.0.0 & g_1 \end{pmatrix} \\ \Pi_{1.2.0} \\ \begin{pmatrix} f_2 \circ 0.2.0 & g_2 \end{pmatrix} \\ \blacklozenge_{1.2.3} \\ \begin{pmatrix} f_3 \circ 0.0.3 & g_3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} f_1 \\ \Pi_{1.2.0} \\ f_2 \\ \blacklozenge_{1.2.3} \\ f_3 \end{pmatrix} \circ_1 \circ_2 \circ_3 \begin{pmatrix} g_1 \\ \Pi_{1.2.0} \\ g_2 \\ \blacklozenge_{1.2.3} \\ g_3 \end{pmatrix}$$

$$\begin{pmatrix} f_3 \blacklozenge_{0.0.3} & g_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} f_1 \circ 1.0.0 & g_1 \end{pmatrix} \\ \blacklozenge_{1.2.0} \\ \begin{pmatrix} f_2 \circ 0.2.0 & g_2 \end{pmatrix} \end{pmatrix}$$

$$f_3 = \begin{pmatrix} f_1 \\ \Pi_{1.2.0} \\ f_2 \end{pmatrix}, \quad g_3 = \begin{pmatrix} g_1 \\ \Pi_{1.2.0} \\ g_2 \end{pmatrix}$$

3.2.4. Contextual modeling

Some *Kindergarten* are not to unify under one principle, there are highly different cultures involved. To make it more clear to the children, and probably even more to their parents, a game of different *jobs* (processes) in the kitchen had been invented.

The main motivation was the experience made before, "*Cooking is not COOKING*". It all depends on the role you take in this exercise of cooking.

To solve this situation, children quickly realized that there is no need for struggles and fights. It resolves nearly automatically if they agree to accept different, albeit irreducible, roles in the game. There was also no need for anarchy, only because the hegemony of the boss was

distributed over different players. Hence, the ultimate understanding of Chef-cooking became a special case of a much more lively and much more realistic scenario. Sometimes it wasn't even easy to decide if it still was cooking and not something else.

Hence, in ordinary life, i.e. in a *unified* cooking situation, cooking is cooking by one and only one instance, i.e. by the chef.

$$\left(\text{boil potato then salt potato--as a chef } (l) \right), \text{ while, } \left(\text{fry carrot then pepper carrot--as a chef } (m) \right) \parallel x$$

$$\parallel$$

$$\left(\text{boil potato while fry carrot--as a chef } (u) \right) \text{ then, } \left(\text{salt potato while pepper carrot--as a chef } (n) \right) \parallel y.$$

Scheme of (2, 2) – cooking

$$\text{nr. } l \quad m \quad \left(\cdot, \Pi, \parallel \right)$$

salt	pepper	u
boil	fry	n

$$5. \text{ chef } \text{chef} \quad \left(\cdot, \Pi, \parallel \right)$$

salt	pepper	chef
boil	fry	chef

It became quickly clear that there are many ways of cooking, not only the ultimate but also the poly-mate way. There was some analytical desire to fix the few possible games in a table of types of cooking scenarios.

1. $l \neq m \neq u \neq n$

boil potato *then* salt potato—as a chef (l), *while*, fry carrot *then* pepper carrot—as a guest (m)

||

boil potato *while* fry carrot—as a boss (u), *then*, salt potato *while* pepper carrot—as an apprentice (n).

2. $l = m \neq u \neq n$

boil potato *then* salt potato—as a chef (l), *while*, fry carrot *then* pepper carrot—as a chef (m)

||

boil potato *while* fry carrot—as a boss (u), *then*, salt potato *while* pepper carrot—as an apprentice (n).

3. $l = m \neq u = n$

boil potato *then* salt potato—as a chef (l), *while*, fry carrot *then* pepper carrot—as a chef (m)

||

boil potato *while* fry carrot—as an apprentice (u), *then*, salt potato *while* pepper carrot—as an apprentice

4. $l \neq m = u \neq n$

boil potato *then* salt potato—as a chef (l), *while*, fry carrot *then* pepper carrot—as a guest (m)

||

boil potato *while* fry carrot—as a chef (u), *then*, salt potato *while* pepper carrot—as a guest (n).

