Gotthard Günther[*)]

– Introductory note by *Charles PARSONS*[**)] –

Gotthard Günther (1900-1984) began an academic career in Germany and published a book on Hegel's logic (*Günthier 1933*), based on his 1932 dissertation. He left Germany in 1937 and after brief stays in Italy and South Africa came to the United States in 1940. In the 1940s he taught for a time at Colby College, but he did not have a regular academic position again until 1961, at the end of his correspondence with Gödel, which begins in 1953. During that correspondence he lived in Richmond, Virginia, where his wife seems to have been employed. He had research grants and earned money as a flying instructor and by freelance writing. During the correspondence his relation with the University of Hamburg began, where he obtained a visiting position for the winter semester of 1955-1956. In 1961 he became a research professor in the department of electrical engineering at the University of Illinois in Urbana. He was given the title of Professor Emeritus at Hamburg, and in 1971, after his retirement from Illinois, he moved permanently to Hamburg (after further visits) and continued to give lectures there until the 1982-1983 academic year.

Günther's body of writing is considerable, but it is unlikely to be known to most readers of Gödel's works. His original philosophical background was Hegelian, and he continued to see philosophy from that point of view, though he was also influenced by Leibniz and by twentieth-century German figures. Moreover, although he lived for 30 years in the United States, even during that period his philosophical writing was mostly in German.[[a]] A project that he pursued for many years, which is one of the themes of his correspondence with Gödel, was how formal logic ought to be revised to accommodate what he took to be insights about the nature of thought and its relation to reality from the German idealist tradition. He also became

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Professor Parsons has published papers on mathematical logic, philosophy of mathematics, philosophy of logic and language, Kant, and historical figures in the foundations of mathematics, such as Frege, Hilbert, and Gödel. Some of his philosophical papers are collected in Mathematics in Philosophy (Cornell, 1983). He is editor, with Solomon Feferman and others, of Volume III of the Collected Works of Kurt Gödel, Unpublished Essays and Lectures (Oxford, 1995), and of the two forthcoming volumes of correspondence.

[a] There is a bibliography of Günther's published writings in Günther 1980, pp. 305-310. Günther 1975 is partly autobiographical.

interested in and wrote about "cybernetics". Norbert Wiener, who publicized the term, characterized cybernetics as the science of "control and communication, in the animal and the machine."[b] Its concerns derived from engineering and theoretical biology, but what seems to have most interested Günther was the idea of artificial intelligence. He was one of the earlier thinkers to write from a philosophical point of view on that subject.[c] He was thus a very unusual intellectual figure for his time, a Hegelian philosopher with an interest in modern logic and involvement in what later came to be called computer science.

The occasion for Günther´s correspondence with Gödel was an inquiry Gödel received from the American Committee for Emigré Scholars, Writers and Artists, which wished to support an application by Günther for a grant from the Bollingen Foundation. Günther followed this up by writing to Gödel on 2 August 1953 and sending him several papers. Shortly thereafter he took the liberty of using Gödel´s name as a reference. Gödel wrote what was evidently a supportive letter, of which handwritten drafts survive in his papers.[d] On 12 December 1953 Günther wrote to inform Gödel that he had received a three-year grant and to thank him for his support. The more substantive correspondence began the following spring. On 29 April 1954 Günther wrote expressing some views and raising some questions about the law of excluded middle. The exchange continued for several years, with Günther, however, writing more and longer letters than Gödel. On 17 September 1956 they met in Gödel's office and had a morning of discussion, apparently largely of Günther's work.[e] Gödel's last known letter to Günther was in January 1959, but Günther continued to write to Gödel through 1959 and 1960. There is no evidence known to us that G6del sent any further replies.

1. Günther on metaphysics and logic

Before we describe the course of the correspondence, it is necessary to say something about Giinther's point of view at the time and about the philosophical project, aspects of which he lays out in his letters, in whose service he was making inquiries with Gödel. The conviction with which he began and which animated his whole involvement with logic is that the philosophical insights of German idealism from Kant through Hegel required a revision of logic. This view was already expressed in the published version of his dissertation.[f] Probably by the time he left Germany he was convinced that modern mathematical logic was relevant to the project of such a

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b In the title of Wiener 1948.

c *Günther 1952* is his most direct discussion of whether machines can be conscious. It seems to be presupposed in Günther *1957a,* the clearest and most accessible presentation of the ideas Günther discussed with Gödel, as GödeI himself seems to have thought (letter 13, but see note aa below). Since Günther 1963 incorporates the latter and contains a reprint of the former, it is the best introduction to his ideas.

d The Bollingen Foundation acknowledged receipt of the letter, which was dated October 24, 1953 (letter of Ernest Brooks, Secretary of the Foundation, to Gödel, 26 October, 1953). Correspondence of Gödel with others cited in this note is from the Gödel papers, filed under Günther.

e In letter 9, 20 September 1956, Günther thanks Gödel for such a discussion the previous Monday. The date can be inferred from the fact that 20 September was a Thursday. I know of no evidence that they met on any other occasion.

f Günther 1933; cf. Günther 1978.

revision,[g] but he was never convinced that in its classical version it was what was required. He considered Hegel's logic an effort in this direction, although he was aware that it was not a logic in the sense in which the systems constructed in modern logic are logics. He thought Hegel's logic a grand failure, but he remained interested in the project of expressing it in more formal terms. That logic should be intimately related to metaphysics was a lifelong conviction of his and no doubt a point of agreement with Hegel. Moreover, to express his view of metaphysics he constantly uses as basic categories thought or consciousness, its relation to objects, and self-consciousness. In this respect the idealist tradition determines how he describes even non-idealist philosophy. Both the perspective from idealism and the metaphysical conception of logic are epitomized in his remark, "A logic is the metaphysical self-definition of a subject" (1957, p. 29).

Günther describes his project repeatedly as that of constructing a "non-Aristotelian logic." By "Aristotelian" he means what we would call classical, so that most of the vast extension of logic that has taken place since the mid-nineteenth century still counts as Aristotelian.[h] However, what is decisive for him is a certain metaphysical interpretation of the foundations of logic. His own preferred means for carrying out the project of a non-Aristotelian logic is many-valued logic. That is not itself non-Aristotelian, but it offers the technical means of carrying out the idea of a non-Aristotelian logic. Or so Günther thought in the 1950s at the time of his exchange with Gödel.[i]

Günther's conception of classical logic is intimately bound up with a view of the relation of logic and metaphysics. He sees the metaphysical tradition from Plato and Aristotle at least through pre-Kantian modern philosophy as a unity and classical logic as obtaining its rationale and metaphysical foundation from that tradition. He describes as "axioms" of the classical tradition in logic the principles of identity, contradiction, and excluded middle (1957, p. 5). Whenever he discusses the foundations of logic, however, the framework is that of a theory of thought, in particular of a subject thinking about an object, or being conscious of an object.

What is most distinctive in Günther's point of view comes out in his remarks about the principle of identity. Repeatedly he says that the metaphysical tradition is based on the presupposition of the identity of thought and being. He also took this as a metaphysical presupposition of classical logic. As he said in his letter of 23 May

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g See Günther 1940 and 1957, of which the latter appears to have been drafted in 1935; see Günther 1980, p. 305 n. Unlike other publications of Günther in the 1950s dealing with logic, it makes no mention of three-valued logic. Günther sent Gödel the published version without having mentioned the paper earlier in the correspondence; that would suggest he was not working on it during their exchange.

h "We can leave out of account the 'logistic' criticism [of Aristotelian logic] beginning with Leibniz, because it represents no philosophical critique of the metaphysics of this logic but a generalization and extension (functional calculus) of the classical ways of proceeding" (1957, p. 5 n.). He does go on to say that his own work would not be possible without modern logic, and already in 1940 he argued that philosophers interested in "transcendental logic" needed to pay attention to modern logic.

i Günther 1958 sets forth what was to be the basic logical construction for the second volume of the work of which 1959 was the first. He states (1978a, p. xxii) that the calculus was not able to bear the philosophical weight it was meant to carry. He apparently came to this conclusion through exchanges with cyberneticists early in his time at the University of Illinois. No attempt is made here to follow Günther's thought after his exchange with Gödel.

1954 to Gödel, "Classical logic presupposes the metaphysical identity of thought and being" (letter 3, p. 17). The "original phenomenon" of thought is expressed by "I think something." The relation of "I" and "something" is characterized as identity. What this comes to is that the object, the "something" is identical with itself, but also that the ego is in the end identical with its object.

However, without some further elaboration we do not do justice to Günther's thought. What he took the metaphysical meaning of the principle of identity to be is first of all that thought aspires to complete objectivity:

> While consciousness in the judgment "I think something" determines its definitive and final subject matter, that is the "something", simply as "identity", it claims implicitly that all thinking that is possible at all intends as its definitive metaphysical goal the objective In-itself, identical with itself. The transcendent essence of all self-identical In-itself, however, is Being. Consequently Being is the only, original, and last metaphysical subject matter of reflecting consciousness (1957, p. 8).

The object of thought can preserve its identity through the different perspectives from which it is experienced and thought about. Thus that it is identical with itself means not simply what is expressed by the logical truth that everything is identical with itself, but, one might say, that the concept of identity has a non-trivial application, so that the same object can be presented to consciousness under different circumstances and in different ways, which we might call modes of presentation.

A clear conclusion from the identity of thought and Being that Günther draws in his principal systematic work, *Idee und Grundriss einer nicht-aristotelischen Logik,*[j] is that what is objective or what is true will be perfectly intersubjective (1978a, p. 11). If a subject has a "true concept" of an object, that will in principle be communicable to any other subject. But Günther sees this as implying that the division of subjectivity in general into individual subjects is only "provisional and apparent." I think what he means by this is that an intrinsic goal of the thinking of individuals is to converge on some absolute thinking, in which each one's thoughts would represent the world as it is and thus not in any way differently from those of any other. This train of thought leads to the idea of an absolute subject, in effect God, in whom the relativity of consciousness to a perspective would be overcome. One of the challenges to this way of thinking is the thought, prominent in post-Hegelian philosophy, that actual subjectivity is formed by history (1978a, p. 10).

To get a fuller idea of Günther's conception of classical metaphysical thinking, we have to see a little of what he thought overthrew it. One development he mentions but does not give a prominent place in his philosophical analysis is the gradual divorce of fundamental science from philosophy. It is because modern logic arose primarily within mathematics, and thus on the scientific side of this divide, that Günther thought it did not break with the classical logical tradition on the metaphysical level.

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[j] Günther 1959. This work was Günther's principal project during his exchange with Gödel, and its Preface contains a generous acknowledgment to him (1978a, p. xxi). Quotations from this work are from the second edition, Günther 1978a. Translations of Günther's published writings are my own, of his letters to Gödel by Thomas Teufel and me.

What is central to Günther's story is German idealism beginning with Kant but especially as embodied in Fichte, Schelling and Hegel. His picture of philosophy before Kant clearly owes much to idealism; for example he views the classical tradition as in Kantian terms transcendental realist. However, if one thinks of pre-Kantian philosophy as transcendental or metaphysical realism, one will see the main revolutionary element in German idealism as Kant's Copernican turn and its further development by his successors. That is for Günther only part of the truth. What is central, though related, is the role self-consciousness plays in the thought of these philosophers. It is attention to subjectivity that was lacking in earlier thought. To Gödel he writes the following:

> In the classical tradition the subject of thought and the process of reflection do not count at all. The goal of thought is to grasp the sense of *absolutely objective being*. And truth means absolute agreement of thought with the absolutely objective object. That is, all categories of logic must, if they are to be true, be absolutely objectively definable. Everything "subjective" is quite simply to be eliminated (letter 3, p. 3).

Now, whatever one might think of this as a characterization of ancient and medieval thought, it is a commonplace that subjectivity becomes a philosophical theme with Descartes. Günther has much more to say about what is new in Kant and his successors; the central concept that he uses to describe it is "reflection" - It is not easy to say what he means by it. A basic meaning is certainly "self-consciousness". More generally, it is represented as a feature of thought about objects when the conception of the objects takes into account the subject's thought about them. In this way it comes also to cover what I myself would call semantic reflection, that is, the passage from the straightforward use of words "taken at face value" to discourse in which they are mentioned and something is said about their reference, truth or meaning.

Some helpful explanations are found in *Günther 1957* (already quoted above). In this paper Günther distinguishes stages of reflection, which are fundamental stances of consciousness to the world and itself, which, however, also have some relation to stages in the history of thought. The stages are described as "R-levels", levels of reflection. The idea seems to have some inspiration from the theory of types (1957, p. 5 n. 1), but also from Hegel's *Phenomenology of Spirit.*

What he calls the 0^{th} R-level is one at which there is no self-consciousness at all; consciousness simply reflects the world:

> This elementary state or Oth R-level of consciousness mirrors wholly immediately the objectually closed connection of being and produces, with the naturalness of an optical camera a simple image of the objective world (1957, p. 19).[k]

The first R-level is, as one might expect, that at which the 0^{th} level becomes an object. Günther describes it as arising from an "existential contradiction" in the 0^{th} level, because at that level the subject realizes itself as *negation,* as mere *object,* that is as non-subject. But at the first level it is able to distinguish being and

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k This stage is identified with the "sense- certainty" of Hegel's *Phenomenology*.

consciousness. It discovers that being is mirrored in consciousness, but nothing else is so reflected. So consciousness is in reality being.

This, according to Günther, is the position of Aristotle and therefore of the origins of logic. But it is an unstable position:

> For Aristotle consciousness knows of itself and of its experienced opposition to being, but it relativizes this opposition and rescinds it by means of a reflection. The self-consciousness of man begins its history with a denial of itself (1957, p. 20).

A second R-level, which is conscious of the first level and its limitations, seems to arise at the end of the Middle Ages. Although he agrees with the historians of logic that nothing came of it for formal logic, he seems to attribute some awareness of the limitations of Aristotelian logic, which as belonging to the first level can only be about the 0^{th} level, to those who rebelled against medieval logic during the Renaissance. But no new logical system was possible at the time.

Every R-level, according to Günther, can itself be object of a further reflection. Thus the iteration involved is infinite. Günther draws the further conclusion that it cannot at this point be characterized axiomatically, as a new logic would require, "because it is in no way possible to arrive at final, most general propositions about this open subject and to define it as self-consciousness (thus as a closed whole)" (1957, p. 23). Günther had stumbled on a kind of paradox, which is familiar from reflection on the theory of types. If "reflection", whatever that is, is iterated an arbitrary finite number of times, one arrives only at a particular stage of reflection from which it is possible to go a step further. Therefore general propositions about, all stages can't be formulated.

Günther formulates the problem as one about self-consciousness. At the second level consciousness is related to a relation of consciousness and objects, but self-consciousness is not part of what one is conscious of. It seems that at each level from the first on one has self-consciousness which is, however, limited to its own consciousness at lower levels and so does not really take in itself. Günther argues that this leads to a predicament in which metaphysics is impossible.

In the theory of types, eventually a way was found of so formulating the theory that the iteration of progression to higher types can be iterated into the transfinite, and of course with variables of type W one can make statements about entities of arbitrary - finite types. This does not abolish the dilemma, however, because the progression to higher types can be carried still further. The understanding of the language of set theory that comes most naturally takes the quantifiers as ranging over absolutely all sets, and thus as encompassing all types in the simple type hierarchy. That is not the end of the story, however, since that interpretation is not beyond question, and one still has to cope with another form of ascent, the semantic ascent that, in a perfectly classical setting, parallels ascent in Russell's ramified hierarchy.[1]

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[1] If, as I have suggested, "reflection" as Günther understands it includes semantic reflection, then complete closure, that is a theory that would express its own semantics without any remainder, is impossible. "Total reflection" is not so clearly defined that one can say definitely whether it implies semantic closure in this sense. An affirmative answer is suggested in a passage in Günther 1957a (1963, pp. 78-79). It is somewhat difficult to interpret because it is not always

Günther sees the achievement of idealism as at least posing the task of developing a conception of self-consciousness that would have the required closure property, so that self-consciousness would be truly consciousness of *itself* and not just a representation of itself as an object. In developing his own scheme he does not stay with the idea that there will be an nth R-level for every n, and on into the transfinite. That there is a higher standpoint is, according to Günther, shown by the fact that it is possible to conceive such an iteration and make judgments about it, whether or not one holds that there is a higher standpoint.

> With this thought of the infinite iterability of the reflected consciousness, we have already elevated ourselves above the infinite series of reflections proceeding from the second R-level and made it the "object" of a reflection that by definition cannot itself belong to this sequence. The content of this new reflection is thus the idea of the totality of the infinite sequence of iterations (and not itself an iteration on which others can follow) (1957, p. 27).

Günther concludes that we have a true third R-level that is not just a term in the sequence of iterations of reflection, in which the subject reflects on itself and thus "defines the ego as total self-reflection". It might seem that Günther denies Hume's famous point and thinks that one can directly capture oneself in one's thinking. If that were so, the elaborate story about levels of reflection would be unnecessary. On the contrary, following Hegel, Günther maintains that the compulsion of consciousness toward objectivation can itself be "reflected" and thus seen as a feature of consciousness. It is that that makes "total reflection" possible.

In other writings Günther uses a scheme that he finds in Hegel, which finds three different levels of reflection: "Reflexion-in-anderes", the simple thought of an object by a subject; "Reflexion-in-sich", the thought by a subject of a subject that, however, plays the role of object; and "Reflexion-in-sich der Reflexion-in-sich-und-anderes",[m] which seems at first sight to be just the thought by a subject of a subject's thinking of an object, where both the subject thought of and the object are objects of the first's subject's thought.[n] There is a rough correspondence between these and the 0^{th}, 1^{st} and 2^{nd} R-levels, but Günther uses these concepts for different purposes, in particular explaining the truth-values of his three-valued logic, as we shall see shortly.

One of the other purposes is the "deduction" of the notion of the Thou (Du). Given the fundamental role of consciousness in his thinking, and its distinctness from "being", it is not surprising that other minds should be an ontological category in their own right. The Thou is a subject, thus with the same reflective closure we have been considering. That taking the Thou seriously should lead to a fundamental

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clear when he is rendering Hegel and when speaking for himself. Although the I/R/D three-valued logic in a way renders total reflection, it describes "a thinking that is not and cannot be thought by anyone" (p. 79). To capture the subject of this thought, one must ascend to a four-valued logic, and indefinite further ascent can be forced. Thus Günther writes in letter 11 that "total reflection is not three-valued but indefinitely n-valued, where n > 2 always holds". In the end he seems not to claim to escape an ascent like the ascent of types.

[m] One might translate these roughly as "reflection into other", "reflection into self" and "reflection into self of reflection into other".

[n] Günther 1978a, p. 98; cf. 1953, p. 48, and 1958, pp. 390-391.

change follows from Günther's view of earlier philosophy, because the identity of thought and being tended in the end to abolish the differences between subjects.

Günther expresses the view that takes the Thou seriously in the form of two "metaphysical theorems":

I. Being and Thought are only partially identical.
II. The object has one metaphysical root, the subject has two (1978a, p. 8 5).

This view leads, according to Günther, to a questioning of the law of excluded middle. In some way he identifies the rejection of a "third" between subject and object and the rejection of a third truth-value, or an alternative to being true and being false. The "home" of the law of excluded middle is a classical subject-object schema in which one or more subjects relate to an object. But once one accepts theorem I, two subjects will not be equivalent in the way the tradition had it. If two subjects think of an object, one will be able to think of the other as thinking of the object. That is what Günther calls "Reflexion-in-sich der Reflexion-in-sich und anderes", schematized as $S^S \rightarrow (S^O \rightarrow O^S)$. This suggests an asymmetry that is, however, not the final point of view because it leaves out the fact that the other subject is also capable of thinking of me. We have, in Husserl's well-known phrase, transcendental intersubjectivity.[o]

Günther argues that from this point of view one must distinguish two negations, one of which is expressed in the statement that the subject is not its object, another in the statement that the subject (as ego) is not the Thou. Günther seems to be driven toward many-valued logic by the fact that he doesn't consider an alternative to a truth- functional interpretation of propositional logic, and at least a third truth-value is needed in order to make the distinction between the two negations.

How, then, does he interpret his "truth"-values? At this point he does something that is from a logician's point of view crazy, because the values seem not to be truth-values at all. He uses the designations I, R and D, which he reads as "irreflexive", "reflexive" and "double-reflexive". In other words, they represent stages of reflection coming out of the analysis we have discussed. He even says in one place that all the values are "true" (1953, p. 48). The concepts of truth and falsity should "disappear without remainder" from the sort of logic he is constructing because they exclude a genuine third.[p] It seems that he has simply changed the subject, as a result of taking the relation of the I to what is not I as the paradigm of all negation.

Günther is, however, a somewhat slippier target, and I don't think I have grasped his thought at this point. He says that our thought is in a way necessarily two-valued. What the three-valued logic does is allow for the fact that two-valued thought can occur at different levels of reflection. How he conceives this is not at all clear to me. But he does say something about how it works in propositional logic. He singles out pairs of values and notes that one might treat that pair as truth and falsity, and certain

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[o] The symmetry is illustrated by Günther's diagram in 1978a, p. 98. He also remarks that Hegel "defines total reflection as 'Reflexion-in-sich der Reflexion-in-sich-und-Anderes"' (1958, p. 379). I take Günther's analysis of the Thou as implying that he regards this characterization as inadequate.

[p] 1953, p. 47. Cf. the remarks about truth and falsity in letter 3, pp. 8-9, and letter 8, pp. 1-2.

functions might behave like, say, conjunction when just these two values are considered, perhaps behaving differently when the third value is taken into account. He saw the fact that two-valued structures can occur in different places in a three- or more-valued system as analogous to the place-value feature of Arabic or binary notation for numbers:

> A many-valued logic is now nothing but a system that allows us to give to our single "actual" logic different place-values in the system of consciousness of such a kind that each place-value is connected with a different semantic meaning of the two-valued calculus that thus repeats itself. Such a many-valued system allows us thus to read off the structural interrelation of the different two-valued stages of consciousness.[q]

This remark would suggest that the two truth-values retain their status as genuine truth-values and that the values of the many-valued system have a quite different role. It is not clear how this would be reconciled with Günther's claim that his constructions constitute a genuine revision of logic.