5. $l = m = u = n$

boil potato *then* salt potato—as a chef (l), *while*, fry carrot *then* pepper carrot—as a chef (m)

||

boil potato *while* fry carrot—as a chef (u), *then*, salt potato *while* pepper carrot—as a chef (n).

Realizations

1. chef guest ($\ast, \Pi, \Pi$) salt pepper boil fry boss boy	2. chef chef ($\ast, \Pi, \Pi$) salt pepper boil fry boss boy	3. chef chef ($\ast, \Pi, \Pi$) salt pepper boil fry boy boy
4. chef guest ($\ast, \Pi, \Pi$) salt pepper boil fry chef guest	5. chef chef ($\ast, \Pi, \Pi$) salt pepper boil fry chef chef	6. chef guest ($\ast, \Pi, \Pi$) salt pepper boil fry boss boy

General formula

$$\left(\begin{array}{c} (g \circ f) \\ \text{II} \\ (k \circ h) \end{array} \right) \Bigg| (l \parallel m) = \left(\begin{array}{c} g \\ \text{II} \\ k \end{array} \right) \circ \left(\begin{array}{c} f \\ \text{II} \\ h \end{array} \right) \Bigg| (u \parallel n)$$

$$\text{nr. } l \ m \ \left(\begin{array}{c} a, \text{II}, \mid, \text{II} \end{array} \right)$$

$$\begin{array}{cc} g & k \\ f & h \end{array}$$

u
 n

But the game doesn't end here.

There are always some ultra-clever girls who are not happy with the achievement. Why are the operators “while” and “then” the same in each constellation? That's nonsense! The “while” for ‘potatoes’ can't be the same as the “while” for ‘thinking’ and ‘evaluation’. Obviously not! It was agreed that this was a very late question and had to be put on the table for the next session.

For adults, see: “*Chez Maxime's. Human rights in a polycontextural world*”

www.thinkartlab.com/pkl/media/Chez_Maxime/Chez_Maxime.html

3.3. Polycontextural monoidal categories

3.3.1. Interplay between universes and contexts

Considered from the just sketched background it seems to be not too crazy or revolutionary to introduce a new mechanism for a production of multitudes. A universe might be identified as a single, i.e. an elementary contexture.

Obviously, the unique status of a Grothendieck universe is producing a hierarchical order between the universe as such and its classes and sets. This is properly stated by Proposition 1: If $x \in U$ and $y \subseteq x$, then $y \in U$.

Without doubt, the transition from categories to a multitude of contextures is not definable by categorical operations.

This is easy shown by the fact that categories are based on a single universe of construction, while contextures are based on a multitude of discontextural but mediated universa, i.e. a poly-verse, building the dynamics of polycontexturality.

There are no means to construct poly-versa from universes.

As Abramsky pointed out that monoidal categories are framing serial and parallel compositions but are not enabling interactions between composition and juxtaposition and their objects. Bifunctoriality is just defining this kind of separability.

As much as a hierarchical order is not legitimizable by intrinsic arguments, it is neither legitimizable to reject other paradigms, modes or strategies of thinking, i.e. of introducing a chiasmic interplay between universes and classes of objects. Such a chiasm is enabling a working construction for heterarchies and an interplay between hierarchies and heterarchies.

What is seen as “parallel” in one system, appears, at once, as “serial” in another system.

If, as in the set-theoretic theory of relations, a relation is defined as a subset of U , i.e. $r \times r \subseteq U$, then the wording that a Universe becomes a set and a set becomes a Universe, gets some simplification and, at first, more plausibility. Hence, an interchange between relation and universe becomes plausible.

This might be generalized to the category-theoretical situation of an interchange between objects and composition.

$$\chi(\text{Univ}, \text{obj}) = \begin{pmatrix} (\text{Univ}, \text{obj}) \\ \diamond \\ (\text{obj}, \text{Univ}) \end{pmatrix}$$

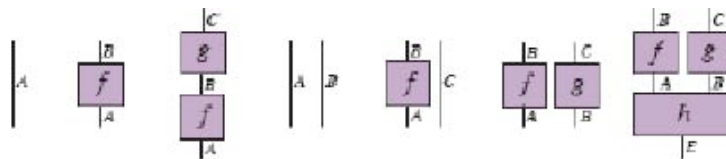
It is just a small step to see that a distributor of contexture can be considered as a radicalization of the operator of juxtaposition. What is set in parallel is now not only the set of objects and the set of compositions by a juxtaposition but the whole frame of the monoidal category itself. Hence, not only the constituents of the category, the objects, morphism, composition, juxtaposition, functors, etc., are distributed but the universe as such. What was in the background as a foundig universe becomes itself an “object” in a calculus of distribution and mediation of contextures, i.e. universes.

As much as monoidal categories get their attraction by the bifactoriality of composition and juxtaposition, polycontextural dissemination of categories are getting their novelty by the principle of interchangeability of composition (juxtaposition) and mediation.

But this is not more than a small door-opener to a field of *tabular* distribution and mediation of interacting operators. Tabularity means, in contrast to linear serial and parallel composition, the possibility of a distribution of the categories over different kenomic loci. This is reflected with new operators, like transposition, iteration, replication and metamorphosis.

3.3.2. Again, some architectonics

The Oxford solution is: parallel and serial composition, i.e. composition and juxtaposition. This might be the Vauxhall solution of production, but ship-building is not working this way! Nevertheless, there are other possibilities too, to interpret the situation of four elements inter-relating than parallel and serial order only. Interaction between relata of different relations might offer a candidate for more flexibility.



1. parallel/serial:

$$f \circ g \equiv \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix} \equiv \begin{array}{c|c} \boxed{g_1} & \boxed{g_2} \\ \hline \boxed{f_1} & \boxed{f_2} \end{array}$$

2. serial/permutation :

$$f \Delta g \equiv \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix} \equiv \begin{array}{c|c} \boxed{g_1} & \boxed{g_2} \\ \hline \boxed{f_1} & \boxed{f_2} \end{array}$$

3. parallel/permutation:

$$f \nabla g \equiv \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix} \equiv \begin{array}{c|c} \boxed{g_1} & \boxed{g_2} \\ \hline \boxed{f_1} & \boxed{f_2} \end{array}$$

4. serial/parallel/permutation

$$f \diamond g \equiv \begin{pmatrix} g_1 & g_2 \\ f_1 & f_2 \end{pmatrix} \equiv \begin{array}{c|c} \boxed{g_1} & \boxed{g_2} \\ \hline \boxed{f_1} & \boxed{f_2} \end{array}$$

5. Permutations

$$\begin{aligned} \text{perm}(f \triangle g) &= \begin{array}{c} \boxed{g_1} - \boxed{f_2} \\ | \quad \square \quad | \\ \boxed{f_1} - \boxed{g_2} \end{array} = \begin{array}{c} \boxed{g_2} - \boxed{g_1} \\ | \quad \square \quad | \\ \boxed{f_1} - \boxed{f_2} \end{array} \\ \text{perm}(f \nabla g) &= \begin{array}{c} \boxed{g_1} - \boxed{g_2} \\ | \quad \square \quad | \\ \boxed{f_2} - \boxed{f_1} \end{array} = \begin{array}{c} \boxed{g_2} - \boxed{g_1} \\ | \quad \square \quad | \\ \boxed{f_1} - \boxed{f_2} \end{array} \\ \text{perm}(f \diamond g) &= \begin{array}{c} \boxed{g_2} - \boxed{g_1} \\ | \quad \times \quad | \\ \boxed{f_2} - \boxed{f_1} \end{array} = (g \diamond f) \end{aligned}$$

The argument, that permutation might be introduced later is not changing the possibility to start with it.

Especially, if both are involved, parallel/serial order and permutation together, which is the case for *metamorphosis*. The results for permutation are not automatically restoring the serial/parallel pattern without reversing the order of the components of different morphisms. Furthermore, there are also some reduced patterns to conceive, where one of the four relations isn't realized.