2. The correspondence

Günther wrote Gödel on 29 April 1954 with questions about the law of excluded middle, prompted by an allusion in *Menger 1933* to Gödel's translations of classical logic and arithmetic into intuitionistic. Although Menger gives a reference to *Gödel 1933e,* Günther says he cannot obtain it and struggles with the issues as best he can. He concerns himself with the difference between a version of the law of excluded middle in propositional logic and the quantificational schema $\neg\forall xFx \rightarrow \exists x\neg Fx$, which he regards as a formulation of the law although it is better described as a consequence of it. His comments on the latter are not very clear, but he seems to hold that its truth in a given case presupposes that an object is given that satisfies $\neg Fx$ if anything does.

Gödel's reply of 15 May to this rather confused letter is a gem, an introductory lesson on the relation of classical to intuitionistic logic and mathematics. In the context of setting forth some of the basic technical facts, he agrees with Günther that the basis of the difference of intuitionist and classical logic is that they apply different conceptions of being. He adopts Günther's term 'aufweisbare Existenz' ('exhibitable existence') for the conception at work in intuitionism. "But the core of the intuitionistic objections lies surely in the fact that it is shown that for exhibitable existence certain statements of classical mathematics are unproved and others are even demonstrably false."

Gödel notes a point of philosophical agreement with Günther, that "the application of the method and results of mathematics [in particular, no doubt, mathematical logic] should not be limited to positivistic philosophy." Günther had remarked that he had been trying, thus far unsuccessfully, to persuade his fellow metaphysicians to pay attention to modern logic.

In his reply of 23 May 1954 Günther does more than before to set forth his basic ideas about the connection of metaphysics and logic. Some of the general ideas

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[q] 1958, p. 393.

discussed above, in particular the connection Günther sees between classical logic and a certain metaphysical tradition, are sketched, and he introduces the idea of three-valued logic, which in his writings of this time is the main technical device for carrying out his logical program. Some of the discussion (following up the earlier exchange on intuitionism) is about different conceptions of being, and early in the letter Günther poses a challenge to Gödel. He remarks on the aim stated in *Gödel 1944* "to set up a consistent theory of classes and concepts as objectively existing entities" (p. 152) and asks in terms of which of the different conceptions of being the "objectively existing entities" should be understood.

One might object to Günther's point of view in many ways. Gödel's response (letter 4, 30 June) is limited in what it takes on. He first denies, that Günther's philosophical claims contradict his own results "although my results make impossible certain forms of a subjectivistic interpretation of mathematics and in general speak strongly against every such interpretation." He remarks on the relevance of undecidability theorems for the law of excluded middle.

Gödel agrees in general terms with Günther's idealistic way of reading intuitionism. But in a very striking remark, he simultaneously agrees about the general importance of some basic ideas from idealism and gives a ringing affirmation of his own realism:

> The reflection on the subject treated in idealistic philosophy (that is, your second theme of thought), the distinction of levels of reflection, etc., seem to me interesting and correct. I even consider it entirely possible that this is "the" way to the correct metaphysics. However, I cannot go along with the denial of the objective meaning of thought that is connected with it, [although] it is really quite independent of it. I do not believe that any Kantian or positivistic argument or the antinomies of set theory or quantum mechanics has proved that the concept of objective being (no matter whether for things or abstract entities) is senseless or contradictory. When I say that one can (or should) develop a theory of classes as objectively existing entities, I do indeed mean by that existence in the sense of ontological metaphysics (pp. 3-4).

Whether idealist philosophers deny "the objective meaning of thought" is a highly controversial matter. I don't think Gödel has really assimilated Günther's conception of "ontological metaphysics", which on his view is an expression of the identity of thought and being that he associates with the classical tradition. Gödel may be affirming a form of transcendental realism, but the very fact that he thinks it can coexist with what he agrees to be insights of idealism implies that it is not exactly ontological metaphysics as Günther understood it. The matter was pursued only to a limited degree in the correspondence. But it illustrates a puzzle about Gödel's philosophy that is not confined to this exchange. Gödel is reported by Hao Wang to have much later described his philosophy as idealistic (Wang 1996, remark 0.2.2, p. 8), and after 1959 he was much attracted to the thought of Husserl, who described his position as transcendental idealism. Yet Gödel, in writing about mathematics and in a few places (including the above) about physics, expresses strong realistic convictions. He gives little explanation of how realism and idealism can coexist.[r]

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[r] Of course this is not to say that they cannot. In particular, Husserl has been interpreted so that his position is at least compatible with "common-sense realism", and many philosophers, including perhaps Husserl, have undertaken to overcome the opposition between realism and idealism or some other opposite of realism. In his introductory note to *1961/? (these Works, vol. III, pp.

Gödel declines to go into Günther's attempts in three-valued logic; he doesn't find a more detailed explanation of the connection of the truth-tables with Günther's philosophical ideas. But I will postpone until §3 a discussion of Gödel's reaction to Günther's own logical ideas.

Günther wrote two letters (2 October 1954 and 19 June 1955) before Gödel replied again. The issues he raises again concern the sense in which objects of different kinds are independent of thought. He mentions again some realistic remarks in *Gödel 1944* and quite reasonably queries the sense in which objects are said to be independent of thought (or of something else belonging to "us", e.g. "our definitions and constructions" (*Gödel 1944*, p. 134)). What concerns him is that different objects might be independent in different senses, although according to the first letter "the previous theory of thought from Aristotle to the present knows only *one* concept of logical (thought-independent) *object!*" This could be questioned, but it might be more relevant to object that classical formal logic does not distinguish different concepts of object or of thought-independence simply because the relevant differences are not in the province of formal logic; that need not make logic inapplicable across these differences. Perhaps intuitionistic logic does in its intended application call for a different conception of object, but it does not follow that classical logic is not applicable to a number of different conceptions.

Somewhat revealing is a remark Günther makes in letter 5 about the ontological status of space:

> If there is only one logically comprehensible form of existence, then the absence of everything "physical" is, as Plato thinks, mere nothing. If, however, we assume two forms of objectivity, then "empty" space is also a genuine "objectual" (*gegenständliches*) object of thought (p. 7).

But he seems to think that two-valued logic forces one to take the first view and thus to hold that space is *"objectively* considered nothing", which he claims to have been the purport of Kant's view that space is a form of intuition.

It's not surprising that in the next letter Günther gives some exposition of three-valued logic. But the context is a different line of thought, the distinction between reflection on objects thought of as independent of thought and reflection on reflection itself. He claims that these two types of reflection require fundamentally different concepts of object and the difference consists in whether the object is altered by the act of thought directed at it. He maintains that this obtains when one thinks about one's own subjective act of thought.

Gödel finally replied on 10 August 1955 (letter 7). He directs his criticism mainly at the formulation in Günther's second letter; even in intuitionistic logic, which he concedes is "a reflection on thought", objects of thought are as little altered by acts of thought directed at them as the objects of physics. He agrees, however, that objects of the second type of reflection are totally different from objects of the first.

364-373), Dagfinn Føllesdal evidently does not see an opposition between Husserl's "idealism" and Gödel's realism concerning mathematics.

This letter also contains Gödel's most extended comment on Günther's ideas about many-valued logic; see §3 below.

There followed a series of letters of Günther to which we have no replies by Gödel. Only the first (letter 8, 18 September 1955) represents an attempt to respond substantively to Gödel. On 22 June 1956, after returning to Richmond from his visit to Hamburg and a few months' stay in Chicago,[s] Günther asked for Gödel's support in his application for a renewal of his grant from the Bollingen Foundation and sent him a manuscript which appears to have been a preprint of *Günther 1958,* his most extended exposition of the ideas on many-valued logic in relation to metaphysics. On 17 September they met for a morning in Gödel's office, and evidently they discussed this paper.[t] Gödel wrote to the Foundation on 20 October to express his "whole-hearted support.[u] He says that the recent paper "Die aristotelische Logik des Seins und die nicht-aristotelische Logik der Reflexion" (i.e., *Günther 1958) "fully* confirms my favorable judgment".[v] Gödel's final remark is revealing about what attracted him in Günther's project:

> In view of the great interest which a satisfactory logical theory of "total reflexion" would have and in view of the depth of the philosophical problems involved three years, in my opinion, are not an excessively long time for studying these questions.

In the event the grant was extended for two years.[w] On 28 December Günther wrote asking for a recommendation for a professorship in Hamburg, which Gödel did write.[x]

On 26 February 1957, when Günther was in California for surgery, he wrote developing the idea that the concept of reflection really originates with Leibniz, although as a logical term "reflection" derives from Hegel. He then goes on to develop some interpretation of Leibniz, using some of his own conceptual apparatus. When Gödel next wrote, on 4 April 1957 (letter 10), he reacted skeptically, finding

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s See Günther 1963, p. 13.

t See above, in particular note e. In a card sent from Hamburg on 20 December 1955, Günther had said he would return to the United States in April or May and would like to drop in on Gödel at his office in Princeton. No other correspondence discusses a meeting; the actual arrangement could well have been made by telephone. That they discussed a version of *Günther 1958* is shown by Gödel's letter to Günther of 4 April 1957.

u A typed draft of this letter, with corrections in Gödel's hand, is in Gödel's papers (document no. 010767). Quotations are from this draft, whose text is in all probability very close to that of the letter as sent. That 20 October was the date of the actual letter is confirmed by the Foundation's acknowledgment (Nancy Russ to Gödel, 8 November 1956).

v This brief remark replaces the crossed-out statement that the paper "gives a remarkably clear exposition of Dr. Günther's leading ideas within the field of logic and also develops new means for building a system of formal logic on their basis." Gödel wrote much less favorably about the paper to Günther on 4 April 1957. John Dawson has remarked to me that Gödel avoided writing unfavorable letters of recommendation. For that reason what he says in such letters cannot always be taken at face value.

w Günther informed Gödel of this in his letter of 8 December 1956.

x Gödel to Helmut Schelsky, 8 January 1957, marked "abgeschickt". In his letter of 15 January 1957, Günther acknowledges receipt of a letter from Gödel dated 10 January, now lost. He immediately thanks Gödel for the recommendation and makes no other comment. Gödel's letter may have done no more than inform Günther that the recommendation had been sent.

the ideas interesting but doubting their basis in Leibniz's text. Following up their discussion in Princeton in 1956 of the manuscript of *Günther 1958*, he says that its basic ideas need clearer explanation and elucidation by examples. In its present form it is "hardly intelligible". But Gödel expresses interest in an idea Günther had expressed earlier, probably in the manuscript that he sent in 1953, that "total reflection" would be something going beyond all ascent of types. (We will return to this subject in §3.) This pleased Günther enough to overcome the effect of the critical tone of the rest, as his reply of 7 April shows. He then sent Gödel some of his papers. Gödel drafted a short note, dated 2 May, acknowledging their receipt.[y]

It seems likely that Gödel's interest in Günther's ideas had begun to decline. In the remainder of 1957, Günther wrote only one substantive letter (22 November, with copies of *Günther 1957* and *1957a*). Gödel replied on 23 December (letter 13) with rather brief comments on some of the papers Günther had sent. About *Günther 1957*, he says it has some overlap with a manuscript Günther had sent him previously, probably the one sent in 1953.[z] He says he has read some of "your new work", apparently 1957a[aa] and praises its clarity, particularly in contrast to the version of 1958 that he had read.

During 1958 Günther wrote Gödel twice, sending new papers. Gödel did not reply; when Günther wrote on 1 January 1959 expressing concern, Gödel wrote an interesting reply (letter 14, 7 January), again referring to Günther's earlier idea. But that was evidently his last reply. Günther continued to write and send papers through 1959 and 1960 but got no response from Gödel. In 1961 Günther obtained the appointment at the University of Illinois. The last item in the correspondence is an undated notice of the change of his address from Richmond to Urbana, postmarked 27 July 1961. On it Günther wrote, "I have accepted a research professorship at the University of Illinois and I am looking forward to it. Herzlichst, Ihr G.G." He was perhaps letting Gödel know that he too no longer needed to continue the exchange.

3. Gödel's reaction to Günther's logical ideas

Both in the correspondence and in published writings, Günther presses his constructions in many-valued logic, taking as his point of departure the views about metaphysics and logic discussed above. Gödel's remarks about this are rather brief. In letter 4 he writes:

> Unfortunately I can't go into your 3-valued logic more in this letter. What I miss so far is a more detailed explanation of the connection in content of your truth tables with your philosophical ideas.

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y Although no such letter is in Günther's papers, it seems likely that it was sent; Günther may well have thought it not worth saving.

z Gödel does not comment now or later on the issues about minds and machines that are discussed in *Giinther 1952* and *1957a* and are mentioned in Günther's letter, although he had already written on the subject in the Gibbs Lecture **1951.* (Günther had included *1952* in the packet of writings he sent in April; see his letter of 7 April.)

aa It is not clear whether what is being praised for clarity is *Günther 1957a* or *1957.* The fact that Gödel does not more explicitly indicate that he is talking about the same writing as in the previous sentence speaks for *1957a,* as does the contrast with *1958,* since the latter two writings set forth Günther's ideas on three-valued logic, while *1957* does not.

He doesn't reject out of hand the idea of using many-valued logic; in letter 7 he writes that it is suggested by reflection on the paradoxes[ab] and by the fact that "necessary" and "possible" can be thought of as truth-values. But Gödel is not able here or elsewhere to make very much sense of Günther's effort.

Regarding the meaning of the truth-values, Günther had attempted an explanation in his letters and in at least one publication before that time, *Giinther 1953*. But in the same letter Gödel wrote that the manner in which Günther wanted to introduce the third truth-value was "not wholly intelligible" to him and that he missed any explanation of what the three truth-values really mean. Gödel may not have seen enough by August 1955 to have even the limited grasp of Günther's intention expressed in §1. Probably he had by the time he read the manuscript of *Günther 1958,* but this did not lead to a change in his expressed attitude, and his view of that paper was not very favorable. It is easy to guess what the obstacle was: Günther wasn't able to give a motivation for modifying basic logic independent of his particular metaphysical interpretation of classical logic, and although he explains the three values in terms of his ideas about consciousness, he not only does not make clear how they are truth-values but seems to want to cut loose from the concept of truth, more so even than intuitionistic logic does. One might well ask whether what resulted should still be called *logic*.

Günther also seems never to have tried to write down axioms and rules of inference for his many-valued calculi. Probably that would have been necessary to persuade Gödel that this was intelligible logical work. But one can see why not only technical limitations prevented Günther from undertaking the task. If the "truth" values express levels of reflection, what would be the significance of being a theorem of the calculus? Such calculi are formulated in order to characterize conceptions of valid logical inference or logical truth. It's not clear that Günther's scheme has any place for these notions. That the concept of truth is not at center stage for him is indicated also by the fact that when he writes about intuitionism, although it is clear to him that in some sense the conception of existence is different from that in classical mathematics, he never remarks on the fact that there is a more underlying difference about truth. Gödel, however, does not make a remark that would smoke Günther out on this issue.

I have left for the last what is probably the most interesting of Gödel's reactions, on the matter of the relation of levels of reflection and type theories. As I have said, it is something Günther said about this that seems to have piqued Gödel's interest at the outset and made him tolerant of Günther's evident technical limitations. It is a theme that Gödel comes back to a couple of times without being prompted by Günther.

We have to reconstruct what Günther's idea may have been, since I have not been able to locate the manuscript Günther sent in 1953. Gödel found some overlap between that manuscript and *Günther 1957*, and as we saw above, Günther does

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[ab] Such an idea has been explored by a number of later writers on semantical paradoxes, and it was also observed that in some three-valued logical settings universal comprehension can be preserved. For a survey with its own development of these ideas see *Feferman 1984*. In these developments truth and falsity have their usual meanings and the third value has the sense "undefined". That is evidently quite different from what Günther had in mind, as he seems to say explicitly in a parenthetical remark in letter 8, p. 2.

formulate an idea of "total reflection" and claim that the third R-level at which that is achieved does contain within itself all iterations of reflection and so the hierarchy of types. In *Günther 1957* no connection with a type-free logic is made, but it seems that in the manuscript of 1953 such a connection was made. It doesn't appear, however, that any concrete suggestions for such a logic were offered, and Gödel had no success in prodding Günther to work out the idea, even though it was pressing this idea that most pleased Günther in Gödel's letter of 4 April 1957, as he wrote on 7 April. Very likely Günther's invocation of many-valued logic at this point disappointed Gödel.

Evidently Gödel entertained hopes that the kind of analysis of the structure of self-consciousness that Günther undertakes and finds precedent for in the classical idealists would yield not only philosophical illumination but even a basis for constructing a type-free logic that would be philosophically better motivated than what was then available. In his last letter, letter 14, he even says that the correct axioms for such a logic should follow *"with necessity* from philosophical insights about the essence of reflection." Much earlier he had made remarks that suggest that the most satisfactory theory of what he calls "concepts" would be a type-free logic of some kind.[ac] It is not known that Gödel did any substantial work toward the construction of such a logic. If he had something in hand early in his exchange with Günther, it is imaginable that he would have brought it up. But it seems to me more likely that he was very uncertain of how one might proceed and therefore perhaps disposed to entertain hopes that light might come from somewhat unlikely sources. Gödel seems never to have resolved this uncertainty, as is indicated by his remarks in his conversations with Wang about the theory of concepts and the unsolved nature of the "intensional paradoxes".[ad]

4. Conclusions

What does this exchange tell us about Gödel's thought during this period, beyond what one can learn from remarks he makes in his letters? What he did not say is revealing. He nowhere says that in order to attain a philosophical understanding of thought and its relation to objects, one might well begin with logic rather than taking "subject" and "object" or "thought" and "being" as fundamental categories. Such an outlook might, indeed, lead us to hold out greater hopes for understanding self-consciousness with the help of the logical analysis of semantic reflection, as it is carried out in theories of truth, than for the reverse procedure of expecting to motivate the axioms of a logical theory from an analysis of self-consciousness.

Still less does Gödel invoke the linguistic turn, for example what Michael Dummett regards as an axiom of analytical philosophy, "first, that a philosophical account of thought can be attained through a philosophical account of language, and, secondly, that a comprehensive account can only be so attained."[ae] One shouldn't read too much into the absence of something from correspondence, but Günther's ideas seem so obviously to call forth a response of some such kind that we might discern at least

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ac 1944, pp. 132, 140; cf. pp. 109-110 of the introductory note in these Works, Vol. 11.

ad *Wang* 1996, §§8.5-8-6.

ae Dummett 1993, p. 4.

an indication that the linguistic turn, at least as Dummett understands it, was not central to Gödel's practice as a philosopher.[af]

Given Günther's obvious technical limitations and the difficulty Gödel had in making sense of his proposals, we might find it surprising that Gödel maintained his interest in Günther for as long as he did and even up to early 1957 wrote supportive letters for him (see §2 above). There seems to have been some mutual personal sympathy; for example both write about their health problems and respond sympathetically to the other's. Both were outsiders to the American philosophical world and shared the project of dissociating modern logic from positivistic philosophy. But a deeper reason is very likely that Gödel was at the time occupied with philosophical problems about concepts (it was the time during which he worked on **1953/9*) and was very unsure about how to approach them.[ag] He was evidently prepared to entertain the possibility that post-Kantian idealism, to which he had apparently not had a lot of exposure, would be a source of illumination. He found Günther a clear expositor of ideas from that tradition.[ah] But he does not seem to have been disposed to work out himself a line of thought in which self-consciousness is a central concept, and when Günther did not pursue what Gödel thought the most promising direction, he lost interest. Not long after his last letter he began his study of Husserl, whose version of idealism he seems to have found much more satisfactory.

Charles Parsons[ai]

A complete calendar of the correspondence with Günther appears on pp. 563-564 of this volume. The editors are indebted to Mr. Lothar Busch for locating Gödel's letters in Günther's papers while they were being catalogued, to Professor Tilo Brandis, Director of the Handschriftenabteilung of the Staatsbibliothek zu Berlin-Preussischer Kulturbesitz, for making copies of these letters available early in our work, as well as to Delia Graff and Øystein Linnebo for preparing typescripts of handwritten letters. The translation is by Thomas Teufel and Charles Parsons, revised using suggestions of John W. Dawson, Jr.

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af From Wang's reports of their conversations, one would conclude that in the 1970s Gödel was actively opposed to the linguistic turn as Dummett and many others would understand it. See Wang 1996, remarks 5.5.7-5.5.9, pp. 180-181, and the sharp distinction between the semantic and intensional paradoxes made in §8.5.

ag That this uncertainty was a source of his interest in Günther was suggested by Warren Goldfarb.

ah Letter to Schelsky (see note *x* above).

ai I am indebted to Klaus Oehler, Claus Baldus and Markus Stepanians for information about Günther, to Øystein Linnebo and Thomas Teufel for assistance in locating writings of Günther and for discussion of his ideas, and to John Dawson, Solomon Feferman, Warren Goldfarb, Sally Sedgwick and Wilfried Sieg for helpful comments.

Briefwechsel-Gödel-GG_01

1. Günther to Gödel

Gotthard Günther
101 Oronoco Ave. (Apt. 2)
Richmond 22, Va.
April 29. 1954

Sehr geehrter Herr Professor Gödel:

Ich arbeite gerade an einer philosophischen Analyse des Satzes vom ausgeschlossenen Dritten und bin dabei auf eine Bemerkung Karl Mengers über Sie gestossen, die mir nicht klar ist. Menger schreibt (Krise und Neubau in den exakten Wissenschaften, Die neue Logik, Leipzig. Wien 1933,[a] S.11): "Nun hat ... Gödel kürzlich gefunden, dass nicht nur die intuitionistische Mathematik ein Teil der klassischen ist, sondern auch der gesamte klassische Aussagenkalkül und die gesamte klassische Zahlentheorie samt dem Satz vom ausgeschlossenen (Dritten) als Teil des Intuitionismus aufgefasst werden können, indem man durch ein einfaches Wörterbuch jeden klassischen Satz in einen intuitionistischen übersetzen kann. ... Die Ablehnung des Satzes vom ausgeschlossenen Dritten hat also (da die Intuitionisten Unmöglichkeiten von Allaussagen zulassen) in Wahrheit gar keine Einschränkung, sondern *bloss eine Umbenennung* der klassischen Sätze zu Folge." Menger weist dabei auf <"Ergebnisse eines mathematischen Kolloquiums", 4, Leipzig 1933> [b] hin, das mir hier aber leider nicht zugänglich ist, weshalb ich mich | an Sie direkt mit der Bitte
2 um freundliche Auskunft wende.
Die intuitionistische Mathematik ist ein Teil der klassischen. Ich verstehe das. Wenn es dann aber weiter heisst, dass der klassische (also zweiwertige) Aussagenkalkül als *Teil* des Intuitionismus aufgefasst werden kann, so müssen doch wohl *zwei* verschiedene Konzeptionen des Tertium non datur im Spiel sein.

1. wenn die intuitionistische Math. Teil der Klassischen ist, dann steht sie *unter* dem *generellen* klassischen Tertium non datur
2. wenn das Tertium non datur Teil des Intuitionismus ist, muss hier eine *speziellere* Interpretation des Drittensatzes in Frage kommen.

1. Günther to Gödel

Gotthard Günther
101 Oronoco Ave. (Apt. 2)
Richmond 22, Va.
April 29. 1954

Dear Professor Gödel,

I am currently working on a philosophical analysis of the law of the excluded middle and in the process encountered a remark about you by Karl Menger that is unclear to me. Menger writes (Krise und Neubau in den exakten Wissenschaften, Die neue Logik, Leipzig. Wien 1933[a] p. 11):
"Now... Gödel has recently discovered not only that intuitionistic mathematics is a part of classical mathematics, but also that the entire classical propositional calculus and the entire classical number theory, as well as the law of the excluded middle, can be understood as part of intuitionism, by translating every classical proposition into an intuitionistic one using a simple dictionary.... The rejection of the law of the excluded middle, therefore (since the intuitionists allow impossibilities of universal quantifications), in fact implies no limitation at all but *merely a renaming* of classical propositions." Menger refers to "Ergebnisse eines mathematischen Kolloquiums 4, Leipzig 1933"[b], which is unfortunately not accessible to me here. That is why I am turning directly to you to ask if you would be so kind as to give me information.
Intuitionistic mathematics is a part of classical mathematics. That I understand. But when the passage says in addition that the classical (that is, two-valued) propositional calculus can be conceived as *part* of Intuitionism, this can only mean that *two* different conceptions of the tertium non datur are at play here.

1. if intuitionist mathematics is part of classical, then it is *subject* to the *general* classical tertium non datur;
2. if the tertium non datur is part of intuitionism, then a *more specific* interpretation of the law of the excluded middle must be in question.

Now, the law of the excluded middle can

Nun lässt sich der Drittensatz in der Tat auf verschiedene Weise formulieren, so dass wir einmal eine allgemeinere (schwächere) Formulierung, das andere Mal eine stärkere (aber speziellere) Version erhalten. Als allgemeinere Formulierung möchte ich anführen:

$$[(A \rightarrow B) \cdot (\sim A \rightarrow B)] \rightarrow B \qquad (1)$$

Die andere erhalten wir auf die folgende Weise. Wir gehen von dem logischen ε-Axiom

$$A(x) \rightarrow A(\varepsilon A)$$

aus. Dabei soll ε(A) ein Objekt bezeichnen für das A(x) wahr ist, wenn A(x) überhaupt für irgend ein Objekt wahr ist. Mit "ε" lässt sich nun sowohl der Existenz wie der Alloperator definieren: |

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$$(Ex)A(x) =_{Def} A[\varepsilon(A)]$$
$$(x)A(x) =_{Def} A[\varepsilon(\sim A)]$$

Auf Grund dieser Definition liefert uns "ε" die zweite Formulierung für das Tertium non datur:

$$\sim[(x)A(x)] \rightarrow (Ex)\ \sim A(x) \qquad (2)$$

Was ich gern wissen möchte ist: ist die zitierte Bemerkung Mengers so zu verstehen, dass die intuitionistische Mathematik ein *Teil* des Systems ist das durch (1) umschrieben wird und enthält sie andererseits (2) als einen Teil ihrer selbst?
Zwischen (1) und (2) besteht ja in der Tat ein beträchtlicher ontologischer Unterschied. Lassen Sie mich, bitte, diesen Unterschied auf folgende Weise beschreiben: Paul Hofmann hat in einer Studie, die zum Besten gehört, was über den Drittensatz geschrieben worden ist, die "unklare Fassung" des Tertium non datur gerügt.[c] Es fehle diesem logischen Axiom ein oberster Bestimmungsgesichtspunkt unter dem sich "A" und "~A" kontradiktorisch ausschliessen. Das ist in der Tat für Formulierung (1) der Fall.
Dagegen ist für (2) ein solcher oberster Bestimmungsgesichtspunkt vorausgesetzt. Derselbe ist durch ε(A) gegeben. D.h. es muss ein Objekt *vorhanden* sein, auf das sich die Aussage bezieht. Der oberste Bestimmungsgesichtspunkt ist also: objektiv vorhandenes Sein ... oder (wenn Sie sich an | dem metaphysischen Terminus <Sein> stossen) gegenständliche Existenz.

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Der Kern der intuitionistischen Einwände

indeed be formulated in different ways, such that we either obtain a more general (weaker) formulation, or a stronger (but more specific) version. As a more general formulation I would like to offer:

$$[(A \rightarrow B) \cdot (\sim A \rightarrow B)] \rightarrow B \qquad (1)$$

The other formulation is obtained in the following manner. We start with the logical ε-axiom:

$$A(x) \rightarrow A(\varepsilon A)$$

Here ε(A) is meant to denote an object for which A(x) is true, provided A(x) is true of any object at all. With "ε" the existential quantifier as well as the universal quantifier can be defined:

$$(Ex)A(x) =_{Def} A[\varepsilon(A)]$$
$$(x)A(x) =_{Def} A[\varepsilon(\sim A)]$$

On the basis of this definition "ε" provides us with the second formulation of the tertium non datur:

$$\sim[(x)A(x)] \rightarrow (Ex)\ \sim A(x) \qquad (2)$$

What I would like to know is: should we understand Menger's quoted comment as [saying] that intuitionistic mathematics is a *part* of the system characterized by (1), and does it, on the other hand, contain (2) as part of itself?
Between (1) and (2) there is indeed a significant ontological difference. Please let me describe this difference in the following way: in a study that belongs to the best things that have been written about the law of the excluded middle Paul Hofmann criticizes the "unclear formulation" of the tertium non datur.[c] [He points out that] this logical axiom lacks a highest determining perspective under which "A" and "~A" exclude each other as contradictories. This is indeed the case for formulation (1).

In contrast, for (2) such a highest determining perspective is presupposed. It is given by ε(A). I.e., an object must be *given* to which the proposition refers. The highest determining perspective is therefore: objectively existing Being ... or (if you take exception to the metaphysical term "Being") existence as an object.

The core of the intuitionistic objections now appears to me to be, that for any

scheint mir nun darin zu liegen, dass für jede *positive* Formulierung von (2) eine allgemeinere gefunden werden kann, die die vorangehende positive Formulierung des angeblichen obersten Bestimmungsgesichtspunkts desavouiert. <Sein> ist kein absolutes Datum, sondern ein Komplementärbegriff zu dem jeweiligen logischen Interpretationssystem, das wir benutzen. Deshalb ist (2) immer nur relativ gültig für *aufgewiesene Existenz* für die der Intuitionismus den Drittensatz ja in der Tat zulässt.

Die Formulierung (1) aber reprdsentiert die nach Hofmann "unklare Formulierung" des Tertium non datur. Der oberste Bestimmungsgesichtspunkt für die totale Disjunktion fehlt. Nur ist das m.E. *kein Mangel.* Denn dieses Fehlen indiziert dass (1) die Formulierung des Drittensatzes für eine unendliche Hierarchie aufeinander folgender *möglicher* Bestimmungsgesichtspunkte ist. In diesem Sinn würde (1) den Intuitionismus einschliessen.

Habe ich Unrecht, so wäre ich für entsprechende Berichtigung durch Sie herzlich dankbar. Bitte vergessen Sie nicht, ich bin meiner Ausbildung nach Historiker und Metaphysiker. Ich versuche das,
5 was heute in | Logik und Mathematik geschieht philosophisch zu interpretieren, aber nicht es besser zu machen.

Vorläufig versuche ich allerdings meine Kollegen in der Metaphysik ohne jeden Erfolg zu der Ansicht zu bekehren, dass man heutzutage nicht mehr Metaphysik treiben kann, ohne die Arbeit der letzten hundert Jahre in symbolischer Logik und Mathematik zugrunde zu legen.

Jedenfalls bemühe ich mich für meine Person das zu tun und wäre Ihnen deshalb für Ihre Hilfe herzlich dankbar.

Mit freundlichen Grüssen und besten Dank im Voraus

Ihr

Gotthard Günther

positive formulation of (2) a more general one can be found which disavows the preceding positive formulation of the allegedly highest standpoint of determination. "Being" is not an absolute datum but a notion complementary to the particular logical system of interpretation we use. Because of that, (2) is always only relatively valid for *exhibited extstence,* for which intuitionism indeed admits the law of the excluded middle.

Formulation (1), however, represents the "unclear formulation" of the tertium non datur according to Hofmann. The highest determining perspective for the total disjunction is missing. But in my view this is *not a shortcoming.* For this lack indicates that (1) is the formulation of the law of the excluded middle for an infinite hierarchy of successive *possible* determining perspectives. In this sense (1) would include intuitionism.

If I am mistaken, I would be most grateful for relevant correction on your part. Please do not forget that I am a historian and metaphysician by training. I try to interpret philosophically what is happening in contemporary logic and mathematics, not to improve upon it.

For the time being, though, I am trying without any success to convert my colleagues in metaphysics to the view that these days one can no longer pursue metaphysics without taking as the basis the work of the last hundred years in symbolic logic and mathematics.

In any case, I for my part strive to do this and would therefore be most grateful for your help.

With cordial greetings and many thanks in advance,

Yours,

Gotthard Günther

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[a] Menger 1933.

[b] I.e. *Gödel 1933*e. These pointed brackets in the German text are in the letter and so do not signify an insertion. Günther fairly frequently uses pointed brackets as another style of quotation marks. For his pointed brackets we use < and >; to signify insertions we use (and). In the translation, however, the usual American conventions concerning quotation have been followed.

[c] Hofmann 1931. Günther may refer to pp. 13-14 (= pp. 93-95-of Hofmann 1931a)

2. Gödel to Günther

Princeton, 15./V. 1954

Sehr geehrter Herr Dr Günther:

Um zu erklären, in welchem Sinn die von Ihnen zitierte Mengersche Äußerung zu verstehen ist, möchte ich zunächst feststellen, daß, wörtlich genommen, weder die intuit. Math. ein Teil der klassischen ist, noch umgekehrt. Denn die intuit. Begriffe sind verschieden von den gleichbezeichneten klassischen u. können auch nicht aus diesen definiert werden (noch umgekehrt). Wie Sie richtig bemerken, hat diese Verschiedenheit ihren Grund darin, daß verschiedene Begriffe des Seins verwendet werden. Das hindert aber nicht, daß es gewisse Begriffe in der klass. Math. gibt, die *formal* denselben Gesetzen gehorchen wie die int. Begriffe[1] , u. umgekehrt Begriffe in der int. Math., für die formal die Grundsätze der klass. Math. gelten. Das erstere | gilt
2 z.B. (innerhalb von Aussagen u. Funktionenkalkül 1. Stufe) für die gleichbezeichneten Begriffe. Das letztere gilt (innerhalb der Zahlentheorie), wenn man den klass. Begriffen:

$\sim p,\ p\cdot q,\ p\vee q,\ p\supset q,\ (x)F(x),\ (\exists x)F(x),$

die folgenden intuit. entsprechen läßt:

$\sim p,\ p\cdot q,\ \sim(\sim p\cdot\sim q),\ \sim(p\cdot\sim q),\ (x)F(x),$
$\sim(x)\sim F(x).$

Für das so definierte "oder" gilt offenbar der Drittensatz in der intuit. Math. u. dasselbe gilt für die andern logischen Grundsätze [2] u. daher auch für alle Theoreme. Die klass. Zahlentheorie hat also ein vollständiges formales Bild innerhalb der int. Zahlentheorie, u. da es in der Math. in erster Linie auf die Form (u. nicht den Inhalt) ankommt, so heißt das für die Math. praktisch dasselbe, als wenn die klass. Zahlentheorie Teil der intuit. wäre. | Sie haben Recht daß dabei

2. Gödel to Günther

Princeton, 15 May 1954

Dear Dr. Günther,

In order to explain in which sense one should understand the remark of Menger that you quote, I would first like to state that, taken literally, intuitionism is neither part of classical mathematics, nor vice versa. For intuitionistic concepts are different from the classical concepts of the same name and cannot be defined in terms of them either (nor vice versa). As you correctly observe, the basis for this difference is that different concepts of Being are employed. That does not, however, prevent its being the case that there are certain concepts in classical mathematics which obey formally the same laws as the intuitionistic concepts,[1] and conversely concepts in intuitionistic mathematics for which formally the principles of classical mathematics hold. The former holds, for example, for concepts of the same name (within the propositional and first- order functional calculus). The latter holds (within number theory), if one correlates to the classical concepts

$\sim p,\ p\cdot q,\ p\vee q,\ p\supset q,\ (x)F(x),\ (\exists x)F(x),$

the following intuitionistic concepts:

$\sim p,\ p\cdot q,\ \sim(\sim p\cdot\sim q),\ \sim(p\cdot\sim q),\ (x)F(x),$
$\sim(x)\sim F(x).$

For the "or" thus defined the law of the excluded middle evidently holds in intuitionistic mathematics, and the same is true for the other basic logical principles [2] and therefore also for all theorems. Classical number theory thus has a complete formal image within intuitionistic number theory, and, since mathematics is primarily concerned with form (and not content), this means for mathematics practically the same as if classical number theory were part of intuitionistic

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1 d.h. genauer, die allen Gesetzen der intuit. Math. gehorchen ohne Riicksicht darauf, ob sie vielleicht außerdem noch andern gehorchen.
I.e., more precisely, which obey all the laws of intuitionist mathematics without regard to whether they perhaps also obey others.

2 was nicht so trivial ist wie im Fall des Drittensatzes
which is not as trivial as in the case of the law of the excluded middle.

3 z.B. für das Heytingsche System der int. Zahlentheorie

zwei verschiedene Konzeptionen des Drittensatzes im Spiel sind. Die zweite, die in der intuit. Math gilt, lautet: "p und ~p können nicht beide falsch sein". Was Sie als zweite (speziellere) Konzeption anführen, nämlich:

$$\sim(x)A(x).\supset .(\exists x)\sim A(x) \qquad (2)$$

gilt nicht in der intuit. Math u. ist (zumindest innerhalb der Zahlentheorie) sogar mit $p\vee\sim p$, sowie auch mit der von Ihnen angeführten 1. Konzeption äquivalent. Für *entscheidbare* Eigenschaften A (d.h. solche für die $(x)[A(x)\vee\sim A(x)]$ beweisbar ist), wobei x eine Variable für natürliche Zahlen ist, gilt (2) für *gewisse formaliserte Teilsysteme* $\langle S\rangle$ des Int.[3] in dem Sinn, daß, wenn eine Ableitung eines Widerspruchs *in S* aus (x)A(x) vorliegt, eine Zahl n berechnet werden | kann, für die ~A(n) gilt.

Es ist mir nicht klar, was Sie unter Ihrer "unendlichen Hierarchie möglicher Bestimmungsgesichtspunkte für den Drittensatz" verstehen. Wenn Sie damit meinen, daß es zwischen aufgewiesener u. objektiver Existenz eine unendliche Folge von in verschiedenem Grade aufweisbarer Existenz gibt, so würde ich dem in einem gewissen Sinne zustimmen. Aber der Kern der int. Einwände liegt wohl darin, daß gezeigt wird, daß für aufweisbare Existenz gewisse Sätze der klass. Math. unbewiesen u. andere sogar nachweislich falsch werden (z.B. gilt ja für gewisse Satzklassen *K* im Int.:

$$\sim(p)\ (p\in K.\ \supset.\ p\vee\sim p))$$

Ich bin ganz Ihrer Meinung, daß die Anwendung der Methoden u. Resultate der Math. nicht auf die positivistische Phil. beschränkt bleiben sollte. Ich habe mich sehr gefreut, daß Sie das Stipendium der Bollingen Foundation bekommen haben u. wünsche Ihnen besten Erfolg für Ihre Arbeit.

Mit besten Grüssen
Ihr
Kurt Gödel

number theory. You are right that two different conceptions of the law of the excluded middle are in play here. The second, which holds for intuitionistic mathematics, says: "p und ~p cannot both be false". What you adduce as the second (more specific) conception, namely:

$$\sim(x)A(x).\supset .(\exists x)\sim A(x) \qquad (2)$$

does not hold in intuitionistic mathematics and is (at least within number theory) even equivalent with $p\vee\sim p$, as well as with the first conception you mention. For *decidable* properties A (i.e., for those for which $(x)[A(x)\vee\sim A(x)]$ can be proved), where x is a variable for natural numbers, (2) holds for *certain formalized subsystems* $\langle S\rangle$ of intuitionism, [3] in the sense that, if a contradiction can be derived *in S* from (x)A(x), a number n can be computed for which ~A(n) holds.

It is not clear to me what you mean by your "infinite hierarchy of possible determining perspectives for the law of the excluded middle". If you mean that between exhibited and objective existence there is an infinite sequence of different degrees of exhibitable existence, I would agree with you in a certain sense. But the core of the intuitionistic objections is, rather, that it is shown that for exhibitable existence certain theorems of classical mathematics become unproved and others even demonstrably false (for example, for certain classes of propositions K,

$$\sim(p)\ (p\in K.\ \supset.\ p\vee\sim p)$$

holds in intuitionism).

I fully agree with your view that the application of the methods and results of mathematics should not remain confined to positivistic philosophy. I was very pleased that you have received the stipend from the Bollingen Foundation and wish you the best of success with your work.

With cordial greetings,
Yours, Kurt Gödel

3. Günther to Gödel

Gotthard Günther
101 Oronoco Ave. (Apt. 2)
Richmond 22
May 23, 1954

Sehr geehrter Herr Professor Gödel:

Vor einigen Tagen erhielt ich Ihren eingehenden Brief vom 15. V., und ich möchte Ihnen hiermit recht herzlich für die darin enthaltene Auskunft danken. Ihr Schreiben hat mir in der Tat einige Punkte, die mir in der Literatur unklar geblieben waren, aufgehellt. Speziell die Mengersche Äusserung, hinter der ich etwas ganz Anderes vermutet hatte. – Ich könnte diesen Brief damit schliessen. Wenn ich dies aber nicht tue und Ihnen hiermit einige weitere Gedankengänge unterbreite, bitte ich mir zu glauben, dass ich dies mit Rücksicht auf Ihre kostbare Zeit nur äusserst zögernd tue. Aber ich befinde mich in einer gewissen Zwangslage. Ich bin – abgesehen von den Neo-Thomisten – so ziemlich der einzige Metaphysiker, der davon überzeugt ist, dass man heute nicht Metaphysik treiben kann, ohne die Ergebnisse der symbolischen mathematischen Logik vorauszusetzen. Und die symbolische Logik im Neo-Thomismus (Ivo Thomas z. B.) ist meiner Ansicht nach auf einem Irrweg. Es wird dort nämlich nicht zugegeben, dass der logische Positivismus überzeugend demonstriert hat, dass die *klassische ontologische* Meta-|physik wissenschaftlich unhaltbar ist. Anstatt die Resultate der Logistik für eine *neue* Metaphysik zu verwenden, versucht man dort immer noch die mittelalterliche Kirchenmetaphysik (die Fundamental-ontologie) mit mathematischer Logik zu beweisen. Auf der anderen Seite steht Heidegger, der erst kürzlich wieder die Logistik eine "Ausartung", die sich mit einem "Schein der Produktivität" umgibt, genannt hat.[a] – Von diesen Leuten her, können meine Gedanken also keine Kontrolle erfahren. Ich muss mich also schon an mathematische Logiker wenden.

2

Im Folgenden möchte ich Ihnen einige grundsätzliche Gedanken unterbreiten, in denen mich Ihr Brief noch bestärkt hat. Ich bitte Sie dieselben nur zur Kenntnis zu neh-

3. Günther to Gödel

Gotthard Günther
101 Oronoco Ave. (Apt. 2)
Richmond 22
May 23, 1954

Dear Professor Gödel,

Several days ago I received your detailed letter of 15 May, and I would hereby like to thank you most warmly for the information contained in it. Your letter has indeed illuminated for me several points that had remained unclear to me in the literature. In particular Menger's remark, behind which I had conjectured something entirely different. – I could close this letter at this point. If, however, I don't do so and submit to you here several further lines of thought, 1 ask you to believe me that I do this only extremely hesitantly, in consideration of your valuable time. But I find myself in a certain bind. I am – apart from the neo-Thomists – pretty well the only metaphysician who is convinced that one cannot do metaphysics today without presupposing the results of symbolic mathematical logic. And symbolic logic in neo-Thomism (Ivo Thomas for example) is in my view on the wrong track. For there it is not admitted that logical positivism has convincingly demonstrated that classical ontological metaphysics is scientifically untenable. Instead of applying the results of symbolic logic for a *new* metaphysics, one still tries to prove medieval church metaphysics (fundamental ontology) by means of mathematical logic. On the other side stands Heidegger, who just recently again called symbolic logic a [manifestation of] "degeneracy", which surrounds itself with an "illusion of productivity".' – Thus from these people my thoughts cannot receive any supervision. I must therefore turn to mathematical logicians.