3.3.3. Interchangeability of operators

Interchangeability of operators

$$\chi(\circ, \otimes) = \begin{pmatrix} (\text{obj}, \circ, \otimes) \\ \diamond \\ (\text{obj}, \circ, \otimes) \end{pmatrix}$$

$$\chi(\text{obj}, \circ) = \begin{pmatrix} (\text{obj}, \circ) \\ \diamond \\ (\circ, \text{obj}) \end{pmatrix}$$

$$\chi(\Pi, \circ) = \begin{pmatrix} (\text{obj}, \circ, \Pi) \\ \diamond \\ (\Pi, \circ, \text{obj}) \end{pmatrix}$$

$$\chi(\text{obj}, \Pi) = \begin{pmatrix} (\text{obj}, \Pi) \\ \diamond \\ (\Pi, \text{obj}) \end{pmatrix}$$

3.3.4. Interplay of polycontextural operators

Interchangeability of a 3 – contextural category with composition and mediation (Π)

$$\begin{aligned} \mathcal{U}^{(3)} &= (\mathcal{U}_1 \Pi_{1,2} \mathcal{U}_2) \Pi_{1,2,3} \mathcal{U}_3 \\ (\mathcal{U}_1 \cap_{1,2} \mathcal{U}_2) \cap_{1,2,3} \mathcal{U}_3 &= \emptyset: \\ \mathcal{U}_i &= \{f_i, g_i\}, \quad i = 1, 2, 3 \end{aligned}$$

$$\begin{bmatrix} g_1 & \square & g_3 \\ f_1 & g_2 & \square \\ \square & f_2 & f_3 \end{bmatrix}:$$

$$\begin{pmatrix} (f_1 \circ_{1,0,0} g_1) \\ \Pi_{1,2,0} \\ (f_2 \circ_{0,2,0} g_2) \\ \Pi_{1,2,3} \\ (f_3 \circ_{0,0,3} g_3) \end{pmatrix} = \begin{pmatrix} f_1 \\ \Pi_{1,2,0} \\ f_2 \\ \Pi_{1,2,3} \\ f_3 \end{pmatrix} \circ_1 \circ_2 \circ_3 \begin{pmatrix} g_1 \\ \Pi_{1,2,0} \\ g_2 \\ \Pi_{1,2,3} \\ g_3 \end{pmatrix}$$

Interchangeability of a 3 – contextural category with composition, mediation (Π) and transposition

$$\begin{pmatrix} f_1 \\ \Pi_{1,2} \\ f_2 \diamond_{2,1} f_1 \\ \Pi_{2,3} \\ f_3 \diamond_{3,1} f_1 \end{pmatrix} \begin{bmatrix} \circ_{1,1} & -- \\ \circ_{2,1} \circ_{2,2} & - \\ \square \\ \circ_{3,1} - \circ_{3,3} \end{bmatrix} \begin{pmatrix} g_1 \\ \Pi_{1,2} \\ g_2 \diamond_{2,1} g_1 \\ \Pi_{2,3} \\ g_3 \diamond_{3,1} g_1 \end{pmatrix} = \begin{pmatrix} (f_1 \circ_{1,1} g_1) \\ \Pi_{1,2} \\ (f_2 \circ_{2,2} g_2) \diamond_{2,1} (f_1 \circ_{2,1} g_1) \\ \Pi_{2,3} \\ (f_3 \circ_{3,3} g_3) \diamond_{3,1} (f_1 \circ_{3,1} g_1) \end{pmatrix}$$

Interchangeability of a 3 – contextural category with composition, mediation (Π) and replication

$$\begin{pmatrix} f_1 \circ_{1,2} f_1 \circ_{1,3} f_1 \\ \Pi_{1,2} \\ f_2 \\ \Pi_{2,3} \\ f_3 \end{pmatrix} \begin{bmatrix} \circ_{1,1} \circ_{1,2} \circ_{1,3} & -- \\ - \circ_{2,2} - \\ -- \circ_{3,3} \end{bmatrix} \begin{pmatrix} g_1 \circ_{1,2} g_1 \circ_{1,3} g_1 \\ \Pi_{1,2} \\ g_2 \\ \Pi_{2,3} \\ g_3 \end{pmatrix} =$$

$$\begin{pmatrix} ((f_1 \circ_{1,1} g_1) \circ_{1,2} (f_1 \circ_{1,2} g_1)) \circ_{1,3} (f_1 \circ_{1,3} g_1) \\ \Pi_{1,2} \\ (f_2 \circ_{2,2} g_2) \\ \Pi_{2,3} \\ (f_3 \circ_{3,3} g_3) \end{pmatrix}$$