In the following I would like to present to you several fundamental thoughts, in which your letter has encouraged me. I only ask you to take note of them, and you do not need to answer this letter.

men, und Sie brauchen diesen Brief nicht zu beantworten. Es sei denn, Sie entdecken in meinen philosophischen Theoremen etwas das positiv falsch auf der Basis Ihrer eigenen Untersuchungen und Ergebnisse sein muss. In diesem Fall wäre ich für einen entsprechenden Hinweis äusserst dankbar:

Die oberste Formel der klassischen Philosophie seit Plato/Aristoteles lautet ὄντωζ ὄν. D. h. Sein des Seienden. Wir haben also ein zweistufiges Wissen. Empirisches Wissen vom *Seienden* (Math. & Physik) und apriorisches Wissen vom *Sein* (Logik & Metaphysik).[b] Dem zweistufigen Wissen entspricht eine zweistufige Objektwelt. Wir haben erstens: den Objektraum als die Vielheit der empirischen Dinge. Dahinter aber steht als
3 zweites, totales Objekt: das absolute Sein. |

Kant hat zuerst, *transzendental* gezeigt, dass das absolute Sein kein wissenschaftliches Objekt sein kann. Seine Demonstration aber war nicht überzeugend, weil sich die Kantische Transzendentallogik nicht formalisieren lässt. In der Logistik ist dann dasselbe Resultat erreicht worden, mit dem Hinweis darauf, dass Prädikatsfunktionen eine Variable enthalten und dass der Wert der Variablen nur *empirisch* aufgenommen werden kann. "Sein" ist der faktische Argumentwert einer Variablen. Quine: "To be is to be the value of a variable."

Damit aber tauchte eine neue, bisher nicht dagewesene Schwierigkeit auf. Ich will sie so kurz als möglich beschreiben. In der klassischen Tradition zählen das Subjekt des Denkens und der Reflexionsprozess überhaupt nicht. Das Ziel des Denkens ist den Sinn des *absolut objektiven* Seins zu fassen. Und Wahrheit bedeutet absolute Übereinstimmung des Denkens mit dem absolut objektiven Gegenstand. D.h. alle Kategorien der Logik müssen, wenn sie wahr sein sollen, absolut objektiv definierbar sein. Alles "Subjektive" ist schlechthin zu eliminieren.

Nun kam aber erst die Kritik der reinen Vernunft und erklärte: Dinge an sich sind grundsätzlich keine Objekte des Bewusstseins. Dann kam der logische Positivismus und bestätigte: absolute Objektivität ist eine blosse Fiktion. Überdies hat ca. 1930 Heisenberg in einer bedeutsamen Schrift über die Grundlagen der Quantenmechanik er-

Unless, that is, you find something in my philosophical theorems that, on the basis of your own investigations and results, must be positively false. In that case I would be very grateful for a corresponding suggestion:

The highest formula of classical philosophy since Plato/Aristotle is ὄντωζ ὄν i.e., the Being of existing things. We thus have two-tiered knowledge. Empirical knowledge of *existing things* (mathematics & physics) and a priori knowledge of *Being* (logic & metaphysics).[b] To this two- tiered knowledge there corresponds a two-tiered world of objects. We have, first: the space of objects as the multiplicity of empirical things. Behind this, however, there stands as second, total object: absolute Being.

Kant was the first to show *transcendentally* that this absolute Being cannot be an object of science. But his demonstration was not convincing, because Kantian transcendental logic cannot be formalized. In symbolic logic the same result was then attained, by noting that predicate functions contain a variable and that the value of the variables can only be determined *empirically*. "Being" is the factual value of a variable. Quine: "To be is to be the value of a variable".

But with this a new, unprecedented difficulty emerged. I want to describe it as briefly as possible. In the classical tradition the thinking subject and the process of reflection do not count at all. The goal of thought is to grasp the sense of *absolutely objective* Being. And truth means absolute agreement of thought with the absolutely objective object. That is, all categories of logic must, if they are to be true, be absolutely objectively definable. Everything "subjective" is simply to be eliminated.

But now, first the Critique of Pure Reason came along and declared: Things in themselves are in principle not objects of consciousness. Then came logical positivism and affirmed: absolute objectivity is a mere fiction. Moreover, around 1930 Heisenberg explained in an important paper on the foundations of quantum mechanics: "...the absolutely

klärt: "... der absolut isolierte Gegenstand
4 hat prinzipiell keine beschreibbaren | Eigenschaften mehr.[c] D.h. die Experimentalsituation des Beobachters muss in die Beschreibung des Objekts mit hineindefiniert werden. Schön, wenn das aber so ist, dass das absolut objektive *Sein* nur eine Fiktion ist und wir nur *relativ* objektiven Seienden begegnen, dann ist die klassische Logik im Irrtum, wenn sie annimmt, dass Objektivität (oder Sein) das *einzige* rationale Thema des Denkens ist. Das logische Denken hat dann *zwei* fundamentale Themata: 1.) das empirische (relative) Objekt und 2.) den subjektiven Reflexionsprozess; der sich selbst nur teilweise in pseudo-objektive Kategorien auflösen lässt.

Diese beiden Themata aber werden in der gegenwärtigen Logik in einer höchst unzulässigen Weise miteinander vermischt. In Ihrem Aufsatz <Russell's Mathematical Logic> (Evanston & Chicago 1944)[d] bemerken Sie sehr richtig, dass unser Ziel (aim) ist "... to set up a consistent theory of classes and concepts as objectively existing entities." (S. 152) Andererseits aber bemerken Sie in Ihrem Brief an mich, dass die Differenz zwischen klassischen und intuitionistischen Prinzipien ihren Grund darin hat "dass verschiedene Begriffe des Seins verwendet werden".

Damit aber entsteht die Frage: welcher der verschiedenen Begriffe des Seins soll den "objectively existing entities" zugrunde gelegt werden?

Die Frage ist heute nirgends zureichend be-
5 antwortet, weil man sich nicht genügend Rechenschaft über | die Differenz verschiedener Seinsbegriffe gegeben hat. Um diesen Brief nicht zu lang werden zu lassen, möchte ich das Nächste in Form von Thesen (ohne längere Begründungen) konstatieren:

1) Es gibt zwei, und nur zwei, fundamentale Seinsbegriffe:

a) Sein, das rein objektiv-thematisch definiert werden kann, d.h. ohne das denkende Subjekt in den Objektbegriff mit hinein zu definieren.
b) Sein, doppel-thematisch interpretiert. D.h. der "subjektive" Reflexionsprozess der "Sein" denkt, muss in den Objektbegriff hinein definiert werden.

isolated object does not, in principle, any longer have any describable properties".[c] That is, the experimental situation of the observer has to be defined into the description of the object.

Fine, but if it is the case that absolutely objective *Being* is only a fiction and we only encounter existing things that are *relatively* objective, then classical logic is in error when it assumes that objectivity (or Being) is the only rational topic of thought. Logical thought then has *two* fundamental topics: 1.) the empirical (relative) object and 2.) the subjective process of reflection; which itself can only be partially resolved into pseudo-objective categories.

In contemporary logic, however, these two topics are being mixed together in a highly inadmissible fashion. In your essay "Russell's mathematical logic" (Evanston & Chicago 1944)[d] you very correctly observe that our aim is "... to set up a consistent theory of classes and concepts as objectively existing entities" (p. 152). But on the other hand, you remark in your letter to me that the difference between classical and intuitionistic principles has its basis in the fact "that different notions of Being are being used".

But then the question arises: on which of the different notions of Being should the "objectively existing entities" be based?

Today this question is nowhere sufficiently answered, because the distinction between different notions of Being has not been satisfactorily accounted for. In order not to let this letter go on for too long, 1 would like to state the following in the form of theses (without longer justifications):

1) There are two, and only two, fundamental concepts of Being:
a) Being that can be defined purely objective-thematically, i.e., without defining the thinking subject into the concept of the object.
b) Being that is double-thematically interpreted. That is, the "subjective" process of reflection that thinks "Being" has to be defined into our concept of the object.

Die Differenz dieser beiden Seinsbegriffe scheint mir dem Unterschied von klassisch:

~p, p·q, p∨q, p⊃q, (x)F(x), (Ex)F(x)

und intuitionistisch:

p, p·q, ~(~p·~q), p⊃q, (x)F(x), ~(x)~F(x)

zugrunde zu liegen. Wenn wir nämlich die klassische Disjunktion durch "~(~p·~q)" vertreten lassen, so haben wir damit ausgedrückt, dass der Drittensatz klassisch (p∨q) nur für absolut objektives Sein gilt. Für den zweiten Seinsbegriff gilt er nur abgeschwächt, insofern als wir den durch "~ ..." ausgedrückten Reflexionsprozess in unsere Konzeption des Objekts hineinnehmen müssen.

Zwischen "~... " in der Tafel

p	~p	(I)
W	F	
F	W	

6 |6 und demselben Zeichen "~ ..." in der Formel "~(~p·~q)" existiert nämlich ein Unterschied, der mir analog dem Unterschied von freier und gebundener Variablen im Prädikatenkalkül zu sein scheint. In Tafel (I) ist "~..." sozusagen ungebunden, während in der Formel ~(~p·~q) es ontologisch durch "—•—" <gebunden> ist. D. h. seine Bedeutung Verneinung überhaupt zu sein, ist auf die Verneinung von "—•—" hin präzisiert worden. Dasselbe gibt für die Ersetzung des Existenzoperators durch ~(x)~F(x).

Der Parallelismus zwischen klass. und intuit. Begriffen scheint also anzudeuten, dass wir zwei Kategorien von logischen Entitäten haben: Radikal objektiv thematisiert im Sinn von a) Sein; und doppel-thematisch (objektiv/subjektiv) orientiert im Sinne von b) Sein.

Dem entspricht die Zweiteilung des Prädikatenkalküls, resp. die Teilung in Quantifikationstheorie und Klassentheorie. Ihren eigenen Entdeckungen zufolge ist das Axiomensystem des engeren Funktionenkalküls vollständig (Monatshefte für Math. v. 37 (1930)),[e] aber es existiert Nichtentscheidbarkeit. Für den weiteren Kalkül gilt, gemäss einer zweiten Entdeckung von Ihnen, weder Vollständigkeit noch Entscheidbarkeit.

The difference between these two concepts of Being seems to me to lie at the foundation of the difference between classical:

~p, p·q, p∨q, p⊃q, (x)F(x), (Ex)F(x)

and intuitionistic:

p, p·q, ~(~p·~q), p⊃q, (x)F(x), ~(x)~F(x)

For if we represent classical disjunction as "~(~p·~q)", we have thereby expressed [the thesis] that the law of the excluded middle in ist classical form (p∨q) is only valid for absolutely objective Being. For the second concept of Being it holds only in a weakened form, insofar as we have to take into our conception of the object the process of reflection expressed by "~ ... ".

Indeed, between "~ ... " in the table:

p	~p	(I)
W	F	
F	W	

and the same sign "~ ..." in the formula "~(~p·~q)", there exists a difference that seems to me to be analogous to the difference between free and bound variables in the predicate calculus. In table (I) "~ ..." is so to speak not bound, while in the formula "~(~p·~q)" it is "bound" ontologically by "—•—". That is, its meaning as negation as such has been made more precise as negation of "—•—". The same holds for the replacement of the existential quantifier by ~(x)~F(x).

The parallelism between classical and intuitionistic concepts thus seems to indicate that we have two categories of logical entities: radically objectively thematized, in the sense of Being a); and double-thematically (objective/subjective) oriented in the sense of Being b).

To this corresponds the division of the predicate calculus, more precisely, the division into the theory of quantification and class theory. According to your own discoveries, the axiomatic system of the narrower functional calculus is complete (Monatshefte für Math. v. 37 (1930)),[e] yet undecidability exists. For the rest of the calculus, in accordance with a second discovery of yours, neither completeness nor decidability ob-

Es scheint mir nun (wenn ich Ihre Ergebnisse und die einiger Ihrer Kollegen, z.B. Church, richtig interpretiere [)], dass der Drittensatz in der Quantifikationstheorie eine andere Rolle spielt als in der Klassentheorie. Seine unbedingte, rigorose |7
7 Geltung im platonischen-aristotelischen Sinn ist in beiden Fällen verneint. Es scheint mir aber der folgende Unterschied zu bestehen: Für die Quantifikationstheorie ist das Tertium non datur vorläufig suspendiert. Für die Klassentheorie [[1]] ist es *schlechthin* aufgehoben.

Unter <vorläufig suspendiert> verstehe ich: die Quantifikationstheorie ist ein System, in dem der Drittensatz im klassischen Sinn gelten *würde,* wenn man in der Lage wäre den (inhaltlich) unendlichen Umfang dieses Systems erschöpfend darzustellen. Das ist empirisch nicht möglich. Folglich ist seine Geltung im System beschränkt. In anderen Worten: seine Geltung ist nur eingeschränkt für Teilsysteme der Quantifikationstheorie. Er gilt aber klassisch unbedingt für das (nicht-realisierbare) Gesamtsystem.

In der Klassentheorie aber scheinen mir die Dinge anders zu liegen. Hier bedeutet seine Einschränkung nicht, dass seine klassisch-rigorose Geltung vorläufig suspendiert ist, sondern dass sie *endgültig* aufgehoben ist. D.h. selbst wenn man die Klassentheorie vollständig, als geschlossenes System, darstellen könnte, würde man entdecken, dass das Tertium non datur auch dann nicht gilt.

Für die Quantifikationstheorie ist das Tertium non datur wenigstens noch ein unendliches (wenn auch nicht erreichbares) formales Ziel des Denkens. Für die Klassentheorie existiert ein solches Ziel
8 nicht. In philosophischer Terminologie: für die Quantifikationstheorie hat |8 der Drittensatz noch thematische Bedeutung. In der Klassentheorie existiert er nur noch athematisch. D.h. wenn man die Klassentheorie vollendet und mit generellen Entscheidungsverfahren darstellen könnte, würde man feststellen, dass das Tertium non datur auch für das Gesamtsystem nicht gilt.

Die hier skizzierte Unterscheidung wird heute in der mathematischen Logik noch nicht gemacht (wenigstens habe ich sie nirgends gefunden), weil man sich, wie ich

tain.
Now, it seems to me (provided I interpret correctly your results and those of several of your colleagues, e.g. Church) that the law of the excluded middle plays a different role in quantification theory than in class theory. Its unconditioned and rigorous validity in the Platonic-Aristotelian sense is denied in both cases. It seems to me, however, that the following difference obtains: With respect to quantification theory the tertium non datur is *temporarily* suspended. With respect to class theory[[1]] it is *absolutely* abolished.
By "temporarily suspended" I understand: quantification theory is a system in which the law of the excluded middle would hold in the classical sense, if one were in a position to represent exhaustively the contentually infinite scope of this system. That is empirically impossible. Consequently its validity *within* the system is restricted. In other words: its validity is only limited with respect to subsystems of quantification theory. But, classically, it holds unconditionally for the (unrealizable) entire system.
In class theory, however, it seems to me that things are otherwise. Here the limitation does not mean that its classical-rigorous validity is temporarily suspended, but that it *is definitively* abolished. That is, even if one could represent class theory completely as a closed system, one would find that even then the tertium non datur does not hold within it.

For quantification theory the tertium non datur is still at least an infinite (even if unattainable) formal goal of thought. For class theory such a goal does not exist. In philosophical terminology: for quantification theory the law of the excluded middle still has thematic meaning. In class theory it only exists non-thematically. That is, if one could represent class theory [as] completed and with general decision procedures, one would establish that the tertium non datur does not hold, even for the entire system.

The distinction sketched here is not yet made today in mathematical logic (at least I have not found it anywhere),

glaube nicht über den Unterschied von zweiwertiger und dreiwertiger Logik klar ist. Wird der Drittensatz vorläufig suspendiert, *aber nicht thematisch* aufgehoben, so bleibt man auf dem Boden der zweiwertigen Logik. D.h. die logische Analyse bewegt sich auf der Basis der Hamletschen Alternativ <Sein oder Nichtsein>. Die Korrespondierenden Werte sind: "wahr" und "falsch".

Wird der Drittensatz aber nicht nur empirisch suspendiert, sondern *endgültig* für den gesamten logischen Formalismus aufgehoben, so findet ein Übergang zum (mindestens) dreiwertigen System statt. D.h. das Tertium non datur ist auch für die "Wahrheits"funktionen der Aussagenlogik endgültig beseitigt. Die *endgültige* Aufhebung des Tertium non datur impliziert einen radikalen logischen Themawechsel. Im dreiwertigen System ist das Thema <Sein> aufgegeben. Das bedeutet aber, dass die seinsthematischen Werte "wahr" und "falsch" nicht mehr
9 zuständig |9 sind. – Ich habe gerade Rosser und Turquette's Buch "Many valued logics"[f] gelesen und ich finde es bezeichnend, wie diejenigen Logiker, die sich mit dem Problem mehrwertiger Logiken befassen, immer noch verzweifelt an der klassischen Wertdichotomie festhalten. Sie wird auf die mehrwertigen Logiken dadurch übertragen, dass man dichotomisch zwischen "designierten" und "nicht-designierten" Werten unterscheidet. Das geschieht, weil man sich nicht von dem Gedanken losreissen kann, dass das einzige und alleinige Thema des Denkens objektives "Sein" ist.

Es ist aber falsch anzunehmen, dass alle Eigenschaften von Klassen sich als Eigenschaften eines objektiven Seins darstellen lassen.[g] Erlauben Sie mir an den Heisenbergschen Satz aus der Quantenphysik zu erinnern:
der "absolut isolierte Gegenstand hat prinzipiell keine beschreibbaren Eigenschaften mehr."[h] Der physikalische Sachverhalt ist ein Zusammenhang von Subjekt *und* Objekt.

Eine Logik aber, die einer solchen Physik parallel läuft, hat eine neue Aufgabe. Sie hat sich zu fragen, was ist die objektive (irreflexive) und was ist die subjektive (reflexive) Komponente in einem theoretischen Begriff? Wenn ich eine

because, I believe, one is not clear about the difference between two-valued and three-valued logic. If the law of the excluded middle is temporarily suspended, *but not thematically abolished,* then one remains on the ground of two-valued logic. That is, logical analysis operates on the basis of Hamlet's alternative "To be or not to be". The corresponding values are: "true" and "false".

Yet, if the law of the excluded middle is not only empirically suspended, but *definitively* abolished for the entire logical formalism, a transition to an (at least) three-valued system takes place. I.e., the tertium non datur is also definitively eliminated for the "truth" functions of propositional logic. The *definitive* abolition of the tertium non datur implies a radical logical change of topic. In the three-valued system the topic "Being" is given up. But this means that the values "true" and "false", which thematize Being, are no longer relevant. – I just read Rosser and Turquette's book "Many valued Logics"[f] and I find it telling how these logicians, who work on the problem of many-valued logics, still hold on desperately to the classical dichotomy of values. It is transferred to many-valued logic by dichotomically distinguishing between "designated" and "nondesignated" values. This happens because one cannot break free from the idea that the one and only topic of thought is objective "Being".

But it is wrong to assume that all properties of classes can be represented as properties of an objective Being.9 Permit me to remind you of Heisenberg's dictum from quantum physics: the "absolutely isolated object does not, in principle, any longer have any describable properties". [h] The physical state of affairs is a connection of subject *and* object.

However, a logic that runs parallel to such a physics has a new task. It has to ask itself, what is the objective (irreflexive) and what is the subjective (reflexive) component in a theoretical concept? If I conceive a logical class as a

logische Klasse als ein 100% objektives "Ding" auffasse, hat sie auch keine beschreibbaren Eigenschaften mehr, weil ihr dann die kontradiktorischen Eigenschaften "p" und "~p" gleichzeitig zukommen.[i] Man
10 ist dieser Verlegenheit |10 vermittels der Typentheorie oder der Quineschen Regel R3' (in New Foundations[j]) oder mit anderen Mitteln, die aber alle auf dasselbe hinauskommen, zuleibe gegangen; aber man hat sich nie gefragt ob nicht die Klassentheorie – abgesehen von der seinsthematischen Wahrheit und Falschheit eines Begriffs – noch eine andere logische Fragestellung suggeriert, nämlich die welche Komponenten eines Begriffs irreflexiv gelten und welche reflexive Bedeutung haben.

Ich verstehe dabei als "irreflexiv", das was unabhängig vom denkenden Subjekt gilt (der klassische Begriff der Objektivität!), und als "reflexiv" dasjenige logische Motiv, in das der Denkprozess (das Subjekt nach Heisenberg) mit hinein definiert werden muss.

Nun ist aber der Gegensatz von "irreflexiv" und "reflexiv" selbst der Gegenstand einer (iterierten) Reflexion. Wir erhalten somit einen dritten Wert, der den Denkprozess designiert, der "irreflexiv" und "reflexiv" unterscheidet. Ich nenne ihn – nach transzendentalem Vorbild – doppeltreflexiv. In meinem Essay für den 11. Phil. Kongress in Brüssel habe ich ausgeführt, dass der Unterschied von "reflexiv" und "doppelt-reflexiv" [k] auf die Differenz von "Ich" und "Du" im Denken hinauskommt. D.h. Reflexion ist grundsätzlich zweiwertig. Sie kann als *objektiver* Vorgang in der Aussenwelt interpretiert werden: dann vollzieht sie sich im *anderen* Ich, d.h. im Du. Sie kann aber auch
11 als subjektiver Erlebnis-|11prozess, in dem logischen Bedeutungen innerlich erfahren werden, aufgefasst werden. Dann vollzieht sie sich im Ich.

Klassisch ist das ganz gleichgültig, weil alle denkenden Subjekte metaphysisch (angeblich) zusammenfallen. Die Differenz zwischen Reflexion im Subjekt (Ich) und Reflexion im Objekt (Du) wird aber sofort relevant, sobald man anerkannt, dass die Kommunikationstheorie (C. Shannon) ein Teil der Logik ist. Die mitteilbare (objektive) Bedeutung eines Begriffs unterscheidet sich

100% objective "thing", it too no longer has any describable properties, because then the contradictory properties "p" and "~p" are attached to it at the same time.[i] One has tried to counter this embarrassment by means of type theory, or Quine's rule R3' (in New Foundations[j]), or by other means, all of which amount to the same thing; but one never asked whether class theory – besides the Being-thematizing truth and falsity of a concept – does not suggest yet another logical question, namely, which components of a concept hold *irreflexively* and which have *reflexive* meaning.

I understand as "irreflexive" that which holds independently of the thinking subject (the classical concept of objectivity!), and as "reflexive" the logical motif, in the definition of which the thought process (the subject according to Heisenberg) has to be included.

But the opposition between "irreflexive" and "reflexive" is itself the object of an (iterated) reflection. We thus obtain a third value, which designates the thought process that distinguishes between "reflexive" and "reflexive". I call it – following the transcendental example – double-reflexive. In my essay for the 11th philosophical congress in Brussels I explained that the distinction between "reflexive" and "double-reflexive" amounts to the difference in thought between "I" and "Thou". [k] I.e., reflection is fundamentally two-valued. It can be interpreted as an *objective* event in the external world: in this case it takes place in the *other* I, i.e., in the Thou. But it can also be understood as a process of subjective experience in which logical meanings are experienced internally. In that case it takes place in the I.

Classically, this is a matter of indifference, because all thinking subjects (allegedly) coincide metaphysically. However, the difference between reflection in the subject ("I") and reflection in the object ("Thou") becomes relevant immediately as soon as one acknowledges that communication the-

von der denkbaren (subjektiven) Bedeutung. Und beide unterscheiden sich von der Differenz zwischen Mitteilbarkeit und Denkbarkeit.

Diese drei Bedeutungen werden durch meine Werte "irreflexiv" (1), "reflexiv" (2) und "doppelt-reflexiv" (3) auseinandergehalten. Die Klassentheorie verwickelt uns in Paradoxe weil wir die *objektive* Bedeutung des Klassenbegriffs als Zeichen auf dem Papier und die *subjektive* als Denkprozess im Bewusstsein ignorieren und über dies vergessen, dass die Differenz zwischen der "papiernen" Bedeutung und der in der Reflexion ebenfalls in die Logik eingeschlossen werden muss.[l]

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[In the omitted portion, Günther presents and discusses some three-valued truth tables.]

15 |15

D.h. das Thema dieser Logik ist nicht mehr "ontologisch" Sein, sondern die Differenz zwischen Sein (Objektivität) und Reflexion, die selber als logisches Reflexionsproblem begriffen wird. Das Kriterium, das zwischen einer Seinslogik und einer Reflexionslogik scheidet, ist der Drittensatz. Was nämlich im Tertium non datur ausgeschlossen wird, ist nämlich die logische Möglichkeit auf das Verhältnis von objektiv Gedachtem und Denkprozess (math: Konstruktion) noch einmal *ausserhalb* der (zweiwertigen) Logik, die beides vereint, zu reflektieren.[m]

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[In two omitted paragraphs, Günther makes further remarks about the law of excluded middle and the, object language-metalanguage distinction.1

16 |16 ... Ich bin davon ausgegangen, dass sowohl Kant wie der moderne Positivismus demonstrieren, dass der Begriff eines transzendenten, absoluten Seins wiederspruchsvoll ist. Damit fällt die ontologische Metaphysik fort.

Solange es ein transzendentes, absolutes Sein gibt, kann es auch nur *eine* Logik geben. Die Auflösung der absoluten Substanz aber hat zur Folge dass die klassische Logik sich nicht auf das reale Sein richtet (wie Russell annimmt). Wir wissen ja nicht ob es
17 das |17 überhaupt gibt; sondern sie richtet sich auf – und definiert den *Sinn* von Sein,

ory (C. Shannon) is a part of logic. The communicable (objective) meaning of a concept is distinguished from the thinkable (subjective) meaning. And both are distinguished from the difference between communicability and thinkability.

These three meanings are kept apart by my values "irreflexive" (1), "reflexive" (2) and "double-reflexive" (3). Class theory entangles us in paradoxes because we ignore the *objective* meaning of the concept of class as a sign on paper, and the *subjective* one as thought process in consciousness and in so doing forget that the difference between the "paper" meaning and that in reflection needs to be included in logic as well.[l]

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[In the omitted portion, Günther presents and discusses some three-valued truth-tables.]

That is, the topic of this logic is no longer Being conceived "ontologically", but the difference between Being (objectivity) and reflection, where this difference is itself conceived as a logical problem for reflection. The criterion that distinguishes a logic of Being from a logic of Reflection is the law of the excluded middle. For what is excluded in the tertium non datur is the logical possibility of reflecting once again, *outside* (two-valued) logic which combines the two, upon the relation between what is objectively thought and the process of thought (mathematically: construction). [m]

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My point of departure was that both Kant and modern positivism demonstrate that the concept of a transcendental, absolute Being, is contradictory. With that ontological metaphysics comes to an end.

As long as there is a transcendental, absolute Being, there can only be *one* logic. The dissolution of absolute substance, however, has the consequence that classical logic does not direct itself at real Being (as Russell assumes). Af-

d.h. sie definiert die logische Bedeutung in der identitätstheoretische Motive im Denken verwandt werden.

Das Thema <Sein selbst> kann durch nichts anderes ersetzt werden. Daher nur eine Logik. Dem reduzierten Thema <Sinn des Seins> aber kann als zweites Thema <Sinn der Reflexion> gegenübergestellt werden. So gelangen wir zum Konzept einer zweiten Logik.

Die klassische Logik setzt die metaphysische Identität von Denken und Sein voraus. Folglich kann das Denken kein anderes Thema als objektives Sein haben. Geben wir diese Metaphysik auf (und es ist höchste Zeit dafür), so kann das Denken als zweites ebenbürtiges Thema neben dem Sein (Objekt) das Subjekt (d.h. den Reflexionsprozess) selbst haben.

Haben Sie herzlich Dank, dass Sie mir solange zugehört haben!
Mit herzlichen Grüssen
Ihr
Gotthard Günther

P. S. The aim "to set up a consistent theory of classes and concepts as objektively existing entities" setzt eine Logik voraus in der formal zwischen "objektiv" und "subjektiv" unterschieden werden kann. Das ist aber nur in einer dreiwertigen Logik möglich.

ter all, we do not know if that exists at all; rather it directs itself at-and defines the *meaning* of Being, i.e., it defines the logical meaning in which identity-theoretic motifs are employed in thought.

The topic "Being itself " cannot be replaced by anything else. Hence only *one* logic. But the reduced topic "meaning of Being" can be confronted with "meaning of reflection" as second topic. In this way we reach the conception of a second logic.

Classical logic presupposes the metaphysical identity of thought and Being. Consequently thought can have no topic other than objective Being. If we give up this metaphysics (and it is high time we do so), then thought can have, as a second, equally important topic besides Being (object), the subject (i.e., the process of reflection) itself.

Thank you very much for listening to me for so long!
With warm greetings,
Yours
Gotthard Günther

P.S. The aim "to set up a consistent theory of classes and concepts as objectively existing entities" presupposes a logic which can formally distinguish between "objective" and "subjective". But this is only possible within a three-valued logic.

4. Gödel to Günther

Princeton, 30./VI. 1954

Sehr geehrter Herr Dr. Günther!

Daß irgendwelche Ihrer phil. Behauptungen meinen Resultaten widersprechen kann ich, beim gegenwärtigen Stand der Dinge, wohl verneinen, obwohl meine Resultate gewisse Formen einer subjektivistischen Interpret. der Math. unmöglich machen u. überhaupt stark gegen jede solche Interpretation sprechen. Was Ihre Deutung der Resultate bez. Unentscheidbarkeit betrifft, so möchte ich folgendes sagen: Es ist richtig, daß gewisse Theoreme von Church, Turing u.a. dahin interpretiert werden können (u. von Brouwer auch dahin interpretiert werden), daß der Drittensatz für gewisse Satzklassen A im Sinne der Formel $\sim(p)\ [p\in A.\supset p.\vee\sim p]$ zu negieren ist. D.h. man kann zeigen: Die Annahme, man hätte bewiesen, daß der Drittensatz für alle p aus A gilt, führt auf einen Widerspruch. Es stimmt auch, daß nichtsdestoweniger der Drittensatz als unerreichbares Ziel des Denkens be-
2 stehen bleiben könnte, |2 da trotzdem für jedes einzelne p aus A $p\vee\sim p$ beweisbar sein kann (Fall I). Die letztere Behauptung hat aber intuit. keinen Sinn, sondern setzt den Begriff der objektiven Existenz voraus. Die entsprechende int. formulierbare Behauptung aber, nämlich: $\sim(\exists p)[p\in A.\bullet.\sim(p\vee\sim p)]$ gilt für jedes A. D.h. es kann innerhalb der int. Math. von keinem Satz bewiesen werden, daß er unentscheidbar ist. Nichtsdestoweniger könnte das von gewissen Sätzen der int. Math. objektiv richtig u. in der Existential-Math. sogar beweisbar sein. (Fall II). Aber die Unterscheidung der Fälle I u. II. hat, wie gesagt, intuit. keinen Sinn,[1]

4. Gödel to Günther

Princeton, 30 June 1954

Dear Dr. Günther,

That any of your philosophical claims contradicts my results I can answer in the negative, given the way things stand at present, although my results make certain forms of a subjectivist interpretation of mathematics impossible, and generally count strongly against any such interpretation. With regard to your interpretation of the results concerning undecidability, I would like to say the following: it is correct that certain theorems of Church, Turing, et al., can be interpreted in such a way (and are indeed interpreted in this way by Brouwer) that the law of the excluded middle, in the sense of the formula $\sim(p)\ [p\in A.\supset p.\vee\sim p]$, has to be denied for certain classes of propositions A. That is, one can demonstrate: the assumption that one has proved the validity of the law of the excluded middle for all p in A leads to a contradiction. It is furthermore correct that the law of the excluded middle could nonetheless remain in place as an unattainable goal of thought, since $p\vee\sim p$ can be provable all the same for every individual p in A (case I). But this latter claim has no intuitionistic sense but rather presupposes the concept of objective existence. The corresponding claim that can be formulated intuitionistically, however, namely $\sim(\exists p)\ [p\in A.\bullet.\sim(p\vee\sim p)]$ is valid for every A. That is, for no proposition within intuitionistic mathematics can one prove that it is undecidable. Nonetheless, this could be objectively true of certain propositions in intuitionistic mathematics and even be provable in existential mathematics (case

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1 Ich habe bloß bewiesen: "Jede Formel ist entweder widerlegbar oder es gibt eine Realisierung", nicht aber: "oder die Existenz einer Realisierung ist beweisbar". Das letztere ist für jedes formale System falsch.
I have only proved, "Every formula is either refutable or there is a realization [of it], not, however, "or the existence of a realization is provable". The latter is false for every formal system.

2 Damit will ich natürlich nicht behaupten, daß schon das naive Denken das objektive Sein in allen Punkten richtig erfaßt, wie die ontol. Metaphysik vielfach anzunehmen scheint.
Of course I don't wish by that to claim that naive thought already grasps objective being correctly on all points, as ontological metaphysics often seems to suppose.

ausser daß eventuell einer der beiden Fälle auf Grund gewisser Tatsachen ⟨empirisch⟩ wahrscheinlich werden könnte. Ich glaube auch nicht, daß der "Themawechsel" des Denkens etwas mit den Fällen I, II zu tun hat; denn dieser tritt ja schon in dem Augenblick ein, wo der Begriff der aufweisbaren Existenz eingeführt wird, u. würde auch dann bestehen bleiben, wenn man für diese den Drittensatz beweisen könnte. Ob I die Grenze zwischen I u. II mit der zwischen engerem und höherem Funktionenkalkül zusammenfällt, darüber ist nichts bekannt. Mein Resultat über die Vollständigkeit des eng. Funkt. Kalk. hat, int. interpretiert, nichts mit dem Drittensatz zu tun, sondern betrifft den Satz der doppelten Negation (~~p⊃p). Es ist auch nicht bekannt, ob für die ganze int. Math. der Fall II eintritt. Denn diese liegt ja nicht in formalisierter Gestalt vor u. von Brouwer wird sogar die Möglichkeit einer vollständigen Formalisierung bestritten.

Sie haben durchaus recht, wenn Sie sagen, daß in der int. Math. eine Selbstreflexion des Subjekts vorliegt u. daß der Begriff der Absurdität eine geb. Variable enthält. Er bedeutet ja: "Es gibt (im Sinne der Aufweisbarkeit) eine Widerlegung" . Die in der idealist. Phil. behandelte Reflexion auf das Subjekt (d.h. Ihr II Thema d. Denkens), die Unterscheidung von Reflexionsstufen etc. scheint mir sehr interessant u. wichtig. Ich halte es sogar
3 für durchaus |3 möglich, daß dies "der" Weg zur richtigen Metaphysik ist. Die damit verbundene (in Wahrheit aber davon ganz unabhängige) Ablehnung der objektiven Bedeutung des Denkens kann ich aber nicht mitmachen. Ich glaube nicht, daß irgend ein Kantsches oder positivistisches Argument oder die Antinomien d. Mengenl., oder die Quantenmechanik bewiesen hat, daß der Begriff des objektiven Seins (gleichgültig ob für Dinge oder abstrakte Wesenheiten) sinnlos oder widerspruchsvoll ist. [2] Wenn ich sage, daß man eine Theorie der Klassen als objektiv existierender Gegenstände entwickeln kann (oder soll), so meine ich damit durchaus Existenz im Sinne der ontol. Metaphysik, womit ich aber nicht sagen will, daß die abstrakten Wesenheiten in der Natur vorhanden

II). But the distinction between cases I and II, as I said, has no intuitionistic sense,[1] except that one of the two cases could possibly become empirically probable on the basis of certain facts. Moreover, I do not believe that the "shift of topic" of thought has anything to do with cases I and II; for it already occurs at the moment when the concept of exhibitable existence is introduced, and would remain in place, even if one could prove the law of the excluded middle for this concept. Nothing is known about whether the boundary between I and II coincides with that between the lower and higher functional calculus. My result about the completeness of the lower functional calculus has, in its intuitionistic interpretation, nothing to do with the law of the excluded middle, but rather concerns the law of double negation (~~p⊃p). Moreover it is not known whether case II occurs in all of intuitionistic mathematics. For, after all, it does not exist in a formalized version, and Brouwer even contests the possibility of a complete formalization.

You are absolutely right when you say that in intuitionistic mathematics a self-reflection of the subject is present and that the concept of absurdity contains a bound variable. It means: "There is (in the sense of exhibitable existence) a refutation". The reflection on the subject treated in idealistic philosophy (that is, your second topic of thought) , the distinction of levels of reflection, etc., seem to me very interesting and important. I even consider it entirely possible that this is "the" way to the correct metaphysics. However, I cannot go along with the denial of the objective meaning of thought that is connected with it, [although] it is really entirely independent of it. I do not believe that any Kantian or positivistic argument, or the antinomies of set theory, or quantum mechanics have proved that the concept of objective being (no matter whether for things or abstract entities) is senseless or contradictory.[2] When I say that one can (or should) develop a theory of classes as objectively existing entities, I do indeed mean by that existence in the sense of ontological metaphysics, by which, however, I do not want to say that

sind. Sie scheinen vielmehr eine zweite Ebene der Realität zu bilden, die uns aber ebenso objektiv. u. von unserem Denken unabhängig gegenübersteht wie die Natur. Ich kann in diesem Brief leider nicht mehr auf Ihre 3-wertige Logik eingehen. Was ich bisher vermisse, ist eine nähere Erklärung des inhaltlichen Zusammenhangs der Wahrheitstabellen mit Ihren philosophischen Ideen.

Zum Schluß möchte ich noch sagen, daß ich Ihnen jederzeit gerne mit weiteren Auskünften zur Verfügung stehe.

Mit besten Grüssen
Ihr Kurt Gödel

abstract entities are present in nature. They seem rather to form a second plane of reality, which confronts us just as objectively and independently of our thinking as nature. Unfortunately I can't go into your 3-valued logic more in this letter. What I miss so far is a more detailed explanation of the connection in content of your truth tables with your philosophical ideas.

In conclusion, I would like to say that I will be glad to be available with further information at any time.

With cordial greetings,
Yours, Kurt Gödel

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[a] Günther probably refers to the postscript of 1943 to *Heidegger 1929*. There Heidegger raises the question whether thought stands "in the law of truth" when it "follows that thought that 'logic' grasps in its forms and rules". He mentions "die 'Logik', als deren folgerichtige Ausartung die Logistik gelten darf" (" 'logic', as whose consistent degeneration symbolic logic may count", 1967, p. 104). Heidegger enters into a discussion of calculation, and it is "calculating thought" that is said to be given a "Schein der Produktivität" (illusion of productivity) by the nature of calculation (ibid.). Friedman (2000, note 207) considers that the larger passage is a reply to the famous criticism of Heidegger 1929 in Carnap 1932.

[b] The line 'Empirisches Wissen.. apriorisches" is marked by lines in the margin. It is probable that all such markings are by Gödel.

[c] Heisenberg 1931, p. 182.

[d] Gödel 1944.

[e] Gödel 1930.

[1] Oder wenigstens Teile derselben / Or at least parts of it

[f] Rosser and Turauette 1952.

[g] This sentence is marked in the margin.

[h] Heisenberg 1931, p. 182.

[i] This sentence is marked in the margin.

[j] R3' is the well-known schema of stratified comprehension of Quine 1937; cf. Quine 1980, p. 92.

[k] *Günther* 1953, especially pp. 47-49. See § 1 of the introductory note.

4. Gödel to Günther

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I have only proved, "Every formula is either refutable or there is a realization [of it], not, however, "or the existence of a realization is provable". The latter is false for every formal system.

2 Damit will ich natürlich nicht behaupten, daß schon das naive Denken das objektive Sein in allen Punkten richtig erfaßt, wie die ontol. Metaphysik vielfach anzunehmen scheint.

ausser daß eventuell einer der beiden Fälle auf Grund gewisser Tatsachen ⟨empirisch⟩ wahrscheinlich werden könnte. Ich glaube auch nicht, daß der "Themawechsel" des Denkens etwas mit den Fällen I, II zu tun hat; denn dieser tritt ja schon in dem Augenblick ein, wo der Begriff der aufweisbaren Existenz eingeführt wird, u. würde auch dann bestehen bleiben, wenn man für diese den Drittensatz beweisen könnte. Ob I die Grenze zwischen I u. II mit der zwischen engerem und höherem Funktionenkalkül zusammenfällt, darüber ist nichts bekannt. Mein Resultat über die Vollständigkeit des eng. Funkt. Kalk. hat, int. interpretiert, nichts mit dem Drittensatz zu tun, sondern betrifft den Satz der doppelten Negation (~~p⊃p). Es ist auch nicht bekannt, ob für die ganze int. Math. der Fall II eintritt. Denn diese liegt ja nicht in formalisierter Gestalt vor u. von Brouwer wird sogar die Möglichkeit einer vollständigen Formalisierung bestritten.

Sie haben durchaus recht, wenn Sie sagen, daß in der int. Math. eine Selbstreflexion des Subjekts vorliegt u. daß der Begriff der Absurdität eine geb. Variable enthält. Er bedeutet ja: "Es gibt (im Sinne der Aufweisbarkeit) eine Widerlegung" . Die in der idealist. Phil. behandelte Reflexion auf das Subjekt (d.h. Ihr II Thema d. Denkens), die Unterscheidung von Reflexionsstufen etc. scheint mir sehr interessant u. wichtig. Ich halte es sogar für durchaus |3 möglich, daß dies "der" 3
Weg zur richtigen Metaphysik ist. Die damit verbundene (in Wahrheit aber davon ganz unabhängige) Ablehnung der objektiven Bedeutung des Denkens kann ich aber nicht mitmachen. Ich glaube nicht, daß irgend ein Kantsches oder positivistisches Argument oder die Antinomien d. Mengenl., oder die Quantenmechanik bewiesen hat, daß der Begriff des objektiven Seins (gleichgültig ob für Dinge oder abstrakte Wesenheiten) sinnlos oder widerspruchsvoll ist. [2] Wenn ich sage, daß man eine Theorie der Klassen als objektiv existierender Gegenstände entwickeln kann (oder soll), so meine ich damit durchaus Existenz im

II). But the distinction between cases I and II, as I said, has no intuitionistic sense,[1] except that one of the two cases could possibly become empirically probable on the basis of certain facts. Moreover, I do not believe that the "shift of topic" of thought has anything to do with cases I and II; for it already occurs at the moment when the concept of exhibitable existence is introduced, and would remain in place, even if one could prove the law of the excluded middle for this concept. Nothing is known about whether the boundary between I and II coincides with that between the lower and higher functional calculus. My result about the completeness of the lower functional calculus has, in its intuitionistic interpretation, nothing to do with the law of the excluded middle, but rather concerns the law of double negation (~~p⊃p). Moreover it is not known whether case II occurs in all of intuitionistic mathematics. For, after all, it does not exist in a formalized version, and Brouwer even contests the possibility of a complete formalization.

You are absolutely right when you say that in intuitionistic mathematics a self-reflection of the subject is present and that the concept of absurdity contains a bound variable. It means: "There is (in the sense of exhibitable existence) a refutation". The reflection on the subject treated in idealistic philosophy (that is, your second topic of thought) , the distinction of levels of reflection, etc., seem to me very interesting and important. I even consider it entirely possible that this is "the" way to the correct metaphysics. However, I cannot go along with the denial of the objective meaning of thought that is connected with it, [although] it is really entirely independent of it. I do not believe that any Kantian or positivistic argument, or the antinomies of set theory, or quantum mechanics have proved that the concept of objective being (no matter whether for things or abstract entities) is senseless or contradictory.[2] When I say that one can (or should) develop a theory of classes as objectively existing entities, I do indeed mean by that existence in the

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Of course I don't wish by that to claim that naive thought already grasps objective being correctly on all points, as ontological metaphysics often seems to suppose.

Sinne der ontol. Metaphysik, womit ich aber nicht sagen will, daß die abstrakten Wesenheiten in der Natur vorhanden sind. Sie scheinen vielmehr eine zweite Ebene der Realität zu bilden, die uns aber ebenso objektiv. u. von unserem Denken unabhängig gegenübersteht wie die Natur. Ich kann in diesem Brief leider nicht mehr auf Ihre 3-wertige Logik eingehen. Was ich bisher vermisse, ist eine nähere Erklärung des inhaltlichen Zusammenhangs der Wahrheitstabellen mit Ihren philosophischen Ideen.