Mixed bifactoriality for replication, juxtaposition, composition and dissemination

$$\begin{pmatrix} f_1 \circ_{1,2} f_1 \circ_{1,3} f_1 \\ \Pi_{1,2} \\ f_2 \\ \Pi_{2,3} \\ \left(\begin{pmatrix} f_1 \\ \otimes_3 \\ f_2 \end{pmatrix} \right)_{3,3} \end{pmatrix} \begin{bmatrix} \left[\begin{matrix} \circ_{1,1} & \circ_{1,2} \circ_{1,3} \end{matrix} \right] \dashv \dashv \\ - \circ_{2,2} - \\ \dashv \dashv \circ_{3,3} \end{bmatrix} \begin{pmatrix} g_1 \circ_{1,2} g_1 \circ_{1,3} g_1 \\ \Pi_{1,2} \\ g_2 \\ \Pi_{2,3} \\ \left(\begin{pmatrix} g_1 \\ \otimes_3 \\ g_2 \end{pmatrix} \right)_{3,3} \end{pmatrix} =$$

$$\begin{pmatrix} \left(\left(f_1 \circ_{1,1} g_1 \right) \circ_{1,2} \left(f_1 \circ_{1,2} g_1 \right) \right) \circ_{1,3} \left(f_1 \circ_{1,3} g_1 \right) \\ \Pi_{1,2} \\ \left(f_2 \circ_{2,2} g_2 \right) \\ \Pi_{2,3} \\ \left(\begin{pmatrix} f_1 \circ_{3,3} g_1 \\ \otimes_3 \\ f_2 \circ_{3,3} g_2 \end{pmatrix} \right)_{3,3} \end{pmatrix}$$

Mixed bifactoriality for replication and transposition together with composition and dissemination

$$\begin{pmatrix} f_1 \circ_{1,2} f_1 \circ_{1,3} f_1 \\ \Pi_{1,2} \\ f_2 \diamond_{2,1} f_1 \\ \Pi_{2,3} \\ f_3 \diamond_{3,1} f_1 \end{pmatrix} \begin{bmatrix} \left[\begin{matrix} \circ_{1,1} & \circ_{1,2} \circ_{1,3} \end{matrix} \right] \dashv \dashv \\ \circ_{2,1} \circ_{2,2} - \\ \square \\ \circ_{3,1} - \circ_{3,3} \end{bmatrix} \begin{pmatrix} g_1 \circ_{1,2} g_1 \circ_{1,3} g_1 \\ \Pi_{1,2} \\ g_2 \diamond g_1 \\ \Pi_{2,3} \\ g_3 \diamond g_1 \end{pmatrix} =$$

$$\begin{pmatrix} \left(f_1 \circ_{1,1} g_1 \right) \circ_{1,2} \left(f_1 \circ_{1,2} g_1 \right) \circ_{1,3} \left(f_1 \circ_{1,3} g_1 \right) \\ \Pi_{1,2} \\ \left(f_2 \circ_{2,2} g_2 \right) \diamond_{2,1} \left(f_1 \circ_{2,1} g_1 \right) \\ \Pi_{2,3} \\ \left(f_3 \circ_{3,3} g_3 \right) \diamond_{3,1} \left(f_1 \circ_{3,1} g_1 \right) \end{pmatrix}$$

Mixed interchangeability for replication, transposition and juxtaposition ($\otimes$) in composition

$$\begin{pmatrix} f_1 \circ_{1.2} f_1 \circ_{1.3} f_1 \\ \Pi_{1.2} \\ f_2 \circ_{2.1} f_1 \\ \Pi_{2.3} \\ \begin{pmatrix} f_1 \\ \otimes_3 \\ f_2 \end{pmatrix} \circ_{3.1} f_1 \end{pmatrix} \begin{bmatrix} \circ_{1.1} \circ_{1.2} \circ_{1.3} \\ \circ_{2.1} \circ_{2.2} \\ \circ_{3.1} \circ_{3.3} \end{bmatrix} \begin{pmatrix} g_1 \circ_{1.2} g_1 \circ_{1.3} g_1 \\ \Pi_{1.2} \\ g_2 \circ_{2.1} g_1 \\ \Pi_{2.3} \\ \begin{pmatrix} g_1 \\ \otimes_3 \\ g_2 \end{pmatrix} \circ_{3.1} g_1 \end{pmatrix} = \\
 \begin{pmatrix} ((f_1 \circ_{1.1} g_1) \circ_{1.2} (f_1 \circ_{1.2} g_1)) \circ_{1.3} (f_1 \circ_{1.3} g_1) \\ \Pi_{1.2} \\ (f_2 \circ_{2.2} g_2) \circ_{2.1} (f_1 \circ_{2.1} g_1) \\ \Pi_{2.3} \\ \begin{pmatrix} (f_1 \circ_{3.1} g_1) \\ \otimes_3 \\ (f_2 \circ_{3.2} g_2) \end{pmatrix} \circ_{3.1} (f_1 \circ_{3.1} g_1) \end{pmatrix}$$

3.3.5. Metamorphosis

One of the most intriguing constellation is offered by the interactional operation of metamorphosis. This is nicely introduced with the example of a metamorphosis of types and terms for formal systems.

With this construction a radical departure from the previous categorical systems happens. Before, entities, i.e. objects, morphisms, are considered as identical terms in the sense A is B . Now it turns out that this identity-theoretical presumption, the is/has-abstraction, unmasks itself as a derivative construct from the “as-abstraction”, “ A as B is C ”, hence, A as A is $A \Rightarrow A$ is A .

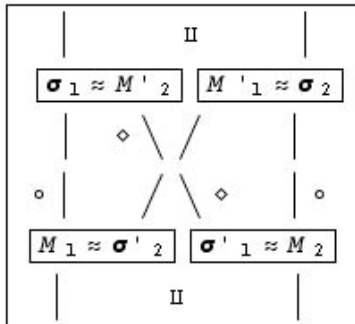
With metamorphosis we are getting more close to what Schelling was speculating for an unrestricted interplay of contextures.

Polycontextuality alone is not enough to realize the interwoven dynamics a new world-view is desperate for. Gotthard Gunther introduced his *proemial* relationship to dynamize his contextures, albeit still restricted to a uni-directional movement. The concept of metamorphosis as part of the *diamond strategies*, based on polycontextuality and disseminated over the *kenomic* matrix, is a further step to realize a radical paradigm change in our way of thinking and designing futures.

As a metaphor or a poietic model, think of *Gregor Samsa* from Franz Kafka. And if you read Gregor Samsa from *Franz Kafka* be aware of its metaphoric complexity as sketched here for you.

<http://www.suite101.com/content/the-metamorphosis-by-franz-kafka-a22435>

Diagram of 2 – fold metamorphosis**Metamorphic chiasm of type and term**

$$\left[(M, \sigma), \approx, \diamond, \circ, \Pi \right]$$
**Interdependence of operators $(\circ, \Pi, \diamond, \approx)$: Metamorphism**

$$\left(\begin{array}{c} (M_1 \circ \sigma_1) \Pi (M_2 \circ \sigma_2) \\ (\sigma'_2 \diamond M'_1) \\ (\sigma'_1 \diamond M'_2) \end{array} \right) \Leftrightarrow \left(\begin{array}{c} M_1 \approx \sigma'_2 \\ \sigma'_1 \approx M_2 \\ \sigma_1 \approx M'_2 \\ M'_1 \approx \sigma_2 \end{array} \right)$$