Zum Schluß möchte ich noch sagen, daß ich Ihnen jederzeit gerne mit weiteren Auskünften zur Verfügung stehe.

Mit besten Grüssen
Ihr Kurt Gödel

sense of ontological metaphysics, by which, however, I do not want to say that

abstract entities are present in nature. They seem rather to form a second plane of reality, which confronts us just as objectively and independently of our thinking as nature. Unfortunately I can't go into your 3-valued logic more in this letter. What I miss so far is a more detailed explanation of the connection in content of your truth tables with your philosophical ideas.

In conclusion, I would like to say that I will be glad to be available with further information at any time.

With cordial greetings,
Yours, Kurt Gödel

Briefwechsel-Gödel-GG_05

5. Günther to Gödel

Gotthard Günther
3407 Montrose Ave.
Richmond 22, Va.

October 2, 1954

Sehr geehrter Herr Professor Gödel!

Zuerst`möchte ich mich entschuldigen dass ich Ihren freundlichen Brief vom 30. Juni erst heute beantworte. Das hatte jedoch seine Gründe. Ich wollte bevor ich erwiderte erst eine Publikation von Ihnen sorgfältig studieren. Und zwar Ihren Aufsatz: Russell's Mathematical Logic (Libr. of Liv. Philos. Evanston, Chicago 1944, S. 123-153.)[a] Und weiterhin wollte ich den Druck eines Aufsatzes von mir: "Achilles and The Tortoise"[b] abwarten, der, wie ich glaube, den Gedanken, den ich in diesem Brief hier zum Ausdruck bringen möchte, in seiner Weise erläutert. Die drei Hefte von "Astounding", die die drei Teile meines "Achilles"-Aufsatzes enthalten, gehen heute mit gesonderter Post als Drucksache an Sie ab.

Nun zum Thema. Sie schreiben in Ihrem letzten Brief: "Wenn ich sage, dass man eine Theorie der Klassen als objektiv existierender Gegenstände entwickeln kann (oder soll), so meine ich damit durchaus Existenz im Sinne der ontol. Metaphysik; womit ich aber nicht sagen will, dass die
2 abstrakten Wesenheiten in der |2 Natur vorhanden sind. Sie scheinen vielmehr eine zweite Ebene der Realität zu bilden, die uns aber ebenso objektiv und vor unserem Denken unabhängig gegenübersteht, wie die Natur."

Diese Feststellung in Ihrem Briefe geht Hand in Hand mit der folgenden Bemerkung in Ihrem Russell-Aufsatz: "Classes and concepts may, however, ... be conceived as real objects; namely classes as <pluralities of things> or as structures consisting of a plurality of things and concepts as the properties and relations of things existing independently of our definitions and constructions. It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical

5. Günther to Gödel

Gotthard Günther
3407 Montrose Ave.
Richmond 22, Va.

October 2, 1954

Dear Professor Gödel,

First I would like to apologize for answering your kind letter of June 30 only today. But there were reasons for that. Before answering I first wanted to study carefully a publication of yours. Namely your article:

Russell's Mathematical Logic (Library of Living Philosophers, Evanston, Chicago 1944, p. 123-153.)[a] In addition I wanted to wait for the publication of an article of mine: "Achilles and the Tortoise",[b] which, I believe, elucidates in its own way the thought I would like to express here in this letter. The three issues of "Astounding" which contain the three parts of my "Achilles" article, will be sent to you as printed matter today.

Now to the issues. You write in your last letter: "When I say that one can (or should) develop a theory of classes as objectively existing entities, I do indeed mean by that existence in the sense of ontological metaphysics, by which, however, I do not want to say that abstract entities are present in nature. They seem rather to form a second plane of reality, which confronts us just as objectively and independently of our thinking as nature."

This statement in your letter goes hand in hand with the following remark in your Russell article: "Classes and concepts may, however, ... be conceived as real objects; namely classes as 'pluralities of things' or as structures consisting of a plurality of things and concepts as the properties and relations of things existing independently of our definitions and constructions. It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical bodies and there

bodies and there is quite as much reason to believe in their existence." (S. 134)

Um ein Missverständnis von vornherein auszuschliessen: *so weit* Ihre Sätze gehen stimme ich mit Ihnen überein – aber es scheint mir, dass sie eine Problematik offen lassen und es ist gerade diese Problematik, in der sich meine eigenen Bemühungen von allem, was momentan auf der Gebiet der Logik ereignet, unterscheidet.

Die bisherige Theorie des Denkens von Aristoteles bis zur Gegenwart kennt nur *einen* Begriff des *logischen* (denkunabhängigen) *Objekts!* Und alle Objekte werden – qua Objektivität – logisch gleich behandelt!! Nun zeigt die Geschichte
3 der Logik, dass man mit dieser |3 Thesis (die auf dem Prinzip der metaph. Identität des Seins mit sich selbst beruht) von vornherein in logische Schwierigkeiten gekommen ist. In der mittelalterlichen Logik hat man sich seit Scotus Eriugena dadurch zu helfen gesucht, dass man innerhalb des Seins Grade der Realität angenommen hat. Nach Eriugena hat der Mensch "mehr" Sein als ein Stein und ein Engel "mehr" Sein als der Mensch. Die logische Unhaltbarkeit dieser Auffassung ist zum ersten Mal von Kant in seiner transzendentalen Dialektik aufgedeckt worden.

In neuerer Zeit hat man es mit der Unterscheidung von "Ding" und "Name" versucht. Quine's Beispiel:

Boston hat 800 000 Einwohner.
"Boston" hat sechs Buchstaben.

Um nicht missverstanden zu werden, möchte ich ausdrücklich darauf hinweisen, dass ich die sekundäre Legitimität der Unterscheidung zwischen einem Gegenstand und seinem Namen nicht bestreite. Ich behaupte aber sie geht nicht tief genug! Beide sind logische "Objekte" und als solche denkunabhängig. Aber auch die moderne Logik gibt uns keine Antwort darauf, worin sich ihre Denkunabhängigkeit unterscheidet. Und mangels einer solchen Unterscheidung wird stillschweigend angenommen, dass diese Denkunabhängigkeit dieselbe (Identität) ist. Ich habe vor vielen Jahren Quine einmal gefragt: "What is the difference when you say <A stone exists independent of my act of thinking it and a name exists equally
4 independently?[>]" Er wusste mir keine |4

is quite as much reason to believe in their existence." (p. 134)

In order to avoid *one* misunderstanding right from the start: as far as your propositions go I agree with you – but it seems to me that there is one difficulty that you leave open, and it is just this difficulty in which my own efforts differ from everything that is currently happening in the field of logic.

Up to now the theory of thought from Aristotle to the present knows only *one* concept of logical (thought-independent) *object!* And all objects are – qua objectivity-treated logically in the same way!! Now, the history of logic shows that right from the start one has run into logical difficulties with this thesis (which rests on the principle of the metaphysical identity of Being with itself). In medieval logic since Scotus Eriugena one sought to help matters by assuming degrees of reality within Being. According to Eriugena human beings have "more" Being than a stone, and an angel has "more" Being than human beings. The logical untenability of this view was first revealed by Kant in his transcendental dialectic.

In more recent times one tried [to do] it by the distinction between "thing" and "name". Quine's example:

Boston has 800,000 inhabitants.
"Boston" has six letters.

In order not to be misunderstood I would like to note explicitly that I do not dispute the secondary legitimacy of the distinction between an object and its name. I do claim, however, that it does not go deep enough! Both are logical "objects" and as such are independent of thought. But even modern logic does not provide an answer as to how their thought-independence differs. And in the absence of such a distinction it is tacitly assumed that this thought-independence is the same (identity). Once many years ago I asked Quine: "What is the difference when you say 'A stone exists independent of my act of thinking it' and 'A name exists equally independently'?" He could not give me an answer.

Antwort zu geben.

Ich behaupte nun dass es zwei grundsätzlich unterschiedene Formen der Denkunabhängigkeit gibt. Und folglich zwei prinzipiell verschiedene Identitäten des Objektes mit sich selbst. Ich nenne die erste irreflexive Identität und die zweite Reflexionsidentität. Der Begriff eines Steines kann logisch nur als irreflexive Identität interpretiert werden. Eine Klasse hingegen hat Reflexionsidentität.

Der Unterschied ist bisher nicht gemacht worden weil er auf den Boden der zweiwertigen, klassischen Logik überhaupt nicht festgestellt werden kann. Für zweiwertiges Denken fällt irreflexive Identität und Reflexionsidentität zusammen. Das ist *eine* Interpretation von "p=~~p". Übrigens kommt Hegel diesem Gedanken einmal ganz nahe, wenn er entrüstet darauf hinweist, dass die bisherige Philosophie keinen Unterschied zwischen der Identität eines Steins mit sich selbst und der Identität Gottes mit sich selbst mache. Für "Gott" können wir hier ruhig "Reflexion" setzen. Hegels Lösungsversuch ist selbstverständlich auch wertlos, da er zu seiner Zeit nicht wissen konnte, dass es auf dem Boden der zweiwertigen Logik eine Lösung für den Unterschied von Ding als Denkobjekt und Reflexion als Denkobjekt schlechterdings nicht gibt.

Und doch wissen wir, dass es einen solchen Unterschied gibt. Wenn wir von der Denkunabhängigkeit des Objektes <Stein> sprechen, meinen |5 wir dass dieses Objekt deshalb unabhängig ist, weil es – primitiv gesprochen – von "aussen her" in das Denken tritt. Umgekehrt ist die Klasse deshalb ein denkunabhängiges Objekt weil sie "von innen kommt" und sich vom Denkprozeß abgelöst hat. Das sind zwei vollkommen verschiedene Begriffe der Denkunabhängigkeit. Und meiner Ansicht nach kommen die grossen Schwierigkeiten, die wir mit der Klassentheorie haben, daher, dass dieser Unterschied logisch ignoriert wird. Es sind hier zwei vollkommen verschiedene Begriffe von "Existenz" im Spiel. Die zweiwertige ontologische Metaphysik aber kann das, da sie einfache Identitätsmetaphysik ist ("absolute" Identität von Denken und Sein) nicht zugeben.

Now I claim that there are two fundamentally different forms of thought-independence. And accordingly two in principle different identities of the object with itself. I call the first irreflexive identity and the second reflection-identity. The concept of a stone can, logically speaking, only be interpreted as irreflexive identity. A class, by contrast, has reflection-identity.

This distinction has not been made so far because it cannot be established at all on the basis of classical two-valued logic. For two-valued thought irreflexive identity and reflection-identity coincide. That is *one* interpretation of "p=~~p ". Hegel, incidentally, comes very close to this thought at one point, when he indignantly observes that previous philosophy does not make a distinction between the identity of a stone with itself and the identity of God with himself. For "God" here we may well substitute "reflection". Hegel's attempted solution is of course worthless as well, since in his time he could not have known that, on the basis of two-valued logic, there is absolutely no solution to the distinction between a thing as object of thought and reflection as object of thought.

And yet we know that there is such a distinction. If we speak of the thought-independence of the object "stone" we mean that this object is independent because it – crudely put – enters into thought, "from without". Conversely a class is a thought-independent object because it "comes from within" and has separated itself from the process of thought. Those are two entirely different concepts of thought-independence. And in my view the great difficulties we have with the theory of classes come from the fact that, logically, this distinction is ignored. There are two entirely different notions of "existence" in play here. But two-valued ontological metaphysics cannot admit this, since it is a simple metaphysics of identity ("absolute" identity between thought and Being).

Die Unterscheidung der beiden Formen der Existenz kann aber sofort gemacht werden, wenn wir vom zweiwertigen zum dreiwertigen System übergehen. Klassisch haben wir nur die einfache Alternative "p∨~p", d.h. zwischen Denken-überhaupt und Gedachtem überhaupt. Die dreiwertige Logik aber fordert die Einführung einer zweiten Negation. D.h. wir haben jetzt zwischen "p" "~p" und "~'p" zu underscheiden. D.h. zwischen Denken überhaupt und zwei Formen von Gedachtem (oder Existenz).

Es scheint mir nun, dass gegenwärtig praktisch (aber nicht prinzipiell) die Grenze zwischen den beiden Begriffen von Denkunabhängigkeit ungefähr mit dem Unterschied
6 von engerem und weiterem |6 Funktionenkalkül zusammenfällt. Oder mit dem Gegensatz von Existenz und Klasse. Die Unterscheidung ist nicht präzis, weil gegenwärtig alles in das Prokrustesbett einer zweiwertigen Logik gezwängt wird. Es scheint mir, dass der erweiterte Funktionenkalkül, in seiner gegenwärtigen Gestalt eine Mischform von zweiwertiger und dreiwertiger Problematik ist, wobei aber die dreiwertige Problematik als *solche* nicht anerkannt wird und in der Pseudoform einer zweiwertigen Logik erscheint.

Bei der Interpretation des Klassenbegriffes gehen zwei grundverschiedene Existenzkonzeptionen durcheinander, die getrennt behandelt werden müssten – aber nicht werden. Es scheint mir übrigens, dass meine Idee des doppelten (irreflexiven und reflexiven) Denkobjektes durch eines Ihrer Resultate gestüzt wird. Lassen Sie mich die Fussnote aus Ihrem letzten Brief zitieren: "Ich habe bloss bewiesen: <Jede Formel ist entweder widerlegbar oder es gibt eine Realisierung>, nicht aber: <oder die Existenz einer Realisierung ist beweisbar>. Das letztere ist für jedes formale System falsch." Ich würde sagen, es ist deshalb für jedes formale System falsch, weil jene Beweisbarkeit die präzise logische Eindeutigkeit des Existenzbegriffes voraussetzt. In anderen Worten: die Beweisbarkeit setzt voraus, dass es nur *einen* general durchführbaren Begriff des Denkobjekts gibt. D.h., dass irreflexives Faktum und Reflexionsprozess vollständig
7 ineinander auflösbar |7 seien. Gerade das aber ist nicht der Fall.

The distinction of the two forms of existence can, however, be made right away if we pass from the two-valued to the three-valued system. Classically we have only the simple alternative "p∨~p", that is between thinking in general and what is thought in general. Three-valued logic, however, requires the introduction of a second negation. That is, we now have to distinguish between "p", "~p", and "~'p". That is, between thinking in general and two forms of what is thought (or existence) .

Now, it seems to me that currently the boundary between the two notions of thought independence roughly coincides practically (but not in principle) with the distinction between narrower and wider functional calculus. Or with the contrast of existence and class. The distinction is not precise, because currently everything is forced into the Procrustean bed of a two-valued logic. It seems to me that the extended functional calculus, in its present form, is a hybrid of two- and three-valued problem areas, where, however, the three-valued problem area is not recognized as such and appears in the pseudo-form of a two-valued logic.

In the interpretation of the concept of class, two fundamentally different conceptions of existence are mixed up which should be treated separately-but are not. Incidentally, it seems to me that my idea of double (irreflexive and reflexive) thought-object, is supported by one of your results. Allow me to quote the footnote from your last letter: "I have only proved, 'Every formula is either refutable or there is a realization [of it]', not, however, 'or the existence of a realization is provable'. The latter is false for every formal system." I would say it is false for every formal system, because that provability presupposes precise logical uniqueness of the concept of existence. In other words: the provability presupposes that there is only *one* generally realizable concept of thought-object. I.e., that an irreflexive fact and the process of reflection can be completely resolved into each other. Just that, however, is not the case. –

Ich habe mir erlaubt Ihnen den Achilles-Aufsatz zu schicken, weil er das Problem zweier logisch prinzipiell verschiedener thematischer Objekte von der physikalischen Seite her aufrollt. Nach Plato und der ihm bis zur Gegenwart folgenden Tradition gibt es nur *ein* absolut generelles Objekt des Denkens, Sein-überhaupt. Folglich ist ihm im "Timaios" der leere Raum, der von *allen* Objekten entleert ist, das absolute Nichts. Es kommt ihm nicht die Idee, dass die blosse Leere (die in der Tat kein Objekt im Sinne eines seienden Dinges ist) das Beispiel einer *zweiten* Objekt- und Existenzkategorie für das reflektierende Bewusstsein sein könne. Erst bei Leibniz findet sich eine vage Andeutung des Gedankens, dass der Raum die existierende Klasse aller individuellen Objekte ist. Das setzt aber voraus, dass der "leere" Raum in einem *anderen* Sinne "existiert" als, sagen wir, ein Stein oder ein Planet.

In dem Achilles-Aufsatz mache ich mir den Gedanken in folgender Weise zunutze: Wenn es nur eine logisch begreifbare Form der Existenz gibt, dann ist die Abwesenheit alles "Physischen" eben, wie Plato meint, das blosse Nichts. Wenn wir aber zwei Formen von Objektivität annehmen, dann ist der "leere" Raum auch ein echtes "gegenständliches" Denk-objekt. Und was echt objektiv
8 gedacht (erfahren) werden |8 kann, das kann auch «technisch» behandelt werden. Folglich ist der Raum genau so *manifestierbar* wie die Körperwelt. Noch für Kant ist das unmöglich, da in der Kritik der reinen Vernunft der Raum blosse "Anschauungsform" ist. Das ist eine höfliche Umschreibung des platonischen Glaubens dass der Raum *objektiv* betrachtet Nichts ist.

Zweiwertig kann man gar nicht anders denken: Entweder die Dinge sind Etwas (Physik) oder die Abwesenheit der Dinge ist Etwas (Metaphysik). Das ganze ist ein reines *Umtauschverhältnis,* wie "rechts" und "links". Die prinzipielle Vertauschbarkeit der Parameter "Raum", "Zeit", "Materie" und "Prozess", von der ich im Achilles rede, setzt aber eine dreiwertige Logik voraus und *zwei* Konzeptionen von objektiver Existenz, die in diesem Spezialfall als "quantized" und "not quantized" charakterisiert werden.

Der Aufsatz behandelt ein generelles natur-

I took the liberty of sending you the Achilles essay because it addresses the problem of two logically fundamentally different thematic objects from the physical side. According to Plato and the tradition that follows him up to the present day, there only is *one* absolutely general object of thought, Being in general. Consequently, for Plato in the "Timaeus" empty space, emptied of all objects, is absolute nothingness. He does not entertain the idea that the mere void (which indeed is not an object in the sense of a thing that is) might be an example of a *second* category of object and existence for reflecting consciousness. With Leibniz we first find a vague indication of the thought that space is the existing class of all individual objects. But that presupposes that "empty" space "exists" in *another* sense than, lets say, a stone or a planet.

In the Achilles essay I make use of this thought in the following manner: If there is only one logically comprehensible form of existence, then the absence of everything "physical" is indeed, as Plato thinks, mere nothingness. But if we assume two forms of objectivity, then "empty" space is also a genuinely "object-like" object of thought. And what can be thought (experienced) as genuinely objective, can also be (technically) treated. Consequently space is *manifest* exactly as is the world of bodies. For Kant this is still impossible, since, in the Critique of Pure Reason, space is a mere "form of intuition". That is a polite way of expressing the platonic belief that space, considered *objeetively,* is nothing.

In two-valued terms one cannot think otherwise at all: Either things are Something (physics), or the absence of things is Something (Metaphysics). All this is a pure *relation of exchange,* like "right" and "left".

But the interchangeability in principle of the parameters "space", "time", "matter" and "process", of which I speak in the Achilles, presupposes a three-valued logic and two conceptions of objective existence, which in this specific case are characterized as

philosophisches Problem. Als Konzession an den Leserkreis des Magazins habe ich es allerdings an dem speziellen Problem des "interstellar space-travel" durchgeführt.

In der Hoffnung, dass Sie angenehme Sommerferien gehabt haben bin ich mit warmen Grüssen
Ihr
Gotthard Günther

"quantized" and "not quantized".

The essay treats of a general problem of natural philosophy. As a concession to the readership of the magazine I have, to be sure, discussed it

in terms of the specific problem of "interstellar space-travel".

In the hope that you had a pleasant summer vacation I remain, with warm greetings,
Yours,
Gotthard Günther

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[a] Gödel 1944
[b] Günther 1954

Briefwechsel-Gödel-GG_06

6. Günther to Gödel

3407 Montrose Ave.
Richmond 22, Va.
19. Juni 1955

Sehr verehrter Herr Professor Gödel:

Es scheint mir ziemlich lange her seit wir miteinander korrespondiert haben. Inzwischen hat sich manches ereignet. Die Universität Hamburg hat mich eingeladen im kommenden Wintersemester als Gastprofessor dort in der Philosophischen Fakultät zu lesen. Im Oktober werde ich hinüber fahren. Als Hauptkolleg habe ich "Metaphysik der Geschichte" gewählt. Daneben werde ich ein Seminar für Anfänger über moderne geschichtsphilosophische Theorien halten, und ein Seminar für Fortgeschrittene über transzendentale Logik.

Das aber nur nebenbei. Der Zweck meines Briefes ist ein anderer. Ich habe seit unserer letzten Korrespondenz einen Gedankengang ausgearbeitet, den ich Ihnen gern zur Kritik unterbreiten möchte.

Sie stimmten mir in Ihrem letzten Briefe bei, dass die modernen Bemühungen der mathematischen Logik im Wesentlichen eine Reflexion auf *das Denken* selbst darstellen. Wenn wir darin übereinstimmen, so sollten wir uns auch über das Folgende verständigen können.

2 Die obige Einsicht zwingt uns |2 zwischen zwei inversen Typen von Reflexion zu unterscheiden: 1) die Reflexion auf den bona fide Gegenstand, der als etwas *a limine* vom Denken Unabhängiges gedacht wird, und 2) die Reflexion auf das Reflektieren in 1).

Es ist unvermeidlich, dass diesen beiden Reflexionstypen zwei grundsätzlich verschiedene Konzeptionen des Begriffes <logischer Gegenstand> entsprechen müssen. Diese Unterscheidung wird aber in der heutigen Logik noch nicht durchgeführt.

Ich will zeigen, was ich meine. Die beiden logischen Gegenstände müssen verschiedenen Identitätscharakter haben. In der Terminologie der älteren logischen Tradition:[1]

6. Günther to Gödel

3407 Montrose Ave.
Richmond 22, Va.
19 June 1955

Dear Professor Gödel:

It seems to me a rather long time since we last corresponded with one another. In the meantime many things have happened. The University of Hamburg has invited me to lecture there next winter semester as Visiting Professor in the Faculty of Philosophy. In October I will go over there.