Metamorphic chiasm of type and term

$$\left[(M, \sigma), \approx, \diamond, \circ, \Pi \right]$$

$$\left(\begin{array}{c} ((\sigma_1 \approx M'_2) \circ (M'_1 \approx \sigma_2)) \\ \diamond \quad \Pi \quad \diamond \\ ((M_1 \approx \sigma'_2) \circ (\sigma'_1 \approx M_2)) \end{array} \right)$$

$$\left[\begin{array}{c} (M_2 \approx M'_2) \\ \Pi \quad \diamond \\ (M_1 \approx M'_1) \end{array} \right] \circ \left[\begin{array}{c} (\sigma_2 \approx \sigma'_2) \\ \Pi \quad \diamond \\ (\sigma_1 \approx \sigma'_1) \end{array} \right] =$$

$$\left[\begin{array}{c} (M_2 \circ \sigma_2) \\ \Pi \\ (M_1 \circ \sigma_1) \end{array} \right] \approx \left[\begin{array}{c} (M'_2 \circ \sigma'_2) \\ \diamond \\ (M'_1 \circ \sigma'_1) \end{array} \right]$$

Interdependence of operators $(\circ, \otimes, \equiv)$: Equality

$$\left[\begin{pmatrix} M_1 & \circ & \sigma_1 \\ & \otimes & \\ M_2 & \circ & \sigma_2 \end{pmatrix} \right] = \begin{pmatrix} M_1 \\ & \otimes \\ M_2 \end{pmatrix} \circ \begin{pmatrix} \sigma_1 \\ & \otimes \\ \sigma_2 \end{pmatrix} \iff \begin{pmatrix} M_1 \equiv M_1 \\ \sigma_1 \equiv \sigma_1 \\ M_2 \equiv M_2 \\ \sigma_2 \equiv \sigma_2 \end{pmatrix}$$

Interdependence of the operators $(\circ, \otimes, \simeq)$: Similarity

$$\left[\begin{pmatrix} M_1 & \circ & \sigma_1 \\ & \Pi & \\ M_2 & \circ & \sigma_2 \end{pmatrix} \right] = \begin{pmatrix} M_1 \\ & \Pi \\ M_2 \end{pmatrix} \circ \begin{pmatrix} \sigma_1 \\ & \Pi \\ \sigma_2 \end{pmatrix} \iff \begin{pmatrix} M_1 \simeq M_2 \\ \sigma_1 \simeq \sigma_2 \end{pmatrix}$$

4. Memristics

4.1. Bifunctionality for memristors

It is postulated that memristors have a second-order behavior which is reflecting its states and the history of its states, i.e. the states of the states. Therefore, a combination of memristors is replicating (retrieving, fetching) the inner state of the repeated memristive activity with the succiding memristor or the succeding memristive behavior. This reflects the history-dependence of memristive behaviours, i.e. $M_1 + M_1 = : M_1 + M_1 r_1$.

Such an understanding of memristive behaviors opens up two possibilities to thematize the operator of iteration in the definition of the second-order character of memristive actions. One is remaining in the conceptual and physical domain of a single and ultimate universe, with typed or non-typed specifications. The other is empathizing the *otherness* in the process of iteration and iterability (repeatability) and is demanding its own universe, hence resulting in two discontextual universes. But in holistic actions, the mediation of two contextures (universes) is producing super-additively a third contexture (universe), albeit discontextual nevertheless reflecting the first two situations.

$$M_1 r_1 + M_2 r_2 = M_1 r_1 + (M_2 r_2) r_1$$

$$M_2 r_2 + M_1 r_1 = M_2 r_2 + (M_1 r_1) r_2$$

Hence, additionally to the serial and parallel actions, the *retro-grade reflectional* action has to be reflected by a specific operator of *replication* " $\square$ ". Hence, $M_1 \otimes r_1 + M_2 \otimes r_2 = M_1 \otimes r_1 + M_2 \otimes (r_2 \square r_1)$.

Replication ($\square$) has no corresponding operator in a "clean" monoidal category.

There is also no need for a monoidal category to deal with superadditivity of yuxtapositions.

4.1.1. Monocontextual modeling

$$\mathbf{BIFUNCT}^{(2)} \begin{bmatrix} r_1 & r_2 \\ M_1 & M_2 \end{bmatrix} :$$

$$\begin{pmatrix} r_1 \\ \otimes \\ M_1 \end{pmatrix} \begin{bmatrix} \circ \square \end{bmatrix} \begin{pmatrix} r_2 \\ \otimes \\ M_2 \end{pmatrix} = \begin{pmatrix} r_1 \circ (r_2 \square r_1) \\ \otimes \\ M_1 \circ M_2 \end{pmatrix}$$

$\forall M, r \in \text{Universe } \mathcal{U}_i, i \in \mathcal{N}, i = 1$
 $\square$: replicator
 $\circ$: composition
 $\otimes$: yuxtaposition

4.1.2. Polycontextural modeling

The retro-grade character of memristive iterability is best thematized and formalized in the framework of morphogramatics but it has a polycontextural thematization with the operator “replication” too.