As principal lecture course I chose "Metaphysics of History". In addition I will hold a seminar for beginners about modern theories in the philosophy of history and a seminar for advanced students on transcendental logic.

But this only as an aside. The purpose of my letter is something else. Since our last correspondence I worked out a line of thought which I would like to present to you for criticism.

You agreed in your last letter that modern efforts in mathematical logic are in essence a reflection on *thought* itself. If we agree on that, we should also be able to reach agreement on the following.

The above insight forces us to distinguish between two inverse types of reflection: 1) reflection on the *bona fide* object, which is thought as something *a limine* independent of thought, and 2) reflection on the reflecting in 1).

It is inevitable that two fundamentally different conceptions of the concept "logical object" must correspond to these two types of reflection. But this distinction is not made in contemporary logic.

I will show what I mean. The two logical objects must have a different character of identity. In the terminology of the older logical tradition[1]: the object

der Gegenstand der Reflexion 1) hat Seinsidentität; der Gegenstand der Reflexion 2) aber hat Reflexionsidentität. Der Unterschied findet sich, meines Wissens nach, zum ersten Mal bei Hegel formuliert. Im Anschluss daran haben Sigwart und Benno Erdmann dann analysiert, was Seinsidentität logisch eigentlich bedeutet. Ihr Resultat: Mit sich selbstidentisch sein heisst für den Gegenstand, dass derselbe durch den Denkakt, der sich mit ihm beschäftigt, *nicht verändert* wird. Seine (logischen) Eigenschaften bleiben dieselben, gleichgültig, ob er gedacht oder nicht gedacht wird.

3 Gerade das aber ist, so sage ich |3 nun, ein Charakteristikum, das die Reflexion, die man zum Gegenstand der Reflexion 2) macht, unmöglich haben kann. Die Reflexion ist ebenfalls mit sich selbst identisch. D.h. sie hat (oder: ist?) Reflexionsidentität.

Ich frage nun, was geht vor, wenn wir die Reflexion selbst denken?

Blicken wir noch einmal zurück auf die Situation, in der sich die Reflexion 1) befindet. Dort ist der Gegenstand "da", mit vor-denklich gegebenen Eigenschaften, die seine Identität ausmachen.

Grundsätzlich anders aber ist die Situation für die Reflexion 2). Hier ist kein unabhängig vom Denken gegebener Gegenstand *da,* an den das Reflektieren jetzt herantritt. Es ist ja – per definitionem – der *subjektive* Denkakt selbst (der Reflexions*prozess*), der jetzt gedacht werden soll. Derselbe muss also erst zum Gegenstand *gemacht* werden. D.h., er muss als logisches *Objekt* "gesetzt"[[2]] werden. Indem ich die Reflexion selbst denke, mache ich sie aus einem *subjekthaften* Prozess in einen *objekthaften* Sachverhalt. Resultat: Reflexionsidentität bedeutet, dass der logische Gegenstand, dadurch, dass er gedacht wird, sich in seinen logischen Eigenschaften ändert. Die denkende Reflexion ist nicht die gedachte Reflexion – und wenn die denkende Reflexion selbst gedacht wird, werden ihre ursprünglich "subjektiven" Eigenschaften in inverse
4 "objektive" |4 verwandelt. Denn qua Reflexionsprozess ist die Reflexion 1) undenkbar! Wird Reflexion 1) gedacht, so wandert ihr Prozesscharakter nach Reflexion 2) ab!!

of reflection 1) has identity of Being; the object of reflection 2), however, has identity of reflection. This distinction is to my knowledge first formulated by Hegel. Following this, Sigwart and Benno Erdmann then analyzed what, logically, identity of Being actually means. Their result: To be identical with itself means for the object that it is *not altered* by the act of thought occupied with it. Its (logical) properties remain the same, no matter whether it is thought or not.

But just that, I now claim, is a characteristic which reflection that is made into an object of reflection 2) cannot possibly have. Reflection is also identical with itself. I.e., it has (or: is?) identity of reflection.

I now ask, what happens when we think reflection itself? Let us look once again at the situation of reflection 1) Here the object is "there", with properties given prior to thought that make up its identity.

The situation with respect to reflection 2), however, is fundamentally different. Here no object given independently of thought is *there,* which reflection now approaches. After all, it is – by definition – the *subjective* act of thought itself (the *process* of reflection) that is now supposed to be thought. Thus, the same act of thought first has to be *made* into an object. That is, it must be "posited"[2] as logical *object.* In thinking reflection itself I turn it from a *subject-like* process into an *object-like* state of affairs. Result: identity of reflection means that in virtue of being thought the logical object alters itself in its logical properties. The thinking reflection is not the reflection that is thought-and when thinking reflection is itself thought, its originally "subjective" properties are turned into inverse "objective" ones. For as a *process of* reflection, reflection 1) is unthinkable! If reflection 1) is thought, its process-like character migrates to reflection 2)!!

Thus, we have two logically fundamentally different types of objects of

Wir haben also zwei logisch grundverschiedene Gegenstandstypen des Denkens überhaupt: Gegenstand 1), der dadurch, dass er gedacht wird, keine Veränderung erleidet und Gegenstand 2), bei *dem eine solche Veränderung einkalkuliert werden muss.*

***

[In the omitted portion Günther once again explains his ideas by truthtables. See the introductory note.]

10 |10 ... Ich habe versucht meinen Gedankengang hier in äusserster Kürze wieder zu geben. Im Text meines Manuskriptes nimmt er mehr als 100 Seiten ein. Ich wiederhole noch einmal: mein Grundgedanke ist, dass sich die Reflexion auf unser (seinsthematisches) Denken nicht mehr zweiwertig adäquat ausdrücken lässt, weil sie den Unterschied von Seinsidentität und Reflexionsidentität nicht mehr kalkülmässig bewältigen kann.

Für Ihre Reaktion darauf wäre ich äusserst dankbar.
Mit herzlichen Grüssen
Ihr
Ihnen sehr ergebenen
Gotthard Günther

thought as such: object 1), which does not suffer any changes by being thought, and object 2) *where such a change has to be reckoned with.*

***

[In the omitted portion Günther once again explains his ideas by truthtables. See the introductory note.]

... I have tried to present my line of thought here with utmost brevity. In the text of my manuscript it occupies more than 100 pages. I repeat once again: my basic idea is that reflection on our thought (that thematizes Being) can no longer be expressed adequately in two-valued terms, because in its calculus it cannot come to terms with the distinction between identity of Being and identity of reflection. I would be extremely grateful for your reaction.

With warm greetings,
Yours very sincerely,
Gotthard Günther

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[1] die in *diesem* Punkt auch heute noch nicht überholt ist.
which on this point is not outdated even today.

[2] "Gesetzt" ist ein transzendentallogischer Terminus der Fichteschen Wissenschaftslehre.
"Posited" is a transcendental-logical term from Fichte's Wissenschaftslehre.

Briefwechsel-Gödel-GG_07

7. Gödel to Günther

Princeton, 10./VIII. 1955

Sehr geehrter Herr Dr Günther:

Ich möchte mich zunächst entschuldigen, daß ich so lange nichts von mir hören ließ. Der Hauptgrund dafür war mein schlechter Gesundheitszustand im letzten Herbst u. Winter, der mich daran hinderte Ihren Brief vom 6. Okt.[a] zu beantworten. Ich habe mich sehr über Ihre Berufung als Gastprofessor nach Hamburg gefreut u. gratuliere Ihnen herzlich.

Nun zum Inhalt Ihrer beiden letzten Briefe. Ich bemerke zwischen ihnen zunächst den Unterschied, daß Sie im ersten von "zwei Arten von Denkunabhänggkeit" sprechen, im zweiten aber den Gegenständen der Refl. 2.) ausdrücklich die Denkunabhängigkeit absprechen. Das erste scheint mir im Bereich der Möglichkeit zu liegen, das zweite aber unbedingt unrichtig zu sein. Ich glaube auch nicht, daß die mathematische Logik (oder überhaupt die Logik) eine Reflexion auf das *Denken* ist, höchstens auch auf das Denken
2 |2 in erster Linie aber auf gewisse im Denken erfaßte allgemeinste Gegenstände. Die intuitionistische Logik allerdings ist eine Reflexion auf das Denken u. daher sind ihre Gegenstände nicht denkunabhängig, aber trotzdem werden sie "durch den Denkakt der sich mit ihnen beschäftigt" ebensowenig geändert wie die Gegenstände der Physik. Damit will ich natürlich durchaus nicht leugnen, daß die Gegenstände der Refl. 2) von denen der Refl. 1) toto genere verschieden sind, ja so sehr verschieden, daß man vielleicht mit Recht von zwei verschiedenen "Existenzformen" sprechen kann. Das ist es ja auch, wenn ich Sie recht verstehe, was Sie in Ihrem Achilles-Aufsatz (für dessen Zusendung ich bestens danke) zum Ausdruck bringen wollen. Dass eine mehrwertige Logik das adäquate Ausdrucksmittel ist, um diesen Unterschied darzustellen, liegt durchaus im Bereich der Möglichkeit. Der Gedanke hat zweifellos etwas Bestechendes an sich, insbesondere mit Rücksicht auf die Paradoxien u. auf die Tatsache, daß ja die
3 Prädikate |3 "notwendig" u. "möglich", die man zur Darstellung begrifflicher Verhält-

7. Gödel to Günther

Princeton, 10 August 1955

Dear Dr. Günther:

First I would like to apologize for your not having heard from me for so long. The main reason was the poor state of my health last fall and winter, which prevented me from answering your letter of 6 October.[a] I was very happy to hear about your appointment as visiting professor in Hamburg and extend my warm congratulations.

Now to the content of your last two letters. To begin with, I note this difference between the two of them. In the first you talk about "two forms of thought-independence", but in the second you explicitly deny thought-independence for the objects of reflection 2). The former seems to me to lie within the realm of possibility, the latter, however, to be absolutely incorrect. I also do not believe that mathematical logic (or logic in general) is a reflection on *thought*. At best [it is] a reflection also on thought, but first and foremost on certain most general objects grasped in thought. Intuitionistic logic is indeed a reflection on thought and accordingly its objects are not thought-independent, but in spite of that, they are no more altered "by the act of thought occupied with them" than the objects of physics. By this I of course do not at all want to deny that the objects of reflection 2) are *toto genere* different from those of reflection 1), indeed so very different that one can perhaps justifiably speak of two different "forms of existence". That, after all, is what you want to express in your Achilles essay, if I understand you correctly. (Thank you very much for sending it.) It lies indeed within the realm of possibility that a many-valued logic is the adequate means of expression for representing this difference. The thought without doubt has something intriguing about it, in particular with respect to the paradoxes and the fact that the predicates "necessary" and

nisse braucht, auch als Wahrheitswerte interpretiert werden können. Ein Beweis für die Richtigkeit dieser Auffassung könnte nur durch eine Axiomatisierung der Logik auf dieser Grundlage u. den Erfolg der aus den Axiomen entwickelten Folgerungen erbracht werden. Daß die Mehrwertigkeit der Logik etwas mit den von mir konstruierten unentscheidbaren Sätzen zu tun haben sollte, ist allerdings kaum möglich. Das könnte eventuell für "absolut" unentscheidbare Sätze der Fall sein. Aber die von mir angegebenen sind ja in einem übergeordneten (ebenfalls richtige Gedanken ausdrückenden) Formalismus immer entscheidbar u. beweisen daher nur, daß kein Formalismus «Siehe separates Blatt!» das ganze abstrakte Denken erfassen kann. An dieser Unzulänglichkeit jedes Formalismus kann auch eine mehrwertige Logik nichts ändern. – Die Art u. Weise, wie Sie den dritten Wahrheitswert einführen wollen, ist mir auf Grund Ihrer
4 Briefe u. des Kon-|4greßvortrages leider nicht ganz verständlich. Sie gehen von einem, & u. ∨-vertauschenden, dualen Verhältnis zwischen Ding u. Begriff aus. Nun ist dieser Dualitätsverhältnis [Vereinigung der Merkmale entspricht Durchschnitt der Umfänge] allerdings eine sehr interessante Tatsache, deren tiefere Bedeutung vielleicht noch der Aufklärung harrt, aber ich sehe keinen Zusammenhang zwischen diesem Charakteristikum der Begriffswelt u. ihrer verschiedenen Existenzform. Auch vermisse ich irgend eine Erklärung darüber, was eigentlich Ihre 3 Wahrheitswerte, im Gegensatz zu dem "wahr" u. "falsch" der klassischen Logik, bedeuten. Eine genaue Definition kann man natürlich nicht verlangen, am allerwenigsten im Rahmen der zweiwertigen Logik, aber doch eine Erklärung in demselben Sinn, in dem man auch die Grundbegriffe der zweiwertigen Logik (trotz ihrer Undefinierbarkeit) verständlich machen kann. Eine Analyse des Sinnes Ihrer Wahrheitswerte scheint mir der Kardinalpunkt zu sein, an dem Sie angreifen müssten, um Ihren Lesern verständlich (zu werden) u. den Aufbau einer Ihren Ideen entsprechenden Logik weiter durchzuführen.

Mit besten Grüssen
Ihr Kurt Gödel

5 |5 d.h. genauer: 1.) Kein Formalismus, von dem wir erkennen können, daß er richtiges

"possible", which are used to represent conceptual relations, can also be interpreted as truth-values. A proof of the correctness of this view could only be provided by an axiomatization of logic on this basis and by the success of the consequences derived from these axioms. But that the many-valuedness of logic should have something to do with the undecidable propositions I constructed is, to be sure, hardly possible. That could potentially be the case for "absolutely" undecidable propositions. But the ones given by me are, after all, always decidable in a higher formalism (likewise expressing correct thoughts), and therefore only prove that no formalism (see separate sheet!) can capture the whole of abstract thought. This inadequacy of every formalism cannot be changed by a many-valued logic either.-The manner in which you want to introduce the third truth-value is unfortunately not fully intelligible to me on the basis of your letters and the lecture to the Congress. You take as point of departure a dual relation between thing and concept that interchanges & and ∨. Now, this relation of duality [union of characteristics corresponds to intersection of the extensions] is indeed a very interesting fact, whose deeper meaning perhaps still awaits an explanation, but I do not see a connection between this characteristic of the world of concepts and its different form of existence. I also miss any explanation at all of what your three truth-values actually mean in contradistinction to the "true" and "false" of classical logic. One can of course not demand an exact definition, certainly not within the framework of classical two-valued logic, but still [one can demand] an explanation in the same sense in which one can make the fundamental concepts of two-valued logic perspicuous (in spite of their undefinability). An analysis of the sense of your truth-values seems to nie to be the cardinal point which you should tackle in order to become comprehensible to your readers and to carry out further the construction of a logic corresponding to your ideas.

With cordial greetings

(u. nur richtiges) Denken ausdrückt, kann *unser* ganzes abstraktes Denken erfassen. 2.) Kein Formalismus, in dem nur objektiv Richtiges ableitbar ist, kann, in seinen ableitbaren Formeln, alle objektiv bestehenden begrifflichen Verhältnisse erfassen.

Yours, Kurt Gödel

i.e., more precisely: 1.) No formalism of which we can know that it expresses correct (and only correct) thought, can capture our entire abstract thought. 2.) No formalism in which only objectively correct [propositions] can be derived, can, in its derivable formulae, capture all objectively obtaining conceptual relations.

8. Günther to Gödel

Gotthard Günther
3407 Montrose Ave.
Richmond 22, Va.
Sept. 18 55

Sehr geehrter Herr Prof. Gödel:

Nun sitze ich schon mitten in den Vorbereitungen für meine Reise nach Hamburg, aber ich möchte nicht von hier fortgehen ohne Ihnen für Ihren freundlichen Brief vom 10. VIII. zu danken - und Einiges darauf zu erwidern.

Zuerst: es hat mich *wirklich* bestürzt von Ihrem schlechten Gesundheitszustand zu hören. Ich betrachte es als ein wirklich grosses Unglück für die Wissenschaft, wenn Ihre kostbare Arbeitskraft auf diese Weise beeinträchtigt wird. Ich hoffe ernstlich, dass Ihr Brief an mich ein positives Zeichen war, dass es Ihnen definitiv besser geht.

Nun zu dem sachlichen Inhalt Ihres Schreibens. Es hat mich etwas entmutigt *auch von Ihnen* (wie von so vielen anderen) zu hören, dass Sie nicht einsehen können, warum ich behaupte dass beim Übergang von einem aristotelischen (2-wertigen) zu einem nicht-aristotelischen (3-wertigen) Logiksystem die Werte "wahr" und "falsch" aufgegeben werden müssen, und dass dafür ein anderer Werttypus – der eine andere ontologische Fragestellung involviert – eingeführt werden muss. Dass ich diese anderen Werte "irreflexiv" (contingent), "einfach reflexiv" (iterativ) und "doppelt-reflexiv" (Einheit von "irrefl." und "refl." im *Bewusstsein*) nenne, ist auf dieser Stufe der Betrachtung relativ irrelevant. Meine Terminologie hat *historische* Gründe in der Geschichte der Logik. Aber wenn jemand kommen sollte und mir eine bessere Werttriade demonstriert, so bin ich bereit, die meinige aufzugeben.

2 |2

Was aber relevant ist, ist, dass man das Dualitätsprinzip der logischen Wertigkeit aufgeben muss. Reichenbach hat ausdrücklich festgestellt, dass dasselbe auch in der Wahrscheinlichkeitslogik, die mit beliebig vielen "Mischwerten" arbeitet,

8. Günther to Gödel

Gotthard Günther
3407 Montrose Ave.
Richmond 22, Va.
18 September 1955

Dear Professor Gödel:

Now I'm already in the middle of the preparations for my trip to Hamburg, but I do not want to set out from here without thanking you for your kind letter of 10 August-and saying several things in response to it.

First: I was truly dismayed to hear about the poor state of your health. I consider it a real misfortune for science when your precious capacity for work is diminished in this-way. 1 sincerely hope that your letter to me was a positive sign that you are truly doing better now.

Now to the material content of your letter. It discouraged me somewhat to hear also from you (as from so many others) that you cannot see why I claim that in the transition from an Aristotelian (2-valued) to a non-Aristotelian (3-valued) system of logic, the values "true" and "false" have to be abandoned, and that a different type of value - which involves a different ontological question - has to be introduced. That I call these other values "irreflexive" (contingent), "simply reflexive" (iterative) and "double- reflexive" (unity of "irreflexive" and "reflexive" *in consciousness*) is, at this stage of the investigation, relatively irrelevant. My terminology has a *historical* basis in the history of logic. But if someone should come and demonstrate to me a better value-triad, I am ready to give up my own.

What is relevant, is that one has to give up the principle of the duality of logical values. Reichenbach has explicitly stated that the latter principle is not even given up in probability logic, which works with arbitrarily many

nicht aufgegeben wird. Die "Mischung" enthält immer die dualen Komponenten "wahr" und "falsch" – und ist stets dichotomisch aufteilbar.

3 Nach mehrfachem Lesen Ihres Briefes ist mir der Gedanke gekommen, dass meine Interpretation einer *echten Werttriade* (wahr-unbestimmt-falsch ist *keine* echte Triade!) als eines Wertsystems, in dem die *Reflemonshöhe* eines Begriffs (und *nicht* seine wahr-falsch-Übereinstimmung mit faktischen Daten) bestimmt wird, Ihnen deshalb Schwierigkeiten macht, weil Sie meine Interpretation der hermeneutischen Struktur eines dreiwertigen Logiksystems ignorieren.

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[In the omitted portion, including a P.S., Günther gives the explanation again with some technical details and gives an address in Hamburg at which he can be reached.)

"mixed values". The "mix" always contains the dual components "true" and "false"-and can always be split up as a dichotomy.

After reading your letter several times it occurred to me that my interpretation of a *genuine* value-triad (true-undetermined-false is *not* a genuine triad!) as a system of values in which the *level of reflection* of a concept (and *not* its true-false correspondence with factual data) is determined, creates difficulties for you, because you ignore my interpretation of the hermeneutic structure of a three-valued system of logic.

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[In the omitted portion, including a P.S., Günther gives the explanation again with some technical details and gives an address in Hamburg at which he can be reached.]

Briefwechsel-Gödel-GG_09

9. Günther to Gödel

Gotthard Günther
3407 Montrose Ave.
Richmond 22, Va.
Sept. 20, 1956

Hoch verehrter Herr Dr. Gödel,

[Günther thanks Gödel for a conversation the previous Monday, which he says will have a substantial influence on his work. He remarks on the procedure for the new Bollingen application.]

Zum Schluss gestatten Sie mir bitte noch eine zusätzliche Bemerkung zu unserem Gespräch. Ich erwähnte, dass ich mich nicht gern als Hegelianer klassifizieren lasse. Mein Vorbild wie philosophiert werden sollte, ist nicht Hegel sondern Leibniz! Leibniz hat etwas besessen, was in der Folgezeit völlig verloren gegangen ist: formale Exaktheit verbunden mit metaphysischer Tiefe. Er ist bisher der Einzige, der gewusst hat, dass der reine rechnende Formalismus eine Transparenz besitzt, die uns erlaubt, durch ihn hindurch in uns nicht unmittelbar gegebene Realitätsdimensionen zu blicken.

Ihr Ihnen dankbar verbundener

Gotthard Günther

9. Günther to Gödel

Gotthard Günther
3407 Montrose Ave.
Richmond 22, Va.
20 September 1956

Dear Professor Gödel,

[Günther thanks Gödel for a conversation the previous Monday, which he thinks will have a substantial influence on his work. He remarks on the procedure for the new Bollingen application.]

Finally, please allow me an additional remark about our conversation. I mentioned that I do not like to be classified as a Hegelian. My model of how to philosophize is not Hegel but Leibniz! Leibniz had a quality which got completely lost in ensuing times: formal exactness combined with metaphysical depth. Up to now he is the only one who knew that the pure calculating formalism has a transparency that allows us to look through it into dimensions of reality not immediately given to us.