Super – additivity of a 3 – category with replication for memristors

$m = 3, n = 2$

$$\begin{bmatrix} r_1 & - & r_3 \\ M_1 & r_2 & - \\ - & M_2 & M_3 \end{bmatrix} :$$

$$\begin{pmatrix} \begin{pmatrix} r_1 \\ \sqcup_{1,2,0} \\ M_1 \end{pmatrix} \\ \begin{pmatrix} \sqcup_{1,2,3} \\ M_3 \end{pmatrix} \end{pmatrix} \begin{bmatrix} \circ_1 \square_1 \\ \circ_2 \\ \circ_3 \square_1 \end{bmatrix} \begin{pmatrix} r_2 \\ \sqcup_{1,2,0} \\ M_2 \end{pmatrix} \begin{pmatrix} \sqcup_{1,2,3} \\ r_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} r_1 \circ_{1,0,0} (r_2 \square_1 r_1) \\ \sqcup_{1,2,0} \\ (M_1 \circ_{0,2,0} M_2) \end{pmatrix} \\ \begin{pmatrix} \sqcup_{1,2,3} \\ (M_3 \circ_{0,0,3} r_3 \square_1 (r_2 r_1)) \end{pmatrix} \end{pmatrix}$$

$\forall M, r \in \text{Universe } \mathcal{U}^{(3)} = \mathcal{U}_i, i \in \mathcal{N}, i = 1, 2, 3$
 $(\mathcal{U}_1 \cap_{1,2} \mathcal{U}_2) \cap_{1,2,3} \mathcal{U}_3 = \emptyset$
 $\mathcal{U}^{(3)} = (\mathcal{U}_1 \sqcup_{1,2} \mathcal{U}_2) \sqcup_{1,2,3} \mathcal{U}_3$

4.1.3. Morphogrammatic modeling

Fields of memristive activities might be characterized by the “multiplication” of memristive sub-fields.

Encountered two memristive constellations, $[M \mid r_{1,2,2}]$ and $[M \mid r_{1,2,3,1}]$, the product of both, $\text{MEM}_{(r_{1,2,2}), (r_{1,2,3,1})}^{(3,4)}$, is defined by the following table, following the multiplication rules of monomorphy-based morphogramatics.

Example for (I) :

(1)	(2)	(3)	(4)
$[r_{1,2,2}]$	$[r_{2,1,1}]$	$[r_{3,4,4}]$	$[r_{1,2,2}]$

<http://memristors.memristics.com/Machines/Memristic%20Machines.pdf>

Total memristance of $MEM_{((r_{1.2.2}), (r_{1.2.3.1}))}^{(2,3)}$:

$$MEM_{((r_{1.2.2}), (r_{1.2.3.1}))}^{(3,4)} \left[r_{1.2.2} \right]^{1.4} = \begin{bmatrix} r_{2.1.1} & r_{2.3.3} & r_{3.1.1} & r_{3.4.4} \\ r_{3.4.4} & r_{3.1.1} & r_{2.3.3} & r_{2.1.1} \\ \square & r_{3.4.4} & r_{2.4.4} & r_{2.3.3} \\ \square & r_{4.1.1} & r_{4.3.3} & r_{2.5.5} \\ \square & r_{4.5.5} & r_{4.5.5} & r_{4.1.1} \\ \square & \square & \square & r_{4.3.3} \\ \square & \square & \square & r_{4.5.5} \\ \square & \square & \square & r_{5.1.1} \\ \square & \square & \square & r_{5.3.3} \\ \square & \square & \square & r_{5.6.6} \end{bmatrix}$$

A seminal classification of the possibilities of morphogrammatic and memristic systems is proposed at: "*Orientation*".

<http://memristors.memristics.com/Machines/Orientation/orientation.pdf>