With gratitude,
Sincerely
Gotthard Günther

10. Gödel to Günther

Princeton, 4./IV. 1957.

Sehr geehrter Herr Dr. Günther!

Besten Dank für Ihre beiden Briefe.[a] Ich habe bedauert, daß Ihre Operation nicht ganz glatt verlaufen ist. Ich hoffe, Sie fühlen sich jetzt wieder vollkommen wohl. Was die Separata Ihrer Arbeiten betrifft, so besitze ich nur "Achilles u. die Schildkröte" u. Ihren Vortrag auf dem Philosophenkongreß in [Brüssel][b].[1 /c] Ich bin natürlich weniger an populären Schriften interessiert als an Abhandlungen, die in philosophischen oder wissenschaftlichen Zeitschriften erschienen sind. Von Ihrem freundlichen Angebot, mir einige Ihrer Arbeiten leihweise zu überlassen, werde ich vielleicht später einmal Gebrauch machen. Gegenwärtig möchte ich bloß Separata von Ihnen haben, so weit sie noch verfügbar sind. Was Ihr neues Buch betrifft, so werde ich mich freuen, wenn ich im Herbst die Korrekturbogen des ersten Bandes zu sehen bekomme.

2 |2 Nun zum Inhalt Ihrer Briefe. Was Sie über Leibniz sagen ist interessant, aber ob es sich aus seinen Schriften rechtfertigen läßt, scheint mir sehr zweifelhaft. Sie sprechen von einem "subjektiven Raum" – wenn ich Sie recht verstehe im Sinne von Freiheitsgraden des Denkens – aber ich wüßte nicht wo Leibniz davon spricht.[2] Daß das abstrakt begriffliche Denken nur durch die Zentralmonade in die Einzelmonaden kommt, ist zwar ein recht Leibnizscher Gedanke, aber ob Leibniz das abstrakte Denken als eine 2te Reflexionsstufe im Sinn von Hegel interpretieren wollte, ist eine andere Frage. Es wäre interessant, wenn man Stellen aus seinen Schriften anführen könnte, die darauf hindeuten. Schließlich imputieren Sie Leibniz die Lehre von der Idealität der

10. Gödel to Günther

Princeton, 4 April 1957

Dear Dr. Günther,

Many thanks for your two letters.[a] I am sorry that your operation did not go entirely smoothly. I hope you now feel completely well again. As regards the offprints of your works, I only have "Achilles and the Tortoise" and your lecture at the philosophers' congress in Brussels[b].[1 /c] I am of course less interested in popular writings than in papers that appeared in philosophical or scientific journals. I will perhaps make use later of your kind offer to lend me some of your works. For now, I would like only to have offprints from you, to the extent they are still available. As regards your new book, I will be delighted to receive the proofs of the first volume in the fall.

Now to the content of your letters. What you say about Leibniz is interesting, but whether it can be justified on the grounds of his writings seems very doubtful to me. You speak of a "subjective space" – in the sense of degrees of freedom of thought, if I understand you correctly – but I would not know where Leibniz talks about this[2]. That abstract conceptual thought enters individual monads only through the central monad is a truly Leibnizian thought, but whether Leibniz meant to interpret abstract thought as a 2nd level of reflection in Hegel's sense is another question. It would be interesting if one could cite passages from his works that point to it. Finally, you impute to Leibniz the theory of the ideality of time in the form: A process is the situation of re-

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1 Außerdem besitze ich den Bericht über den "Congrès Descartes" (1937) mit einem Vortrag von Ihnen im 8. Bd.[c]
Moreover I have the report of the "Congrès Descartes" (1937) with a lecture of yours in volume 8.[c]

2 Eine "res extensa" gibt es wohl bei Leibniz überhaupt nicht, aber das ist in diesem Zusammenhang ohne Bedeutung.
There probably is no "res extensa" in Leibniz at all, but that is unimportant in this context.

3 was also gewissermaßen alle möglichen Typen zusammenfaßt
that, as it were, unifies all possible types

Zeit in der Form: Ein Vorgang ist die Reflexionssituation eines Gegenstandes. Diese Formulierung ist sehr interessant u. sollte näher ausgeführt werden, aber es würde mich sehr wundern, wenn man darüber eine Andeutung bei Leibniz finden könnte. Die
3 Behauptung, daß Realität |3 ein *Verhältnis* ist (nämlich zwischen Subjekt u. Objekt) ist mir als eine der idealistischen Grundpositionen verständlich, aber daß sie ein Umtauschverhältnis ist, bedarf näherer Erklärung. Eine solche wäre insbesondere im Zusammenhang mit Ihrer Theorie der Negation vonnöten. Was die Beziehungen betrifft, die Sie zwischen den Reflexionsstufen u. gewissen physikalischen Theorien herstellen wollen, so bin ich auf Grund Ihrer Andeutungen leider nicht imstande, ihnen einen verständlichen Sinn zu geben. Insbesondere sieht man zunächst gar nicht, was die Art der Materie, aus denen die Gehirne bestehen, mit der Möglichkeit einer Verständigung (im geistigen Sinn) zu tun haben kann.

Was Ihr Manuskript über die nicht-Aristotelische Logik betrifft,[d] so habe ich Ihnen ja meine Ansicht in unserem Gespräch in Princeton auseinandergesetzt. Ich glaube, daß der *Sinn* Ihrer Grundbegriffe durch *Beispiele* u., soweit möglich, durch präzise Erklärungen |4 erläutert werden sollte u. daß die Arbeit in ihrer gegenwärtigen Form kaum verständlich sein wird. Ich möchte übrigens bemerken, daß ich vor einigen Jahren ein Manuskript von Ihnen über die Grundlagen der Logik gelesen habe,[e] das einen sehr interessanten Gedanken enthielt, den ich in Ihren neueren Arbeiten über den Gegenstand vermisse. Sie haben nämlich damals die Totalreflexion als etwas über alle Typenbildung hinausgehendes[3] interpretiert. Es ist plausibel, daß die Durchführung *dieser* Idee zu einer nicht Aristotelischen Logik führen muß, da man ja auf diese Weise sofort in die Antinomien der Mengenlehre hineinkommt. Jetzt scheinen Sie eher der Ansicht zuzuneigen, die doppelte Reflexion mit dem zweiten logischen Typus zu identifizieren u. die Aristotelische Logik bereits für den erweiterten Funktionenkalkül aufzugeben, was ich für unberechtigt halte.

Mit besten Grüssen u. Wünschen für eine gute Gesundheit

Ihr Kurt Gödel

flection of an object. This formulation is very interesting and should be elaborated further, but I would be very surprised, if one could find a suggestion of it in Leibniz. The claim that reality is a *relation* (namely between subject and object) is comprehensible to me as one of the fundamental positions of idealism, but that reality is a relation of *exchange* requires further explanation. Such explanation would be particularly necessary concerning your theory of negation. As regards the relations you want to establish between levels of reflection and certain physical theories, I am unfortunately unable to make sense out of them on the basis of your suggestions. In particular, initially one cannot see at all what the kind of matter that makes up brains can have to do with the possibility of communication (in the mental sense).

As regards your manuscript on non-Aristotelian logic,[d] I already explained my views to you in our conversation at Princeton. I believe that the *sense* of your basic concepts should be elucidated through *examples* and, so far as possible, through precise explanations, and that the work in its current form, will hardly be comprehensible. By the way, I would like to remark that a few years ago I read a manuscript of yours about the foundations of logic[e] that contained a very interesting thought I now miss in your more recent works on the topic. For, back then, you interpreted total reflection as something that goes beyond all formation of types.[3] It is plausible that the implementation of this idea must lead to a non-Aristotelian logic, since in this way one immediately runs up against the antinomies of set theory. Now you seem rather to lean towards the view of identifying double reflection with the second logical type and giving up Aristotelian logic for the extended functional calculus, which I regard as unjustified.

With cordial greetings and best wishes for your health,

Yours, Kurt Gödel

[a] Meant are surely Günther's letters of 26 February and an undated letter in all probability written in mid-March, together with a postscript dated "Montag".

[b] Günther 1954 and 1953.

[c] Günther 1937.

[d] Evidently a draft of Günther 1958.

[e] Probably this was the manuscript "Der metaphysische Hintergrund der Logik und die absolute Rationalität," which Günther sent with his letter of 2 August 1953.

Briefwechsel-Gödel-GG_11

11. Günther to Gödel

3407 Montrose Ave.
Richmond 22, Va.
April, 7, 1957

Hochverehrter Herr Prof. Gödel,

[Günther makes remarks about his writings and comparative lack of publication and about three items he is about to send to Gödel]

2 |2 ...

Nun zu dem sachlichen Inhalt Ihres letzten Briefes und warum er mir eine solche besondere Freude gewesen ist! Sie schreiben, dass Sie früher eine Arbeit von mir gelesen haben, in der ich "die Totalreflexion als etwas über alle Typenbildung hinausgehendes interpretiert" habe. Und Sie fahren dann fort: "Es ist plausibel, dass die Durchführung, *dieser* Idee zu einer nicht-Aristotelischen Logik führen muss, da man ja auf diese Weise sofort in die Antinomien der Mengenlehre hineinkommt. Jetzt scheinen sie der Ansicht zuzuneigen, die doppelte Reflexion mit dem zweiten logischen Typus zu identifizieren

Sie wissen gar nicht, wie glücklich mich diese Bestätigung gemacht hat. *Ich bin immer noch der Ansicht, dass die Totalreflexion eine solche ist, die alle überhaupt möglichen Typen zusammenfasst.* Darauf allein beruht ihr Totalitätsanspruch. Und erst wenn man die Technik einer solchen Zuammenfassung besitzt, hat man wirklich eine fertige Nicht-Aristotelik!!! Aber ich bin in den letzten Jahren schüchtern geworden, diesen Gedanken auszusprechen. Ich habe darüber vor vielen Jahren eine Anzahl Gespräche mit Quine gehabt. Er hält diese Idee für falsch und hat sie mir damals fürchterlich verrissen. Er hat mich *nicht* überzeugt, dass sie falsch ist, wohl aber hat (er) mich damals ganz zwingend belehrt, das meine bisherige Weise das Problem anzufassen, völlig unzureichend war und nie hätte zum Ziele führen können. Inzwischen habe ich einiges gelernt, aber immer noch nicht genug, als dass ich es wagen würde die These so in den Vordergrund zu stellen, wie ich glaube, dass sie es verdient. Ich gehe jetzt

11. Günther to Gödel

3407 Montrose Ave.
Richmond 22, Va.
April 7, 1957

Dear Professor Gödel:

[Günther makes remarks about his writings and comparative lack of publication and about three items he is about to send to Gödel.]

...

Now to the material content of your last letter, and why it was such a special pleasure for me! You write that previously you had read a work of mine, in which I "interpreted total reflection as something that goes beyond all formation of types". And you continue: "It is a plausible view that the implementation of *this* idea must lead to a non-Aristotelian logic, since in this way one immediately runs up against the antinomies of set theory. Now you seem rather to lean towards the view of identifying double reflection with the second logical type...."

You don't know how happy this confirmation made me. *I am still of the opinion that total reflection is one which unites all the types that are at all possible.* That alone grounds its claim to totality. And only when one possesses the technique for such unification does one truly have a complete non-Aristotelianism!!! But in recent years I have become timid about articulating this thought. Many years ago I had a number of conversations with Quine about this. He believes this idea is false and back then tore it to pieces in a terrible way. He did *not* convince me that it is false, but he taught me very compellingly that my previous way of conceiving the problem was completely inadequate and could never have achieved its end. In the meantime 1 have learned a thing or two, but still not enough to dare to put the thesis as much to the fore as I believe it deserves. I now proceed cautiously and formulate my (provisional) position

vorsichtig vorwärts und formuliere meinen (vorläufigen) Standpunkt etwa so: Reflexion-in-Anderes (Theorie der Gegenständlichkeit) ist klassisch zweiwertig. Die Reflexion-in-sich (gleichgültig welcher Art) beginnt mit den mehrwertigen Stellenwertsystemen. Der unterste Fall ist die dreiwertige Logik. Die Totalreflexion aber ist nicht dreiwertig sondern unbestimmt n-wertig, wobei immer gilt n > 2. Der limes ist eine unendlich wertige Logik.

[Günther goes on to claim that three-valued logic is a "pure system of consciousness" and says that with four or more values the object, dissolved by idealism, can be restored. He goes on to comment on the antinomies and to defend his interpretation of Leibniz as a reconstruction.]

roughly like this: Reflection-in-other (theory of objecthood) is classically two-valued. Reflection-in-itself (no matter what kind) begins with many-valued place-value systems. The lowest case is three-valued logic. Yet, total reflection is not three-valued, but indeterminately n-valued, where n > 2. The limit is an infinite-valued logic...

[Günther goes on to claim that three-valued logic is a "pure system of consciousness" and says that with four or more values the object, dissolved by idealism, can be restored. He goes on to comment on the antinomies and to defend his interpretation of Leibniz as a reconstruction.]

12. Günther to Gödel

3407 Montrose Ave.
Richmond 22, Va.
November 22, 1957

Hochverehrter Herr Prof. Gödel,

[Günther says he is enclosing two publications, Günther 1957 and 1957a. Brief remarks about the former and about cybernetics.]

2 ...

Aus diesem Grunde kann auch – wie ich in meiner zweiten Beilage dem "Bewusstsein der Maschinen",[a] ausgeführt habe – die Kybernetik nicht mehr zureichend mit den zweiwertigen Mitteln der Aristotelischen Logik interpretiert werden. Denn wenn der Mensch heute allmählich beginnt seine Bewusstseinsfunktionen in einem *electronic brain* abzubilden, dann ist als Grundkategorie wieder das Phänomen der Reflexion involviert durch die jetzt entstehende Frage: wie unterscheidet sich eine "Bewusstseinsanalogie" im Mechanismus von dem Selbstbewusstsein, dass sie produziert. In diesem Sinn ist die Kybernetik die erste konsequente nicht-aristotelische Technik und dementsprechend nur mehrwertig analysierbar.

[Says that the revised first volume of his book (i.e. Günther 1959) is being reviewed by the Deutsche Forschungs-Gemeinschaft]

12. Günther to Gödel

3407 Montrose Ave.
Richmond 22, Va.
November 22, 1957

Dear Professor Gödel:

[Günther says he is enclosing two publications, Günther 1957 and 1957a. Brief remarks about the former and about cybernetics.]

...

For this reason too – as I explained in my second supplement to the "Bewusstsein der Maschinen"[a] – cybernetics cannot any longer be interpreted adequately with the two-valued means of Aristotelian logic. For, if today man begins gradually to map the functions of his consciousness in an electronic brain, then the phenomenon of reflection is again involved as a fundamental category through the question that now arises: How is the "analogy of consciousness" in a mechanism distinguished from the self-consciousness that produces it? In this sense cybernetics is the first consistent non-Aristotelian technique and accordingly only analyzable in many-valued [logic]. –

[Says that the revised first volume of his book (i.e. Günther 1959) is being reviewed by the Deutsche Forschungs-Gemeinschaft.]

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[a] Günther 1957a.

13. Gödel to Günther

Princeton, 23./XII. 1957.

Sehr geehrter Herr Dr. Günther!

Besten Dank für die Zusendung des Sonderdrucks u. des Buches, sowie auch Ihre freundlichen Neujahrswünsche, die ich auf das herzlichste erwidere. Ihre Arbeit: "Metaphysik, Logik u. Theorie d. Reflexion"[a] hatte ich bisher nicht gesehen, außer insoweit als sie mit einem alten Manuskript von Ihnen über die Metaphysik der klassischen Logik übereinstimmt.[b] Ich habe einiges aus Ihrer neuen Arbeit gelesen u. finde, daß sie sich durch besondere Klarheit auszeichnet, insbesondere auch in Vergleich mit dem Manuskript über die arist. Logik des Seins u. die nicht arist. Logik der Reflexion,[c] das ich gelesen habe. Beiliegend sende ich Ihnen mit bestem Dank das Manuskript über die Metaphysik des Todes zurück.[d] Ich habe es mit Interesse gelesen. Über die Arbeit "Logistik u. Transzendentallogik"[e]wollte ich Ihnen
2 |2 einige kritische Bemerkungen schreiben, bevor ich das Separatum retourniere. Ich wurde aber leider im letzten Sommer durch einige unaufschiebbare Angelegenheiten u. durch Krankheit daran verhindert. Doch hoffe ich dies, sowie auch dasselbe bez. Ihrer neuesten Arbeit, demnächst noch nachholen zu können. Heute kann ich Ihnen leider nicht ausführlicher schreiben.

Ich hoffe, daß es Ihnen gesundheitlich und auch sonst gut geht, u. verbleibe mit besten Grüßen

Ihr Kurt Gödel

13. Gödel to Günther

Princeton, 23 December 1957

Dear Dr. Günther!

Many thanks for sending the offprint and the book, as well as your kind wishes for the new year, which I warmly return. Your paper: Metaphysik, Logik, und Theorie der Reflexion[a] I had not seen before, except insofar as it agrees with an old manuscript of yours about the metaphysics of classical logic.[b] I read parts of your new work and think that it is distinguished by special clarity, in particular compared, to the manuscript about the Aristotelian logic of Being and the non-Aristotelian logic of reflection,[c] which I have read. I am returning enclosed your manuscript about the metaphysics of death.[d] I have read it with interest. I wanted to write you several critical remarks on the paper "Logistik und Transzendentallogik",[e] before I return the offprint. But unfortunately last summer I was prevented from doing so by several urgent matters and by illness. Yet I hope soon to be able to make up for this after all, as well as for the same concerning your latest work. Today, unfortunately, I cannot write to you at greater length.

I hope that things are going well for you with respect to health and otherwise, and I remain with cordial greetings,

Yours, Kurt Gödel

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[a] Günther 1957. This is no doubt the reprint referred to in the previous sentence; the book would be Günther 1957a.

[b] Probably again "Der metaphysische Hintergrund der Logik und die absolute Rationalität"; see note d to letter 10 above.

[c] Günther 1958.

[d] Surely a version of Günther 1957b. Günther 1940.

[e] Günther 1940.

13. Gödel to Günther

Princeton, Jan. 7, 1959.

Sehr geehrter Herr Dr. Gunther:

Besten Dank für Ihren Brief sowie den Sonderdruck. Ich habe mich sehr gefreut von den Fortschritten zu hoeren, die Ihre Arbeit gemacht hat, insbesondere darueber dass der erste Band des neuen Buches bereits im Druck ist. Die Frage der geschichtsphilosophischen Implikationen der Kybernetik scheint mir sehr interessant und aussichtsreich zu sein und ich begruesse Ihren Entschluss, sich damit zu beschaeftigen.

Was den logischen Teil Ihrer Arbeit betrifft, so scheint mir der interessanteste und aussichtsreichste Gesicht(s)punkt der zu sein, den Sie in einer Ihrer früheren Arbeiten einnahmen.[a] Sie identifizierten damals die iterierte Reflexion mit der Typentheorie und die totale Reflexion mit einer typenlosen (d. h. alle Typen in eins zusammenfassenden) Logik. Man sollte glauben, dass aus philosophischen Einsichten über das Wesen der Reflexion sich *mit Notwendigkeit* die richtigen Axiome einer typenlosen Logik ergeben muessten, was ein ungeheurer Fortschritt gegenueber dem heute angewendeten Verfahren des "trial and error" waere.

Es tut mir sehr leid, dass Sie Schwierigkeiten mit Ihren Augen haben, und ich wuensche Ihnen vom Herzen gute Besserung. Sehr gefreut hat es mich, zu hoeren, (dass die Bollingen Foundation) Ihnen so wohlgesinnt ist.

Ich wuensche Ihnen das beste für das kommende Jahr und verbleibe mit besten Grüssen.

Ihr
Kurt Gödel

P.S. Bitte entschuldigen Sie, dass ich so lange nichts von mir hoeren liess.

2 |2 P.S. Soeben fand ich Ihren Brief vom 1. Jan. im Institut vor. Ich danke bestens für die freundlichen Neujahrswünsche. Mein Gesundheitszustand ist nicht schlechter als

13. Gödel to Günther

Princeton, 7 January 1959

Dear Dr. Günther,

Thank you for your letter as well as the offprint. I was very pleased to hear of the progress that your work has made, in particular that the first volume of the new book is already in press. The question of the implications of cybernetics for the philosophy of history seems to me very interesting and promising, and I welcome your decision to occupy yourself
with it.

As for the logical part of your work, it seems to me that the most interesting and promising point of view is the one that you took in your earlier work.[a] At that time you identified iterated reflection with type theory and total reflection with a type-free logic, that is, one comprehending all types into one. One should think that from philosophical insights about the nature of reflection the correct axioms of a type-free logic would have to result *with necessity,* which would be an enormous advance compared to the procedure of "trial and error" applied today.

I am very sorry that you have troubles with your eyes, and I sincerely wish you a good recovery. It has pleased me very much to hear that the Bollingen Foundation is so well disposed towards you.

I wish you the best for the coming year and remain with cordial greetings,

Yours, Kurt Gödel

P.S. Please excuse the fact that you did not hear from me for so long.
P.S. I just found your letter of 1 January in the Institute. Thank you very much for the kind New Year's greetings. My state of health is no worse than otherwise. But I had, apart from being occupied with the questions that interested me, all sorts of time-consuming duties

sonst. Aber ich hatte, abgesehen von der Beschäftigung mit den mich interessierenden Fragen, allerhand zeitraubende Verpflichtungen u. sonstige Ablenkungen, was dann in Anbetracht meiner verminderten Arbeitskraft etwas zu viel wird. Der geschichtliche Teil Ihres Buches würde mich sehr interessieren, aber aus den angegebenen Gründen würde es mir kaum möglich sein, ihn im Laufe der nächsten Monate zu lesen.

Nochmals beste Grüsse

Kurt Gödel

and other distractions, which becomes somewhat too much considering my diminished capacity for work. The historical part of your book would interest me very much, but for the reasons [I have] given it would scarcely be possible for me to read it in the course of the next few months.

Once again cordial greetings

Kurt Gödel

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[a] A Gödel appears to have in mind the manuscript about the metaphysics of classical xxxx